DRSTP - Strong Theory and the Swampland

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Contents

1. Introduction
2. Swampland intuition: weak-coupling limits
3. The Weak Gravity Conjecture
4. The Distance Conjecture
5. Relations and underlying Physics
6. Swampland away from flat space
7. Holography and the Swampland
DRSTP - String Theory and the Scampland

See reviews 1903.06289, 2102.01111

References in text.

1) Introduction

Physics is built using effective theories, valid up to a certain energy scale.

e.g. Theory of Blackbody Radiation

Power emitted as a function of wavelength

$$B(\lambda)_{\text{classical}} = \frac{2kT}{\lambda^2}$$

Valid for $$\lambda \gg \lambda_{\text{cutoff}} \sim \frac{hc}{kT}$$.

$$B(\lambda)_{\text{quantum}} = \frac{2hc}{\lambda^3} \left( e^{\frac{hc}{\lambda kT}} - 1 \right)^{-1}$$

Cutoff scale: $$\lambda_{\text{cutoff}} \sim \frac{1}{M_p}$$

Natural units: $$\hbar = c = 1$$

- Sometimes, $$M_p = 1$$.
Modern Wilsonian Effective Theories are built using certain expectations

- Can guess estimate necessary cutoff from the EFT

  For example, gravity becomes strongly-coupled, so highly quantum in nature, at

  \[ E \sim M_p \sim 10^{19} \text{ GeV} \sim 10^{-25} \text{ m} \]

  So \( M_p \) serves as a cutoff for any effective theory (that is not a full-fledged theory of QG).

- Write Lagrangian operators consistent with symmetries of the theory, and suppress irrelevant operators by the cutoff scale:

  \[ \mathcal{L}(\phi) = \frac{1}{2} (\partial \phi)^2 + \frac{\lambda^2 \phi^2}{\lambda^2} + \lambda \phi^4 + \frac{1}{\lambda^2} \phi^6 \]

  These are expectations, there are also consistency checks

  e.g. Anomaly cancellation:
The Swampland Program aims to supplement the consistency rules for any effective theory that is coupled to gravity.

These new rules also often go against the Wilsonian expectations! That is when they are the most interesting.

- How could we have missed such general consistency constraints until now?

Inconsistency does not manifest in the EFT, it is a requirement for an ultraviolet completion to Quantum Gravity.

Definition:

- EFTs that potentially have an ultraviolet completion to Quantum Gravity are in the Landscape.

- EFTs that have no possible ultraviolet completion to Quantum Gravity are in the Swampland.
Methodology of the Swampland Program

The key idea behind the Swampland are two expectations of physics at sufficiently high energies:

1) Unification

2) Uniqueness

We expect that at sufficiently high energies the laws of physics are uniquely fixed.

String theory manifests such features: at high energies there are only 5 string theories, which have no constant parameters.

We now think of these as the possible weakly coupled theories of quantum gravity.

At even higher energies we expect a single unique theory, but which is strongly coupled, M-theory.

So, all effective theories coupled to gravity have a single, unique ultraviolet completion!

(Not true without gravity.)
Different effective theories correspond to different solutions / configurations of this single theory.

E.g., Newtonian friction coefficients vary between materials. But all Newtonian friction comes from electromagnetic interactions of electrons, described by a single theory: QED.

We have this picture:

Swampland constraints, or consistency rules, correspond to universal properties of solutions to the unique theory of quantum gravity.

How do we extract such properties?
Scientific Method - Look for patterns in the data
Set & ST Solutions

Test → Set Theory → Solutions

EFTs = Data → Patterns

Hypothesis/Conjecture → Observational Consequences

This is supplemented by input from:

- Black Hole physics
- Talograph (Lecture 4) ? underdeveloped!
- Amplitudes
2) Swampland intuitions - weak-coupling limits

Let us build some intuition for Swampland constraints by looking at general aspects of String Theory.

