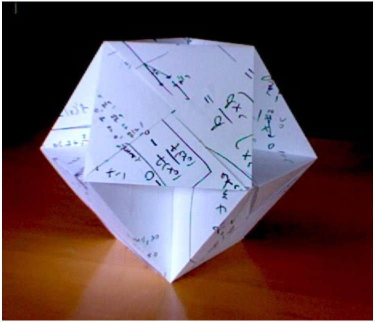
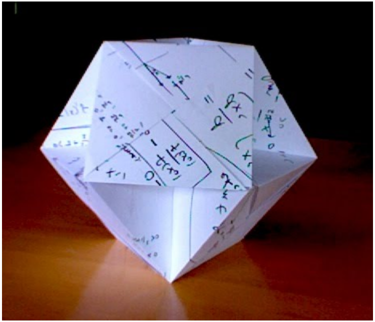
Duality Origami



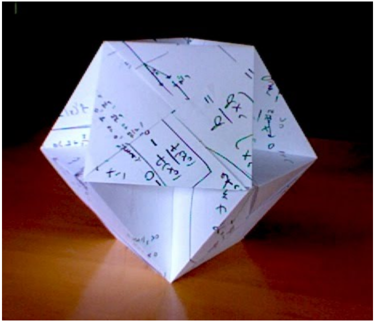
Duality Origami

Emergent Ensemble Symmetries in Holography



Duality Origami

Emergent Ensemble Symmetries in Holography



Jacob M. Leedom

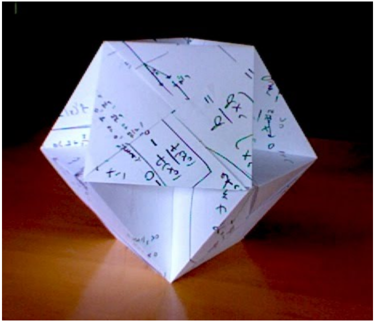
1^{st} DIP Conference

2104.14710, 23XX.XXXXX,+

M.Ashwinkumar, M.