Ten-dimensional type II String Theory has only two parameters:

- String tension, \( [M_s] = 1 \)
- String coupling, \( [g_s] = 0 \)

It has a massless graviton mode, so the low-energy effective action includes gravity:

\[
S_{100} = \int d^10 x \left( \frac{M_s^8}{g_s^2} \right) R + \ldots
\]

The Planck mass in 10 dimensions is always derived as

\[
S = \int d^10 x \frac{1}{2} \left( \frac{1}{M_p^{(d)}} \right)^2 R + \ldots
\]

So we see, \( M_s \sim g_s^{1/4} M_p \)

The spectrum of the string contains an infinite tower of oscillators,

\( M_n \sim \sqrt{n} M_s, \ n \in \mathbb{N} \)
In the weak-coupling limit, an infinite tower of states becomes parametrically lighter than the Planck scale.

$$\text{Hosscillators} \sim \frac{M}{s} \sim 9^{1/4} \text{Mp}$$

**Surprising:** Weakly coupled quantum gravity has an infinite tower of states far below $\text{Mp}$.

Can see similar behavior in classical gravity: dimensional reduction on a circle as pure gravity.

$$dS_5^2 = 2m \, dx^2 dx^6 + R^2 (dy + A dx^5)^2, \quad D = d+1$$

$A$ = graviphobic, $R$ = radius of circle.

$$g_{\text{mix}} = \frac{1}{2\pi R} \left( \frac{1}{2\pi R} \right)^{d-2/2} \quad \text{(in Planck units $M_p^{(2)} = 1$)}$$

There is an infinite tower of Kaluza-Klein states.

$$M_n = \left( \frac{1}{R} \right) \left( \frac{1}{2\pi R} \right)^{d-2/2}, \quad q_n = 2\pi n$$

So can unite:

$$M_n = q_n^2 M_p$$

Weak coupling $g_{\text{mix}} \ll 1$ brings down an infinite tower of states.
The lightest tower of states provides the degrees of freedom for a description of the theory:

- $g_s \to 0$: A strong theory in d dimensions
- $g_{mn} \to 0$: A higher dimensional field theory in d+1 dimensions

Emergent strong coupling: These are the only possible infinite light towers on $g \to 0$ limits. (1904.06344)

Example test: Type IIA string theory in 10D has a graviphoton $A_m$.

$$g_s \sim \frac{1}{g^{3/4} s}$$

Weak gauge coupling $\leftrightarrow$ strong string coupling

Have non-perturbative charged 10D branes.

$$M^2 \sim \frac{1}{g^{3/4}} \sim g_s$$

At strong string coupling they form bound states

$$M_n^2 \sim \frac{n}{g^{3/4}} \sim N g_s M_p$$

Same as KK tower with $g_s \leftrightarrow g_{mn}$. 
Strongly-coupled IIA $\sim$ 11-dimensional supergravity

3. The Weak Gravity Conjecture (4-dimensions)

Consider a theory with gravity and U(1) gauge field

$$ S = \int d^4x \sqrt{|g|} \left( \frac{1}{2} M_p^2 R - \frac{1}{4 g^2} F^2 + \cdots \right) $$

- Electric WGC: There must exist a charged state

  $$ M \leq \sqrt{2} \sqrt{g} M_p $$

- Magnetic WGC: There exists an infinite tower of charged states with

  $$ M_\infty \sim g M_p $$

**Electric-Magnetic Duality**

In general: Weak Electric $\sim$ Strong Magnetic

EM duality:  $M_{\text{monopole}} \sim \frac{1}{g} M_p$

Monopole mass is at least the energy stored in its magnetic field:

$$ M_{\text{monopole}} \sim \frac{\Lambda_{\text{GUT}}}{g^2} \Rightarrow \Lambda_{\text{GUT}} \sim g M_p $$
Relation to Black Holes Stability

Consider an object with mass $M$ and charge $Q$, which decays to a number of objects with mass $M_i$ and charge $Q_i$.

\[
M \geq \sum M_i, \quad Q = \sum Q_i
\]

\[
\frac{M}{Q} \geq \frac{1}{Q} \sum \left( \frac{M_i}{Q_i} \right) Q_i \geq \frac{1}{Q} \left( \frac{M}{Q} \right)^{\min} \leq \frac{Q}{Q} = \left( \frac{M}{Q} \right)^{\min}
\]

So a decay can only occur to objects which have at least one with larger charge-to-mass ratio.

Extremal Black Holes have $M = \sqrt{2} Q M_P$, so WBC says the statement about BHs can decay.

Similarly, the forces acting on a particle with itself

\[
\frac{m}{m^2} \rightarrow \frac{g q}{g q}
\]

WBC says there should be at least one particle on which gravity acts as the weakest force (so that it is self-repulsive).

WBC violation $\Rightarrow$ infinite number of stable states.
May be problematic - but no proof.

4) **Distance conjecture**

In string theory, there are no dimensionless time constants, only expectation values of scalar fields.

\[ g_s = e^{-\phi}, \quad \phi = \text{dilaton} \]

\[ g_s \sim e^{-3/4 \phi} \]

\[ M_{s,c} \sim \sqrt{\phi} e^{-\phi} \]

\[ M_{D,0} \sim e^{3\phi/4} \]

**Distance conjecture:**

For any \( \phi \), canonically normalized, for \( |\phi| > M_{D,0} \),

\[ M \sim e^{-\frac{1}{4} \phi (\phi)} M_{D,0} \sim 0(1) \]

More precisely: Consider kinetic term

\[ L \propto P_{ij} \partial \phi \partial \phi \]

Proper distance on field space between two points

\[ \Delta(\phi_1, \phi_2) = \int \sqrt{P_{ij} \partial \phi \partial \phi} \; \text{d}\tau \]

\( \tau \) geodesic with parameter \( \tau \).
Then,

$$M_{\alpha}(Q) - M_{\alpha}(P) \sim \lambda \Delta(Q,P)$$

as $\lambda(P,Q) \to \infty$.

Distance conjecture in Calabi-Yau moduli spaces

Consider type IIB string theory compactified on $M_4 \times CY_3$

Gives an $N=2$ Supergravity in four dimensions

The geometry of 3-cycles on the CY is controlled by the complex-structure moduli, $z^i$:

$z^i$ are complex scalars with kinetic term:

$$L = \phi^i \partial \phi^i$$

The geometry of the (moduli) field space is determined by the period vector:

$$\Pi_I(z) \quad I = 0, 1, \ldots, 6$$
\[ P_{ij} = \frac{\partial z_i}{\partial \overline{z}_j} \cdot k \quad , \quad k = -\log \left[ \int \pi T \cdot \pi \right] \]

\( \eta \) is the symplectic intersection matrix of 3-cycles

\[ \eta_{ij} = \xi_i \cdot \xi_j \]

Consider a locus at infinite proper distance in the moduli space, denoted by

\[ z \to 0 \]

**Theorem:** Such a locus is a singularity on the moduli space with a monodromy about it.

\[ \pi(z e^{2\pi i}) = T \cdot \pi(z) \]

\( T \) is the monodromy matrix.

Define, \( N = \log T \)

Then, \( N \) is nilpotent:

\[ N^{n+1} = 0 \quad , \quad n \leq 3 \]

**Theorem:** \( \pi(z) = \exp \left( N \log z \right) (a_0 + a_1 z + \cdots) \)

**Theorem:** Infinite distance \( \iff n^{d+1} a_0 \neq 0 \quad , \quad d > 0 \).
To calculate the proper distance, change variables:

\[ t = \frac{\log z}{2\pi i}, \quad t \to +i\infty \]

\[ \Pi(t) = \exp[Nt] \left( a_0 + o(e^{2\pi t}) \right) \]

\[ \sim t^d(N^d a_0) \]

\[ k = -\log \left[ \prod_i \Pi(t_i) \cdot \eta \cdot \Pi(t) \right] \]

\[ \sim -\log (t-t_i)^d \]

\[ P_{tt} \sim \frac{d}{4(N_{\text{int}})^2} \]

\[ \Delta \phi \sim \int_{t_i}^{t_f} \frac{d(N_{\text{int}})}{2} \log \left( \frac{t_f}{t_i} \right) \]

The $t_i$ control the sizes of 3-cycles, and there can be wrapped by D3 branes. The mass of such stacks is:

\[ M(t, q) = |Z(t, q)| = \left| \frac{4 \cdot \eta \cdot \Pi(t)}{\prod_i \Pi(t_i) \cdot \eta \cdot \Pi(t)} \right| \]

\[ = \frac{19 \cdot q \cdot a_0}{(\text{Int})^{3/2}} + \frac{\text{Int} (19 \cdot q \cdot N \cdot a_0)}{(\text{Int})^{3/2}} \]

Lightest states have \( M(t) \sim (\text{Int})^{-\frac{3}{2}} \).
Therefore,

\[ \frac{M(t)}{M(t)} \sim \left( \text{Im} t \right)^{d/2} \cdot e^{-\frac{2\Delta \phi}{\lambda}} \]

So, distance conjecture holds for any infinite distance locus in any Calabi-Yau moduli space.

(0 (10^6) known CY's, having typically d(10) dimensional moduli spaces, each with c(10^10) different infinite distance loci.)

5) Relations and underlying physics

Distance conjecture and gravity as weakest force

A massless scalar field can mediate a Coulomb self-force (attractive), which can be compared to gravity:

\[ \mathcal{L} \sim m(\phi) \overline{\psi} \psi = (\partial \phi \cdot \mathcal{M}) \phi \overline{\psi} \psi \]

\[ \Rightarrow \mu = (\partial \phi \cdot \mathcal{M}) \]

Therefore, generally (also for scalar particles)
Gravity as weakest force:

\[ M = (\Theta + m) > M \]  (Planck units)

For \( \phi \to \infty \Rightarrow M \sim e^{-a\phi} \Rightarrow \text{Distance objection!} \)

(e.g. \( M \sim \phi^p \Rightarrow p > \phi \), violated for \( \phi \to \infty \))

WGC and Distance Conjecture from unification

The Species Scale

Are the infinite towers of states really infinite?

Consider dimensional reduction on a circle: \( D = d + 1 \).

\[
(M_p^{(d)})^{d-2} \int_{M_0} R^{(0)} \sqrt{-G^{(0)}} = (M_p^{(0)})^{d-2} (2\pi R) \int_{M_0} R^{(3)} \sqrt{-G^{(3)}}
\]

\[
= (M_p^{(3)})^{d-2} = (M_p^{(0)})^{d-2} 2\pi R
\]

So, in \( d \)-dimensional Planck units:

\[
M_p^{(0)} \sim \frac{1}{R^{\frac{d-2}{2}}}
\]

This is the true scale of strong coupling of gravity.
The Kaluza-Klein tower mass scale

\[ M_{\text{KK}} \sim \frac{1}{R} \]

So the number of states between \( M_{\text{KK}} \) and \( M_{(0)} \) is

\[ N_s \sim \frac{M_{(0)}}{M_{\text{KK}}} \sim R \]

\[ \Rightarrow M_{(0)} \sim \frac{1}{N_s^{\frac{1}{d-2}}} \]

An example of a general relation:

\[ N_s \sim \frac{M_{(0)}}{N_s^{\frac{1}{d-2}}} \]

In four dimensions \( N_s \sim \frac{M_{(0)}}{N_s^{\frac{1}{3}}} \), so the towers:

\[ N_s \sim N_s \]

Can also understand it from Hoop renormalization:
Strong-coupling is the scale at which the H loop corrections are of order the tree-level result

\[ M_p \sim N_s \Lambda_s \Rightarrow \Lambda_s \sim \frac{M_p}{\sqrt{N_s}} \]

What if one insists that any gauge field must become strongly-coupled at the same scale as gravity? (so they can unify at that scale)

H-loop running from a single charged particle

\[ \frac{1}{g^2_{2e}} = \frac{1}{g^2_{2e}} + \frac{q^2}{g^2_{2e}} \log \left( \frac{M}{\Lambda} \right) \]

Strongly coupled at \( \Lambda \sim e^{\frac{q^2}{g^2_{2e}}} M \Rightarrow M_p \) (Landau pole)

To unify need a larger or charged states:

\[ \frac{1}{g^2_{2e}} = \frac{1}{g^2_{2e}} + \frac{N}{n} \left( qN \right)^2 \log \left( \frac{M_n}{\Lambda} \right) \]

How many states in order:

\[ M_n \sim N M_{\infty} \]
\[ \Lambda \sim N M_{\infty} \]
\[ \Lambda \sim N_s \sim \frac{M_p}{\sqrt{N_s}} \]

\[ \text{Strong-coupling:} \quad \frac{1}{g^2_{2e}} \sim \frac{N}{n} \left( qN \right)^2 \log \left( \frac{M_n}{\Lambda} \right) \sim N^3 \left( \frac{M_p}{M_{\infty}} \right)^2 \]
\[ M_{oo} = 3 \pi \epsilon M_p \]

This is the magnetic Weak brevity Conjecture!

What about scalar fields?

\[ L = \mathcal{P}_{oo} (\partial \phi)^2 \]

\[ \mathcal{P}_{oo} L_{oo} = \mathcal{P}_{oo} \mathcal{L}_W + \sum_n (\partial \phi M_n)^2 \log \left( \frac{M_n}{\Lambda} \right) \]

Strong coupling:

\[ \mathcal{P}_{oo} L_{oo} \sim \sum_n n^2 (\partial \phi M_n)^2 \sim N^3 (\partial \phi M_{oo})^2 \sim \left( \frac{\partial \phi M_{oo}}{M_{oo}} \right)^2 \frac{M_p^2}{M_{oo}} \]

Proper distance in field space:

\[ \Delta \phi = \int_{\phi_i}^{\phi_f} \sqrt{\mathcal{P}_{oo} L_{oo}} \, d\phi \sim M_p \int \Delta \phi (\log M_{oo}) \, d\phi \]

\[ \sim M_p \log \left( \frac{M_{oo} (\phi_f)}{M_{oo} (\phi_i)} \right) \]

\[ \Rightarrow M_{oo} (\phi_f) = M_{oo} (\phi_i) e^{-\Delta \phi / M_p} \]

This is the Distance Conjecture!
6) Swampland away from flat space

String theory is a framework in which we can calculate \( \Lambda \) (for certain solutions) the cosmological constant \( \Lambda \).

It corresponds to the value of the minimum of the full scalar potential:

\[ \Lambda > 0 \Rightarrow \text{de Sitter space} \]
\[ \Lambda = 0 \Rightarrow \text{Minkowski space} \]
\[ \Lambda < 0 \Rightarrow \text{Anti-de Sitter space} \]
6.1 $\Lambda > 0$ : de Sitter

Difficult to extract information from String Theory: no fully understood de Sitter vacua are known.

Best candidate: KKLT, but many open questions.

Possible that this is hunting at a Swampland obstruction to de Sitter.

Refined de Sitter Conjecture

$$|\nabla V| \geq c V \quad \text{or} \quad \min_{M_P} \left( \frac{V}{g_5^4 V} \right) \leq -\frac{c'}{M_P^2}$$

with $c, c'$ constants of order one.

If true, would predict Dark Energy is dynamical.

Maybe true, but more likely holds as an asymptotic statement in parametric limits:

$$\phi \to \infty, \quad g \to 0$$
Dilaton field is exactly a flat direction.

Taking \( N \to 0 \) allows the \( g_s \to 0 \) limit in controlled regime.

3) Flat space limit and light towers

The flat space limit \( N \to 0, \quad R \to \infty \) is accompanied by an infinite tower of states becoming light.

\[
M_{\infty} \sim \frac{1}{R}
\]

Indeed, the radius of curvature of AdS and the \( S^5 \) are equal, so the background is not 5-dimensional.

This is called no scale-separation.

All fully-understood AdS solutions of String Theory have these properties. This motivates an AdS distance conjecture:

The flat space limit of AdS is accompanied by an infinite tower of states which behave as

\[
M_{\infty} \sim \frac{\Lambda}{M_p^{d-3}}, \quad d \sim O(1).
\]
The strong version implies no scale separation:

\[ \text{(Strong ADC)}: \text{ Supersymmetric ADS vacua are such that } \alpha' = \lambda' \]

\[ 1 \sim M_{\infty} \sim \left( \frac{\Lambda}{M_p} \right)^{1/2} \sim \lambda^{1/2} \sim \frac{\Lambda}{M_p} \]

Relation to Distance Conjecture

The ADC views the flat space limit of ADS as a type of infinite distance limit, in analogy with the distance conjecture.

We can make this a little more precise: consider the distance conjecture for Calabi-Yau moduli space. The moduli are variations of the CY metric:

\[ \Delta = \int \left( \frac{\Omega_j \Omega_j'}{2 \epsilon} \right)^2 \frac{1}{2} \left( \frac{g_{\mu \nu} g_{\rho \sigma}}{2 \epsilon} \right) \]

\[ = \int \left( \frac{1}{V_{CY}} \sqrt{g_{\mu \nu} g_{\rho \sigma}} \frac{2 \epsilon}{2 \epsilon} \frac{\partial \phi}{\partial \phi} \right)^2 \]
Apply the AdS metric for vacuum \( R \rightarrow \infty \).

Since \( R \) is an overall factor

\[
\Delta \sim - \int_{R_0}^{R} \tilde{d}^k \sim - \ln R
\]

Then take,

\[
M_{\infty} \sim e^{-\Delta} \sim \left( \frac{1}{R} \right)^{\frac{3}{2}} \sim \lambda^{1/2}
\]

Which is the APC.

Note: there is an (untamable) negative sign for the distance.

6.3. The weak gravity conjecture in AdS (2008.01584)

Consider an action with a negative cosmological constant:

\[
S = \frac{1}{16 \pi} \int d^5 x \sqrt{-g} \left[ \frac{R}{2} + \mathcal{L}_F - \frac{1}{4} \phi^2 - M^2 \phi^2 - \frac{1}{4 g^2} \left( a |\phi|^4 + b |\phi|^2 |D\phi|^2 + \ldots \right) \right]
\]

We have set the AdS scale to 1: \( R_{AdS} = 1 \).

\( \phi \) is charged field, with \( g = 1 \).
In flat space we would require:

\[ M \leq \sqrt{g_{\mu\nu}} \]

However, if we are motivated by the absence of stable bound states we need to consider all the contributions to the possible binding energy.

**All exchange diagrams:**

- **Gaugeton Exchange**
- **Gauge Exchange**
- **Contact terms**

**Binding energy**

\[ \mathcal{V} = \mathcal{V}_{\text{proton}} + \mathcal{V}_{\text{gaugeton}} + \mathcal{V}_{\text{quartic}} \]

\[ \mathcal{V}_{\text{proton}} = \frac{\pi^2 \, N_A^4 \, g^2 \, q^2}{2 \Delta - 1} \]

\[ \mathcal{V}_{\text{gaugeton}} = \frac{2 \pi^2 \, N_A^4 \, \Delta^2 (\Delta - 2)}{3 (\Delta - 1) (2 \Delta - 1)} \]

\[ \mathcal{V}_{\text{quartic}} = \frac{\pi^2 \, N_A^4 \, (a + b \Delta (2-\Delta))}{(\Delta - 1) (2 \Delta - 1)} \]

\[ m^2 = \Delta (\Delta - 4) \]

\[ N_A = \sqrt{\frac{\Delta - 1}{2\pi^2}} \]

In SUSY, when \( \phi \) is BPS:

\[ -g^2 q^2 = a + b \Delta (2 - \Delta) = -\frac{2}{3} \Delta^2 \]

\[ \therefore \mathcal{V}_{\text{quartic}} = 0 \]
In AdS contact terms are not sub-leading because cannot separate the particle from its copy by an infinite distance.

Positive Bundle Conjecture: $\exists 0$ for some particle

1) Holography and the Swampland

String Theory / Quantum Gravity on AdS spaces is expected to be dual to a Conformal Field Theory on the boundary of AdS.

For example, Type II on $\text{AdS}_5 \times S^5 \rightarrow N=4$ Super Yang Mills on $\mathbb{R}^{1,4}$.

There Swampland constraints in AdS should have dual constraints on CFTs.

7.1 Scale separation on CFT gaps

The masses of states on the gravity side are mapped to the conformal dimensions of dual operators on the CFT:

$$M^2 R_{\text{ads}}^2 = \Delta (\Delta - d) \quad (d=C\text{FT dim})$$

So no scale-separation implies a tower of states with mass scale

$$\text{Max} R_{\text{ads}} \sim \frac{R_{\text{ads}}}{R_{\text{ads}}} \sim 0(1)$$
In the CFT dual, this predicts an infinite tower of operators with dimensions of order one:

\[ \Delta_n \sim O(n^n) \]

So the operator spectrum has no gap, where we have a few operators with dimensions \( O(1) \), and then an infinite number of operators with parametrically separated dimensions.

Conjecture: true for any CFT, not only "holomorphic" ones.

(e.g. weakly coupled: \( \phi, \phi^2, \phi^3 \ldots \))

Aside: Holographic Versus Non-Holographic CFTs

In holography, the dual to the radius of \( \text{AdS}_5 \) in Planck units, is the central charge of the CFT:

\[ (\text{Rays } M_p) \sim C \]

\[ (M_{\text{CFT}_4}/M_{\text{AdS}_5}) \]

The central charge (roughly) counts the number of degrees of freedom in the CFT.

Holographic CFTs are those which have gravitational duals that behave like Einstein gravity, so they require (at least) \( C \gg 1 \).
Nonetheless, one can say that every CFT defines a quantum gravity theory in AdS, because:

- CFTs are UV/IR complete.
- Local CFTs have an energy-momentum tensor, which is the operator dual to the graviton (massless spin-2 field).

Often, such "quantum gravities" both nothing like Einstein gravity, say they must have an infinite number of massless higher spin fields. But they are all expected to be part of strong gravity.

7.2 CFT Distance Conjecture

CFTs can come in families with exactly marginal operators:

$$\mathcal{C}_{\text{CFT}} = \{ \mathbf{e} \text{ t} \, | \, \text{deformation parameters} \text{ exactly marginal} \}$$

These are dual to moduli spaces on the gravity side. We therefore expect a dual to the distance conjecture (can define a metric on the deformation space (conformal manifold):

$$g_{ij}(t) = |x-y|^2 \mathcal{O}_i(x) \mathcal{O}_j(y)$$
we can use the metric to define a distance on the conformal manifold: \( d(t, t') \).

Then the conjecture is that as \( d \to 0 \), there is an infinite tower of operators whose dimension goes to zero.

More precisely, if the minimal dimension is the tower \( \Delta_{\text{min}} \), is bounded by \( \epsilon : \Delta_{\text{min}} \geq \epsilon \). Then, \( d \sim \epsilon \).

Further, it was proposed that all infinite distance loci in CFT's are 

**Higher-Spin (HS)** theories. 

That means that the tower of operators have increasing spins \( S \):

\[
\Delta_S = d - 2 + J.
\]

### 7.3 Charge convexity conjecture

Recall that on ADs we formulated a version of the WGC called the Positive Binding Conjecture, which demands the existence of a charged particle, say at charge \( Q \), with positive self-binding energy, \( \Sigma \geq 0 \).

This implies that on the gravity side we have a state of energy \( E_Q \), and charge \( Q \) under U(1) gauge symmetry, such that 

\[
E_Q - 2E_Q \geq 0.
\]
In the dual CFT we have a $U(N)$ global symmetry, and a bound on the dimension of charged operators:

$$\Delta(2\sigma) - 2\Delta(\sigma) \geq 0$$

More generally, the Charge Convexity Conjecture proposes:

$$\Delta(n_1 \sigma_1 + n_2 \sigma_2) \geq \Delta(n_1 \sigma_1) + \Delta(n_2 \sigma_2)$$

where $\Delta(\sigma)$ is the lowest dimension operator of charge $\sigma$, and $n_1$ and $n_2$ are any positive integers, and $\sigma_1 \sim O(1)$ (more precisely, cannot be made parametrically large).

Proposed to hold for $d > 2$.

In $d=2$, $\sigma_0$ can be forced to be parametrically large, but in a very specific way.

Current limit of the global symmetry current:

$$J(2) J(0) \sim \frac{1}{g^2}$$

Then can prove the conjecture, with $\sigma_0 \leq k$. 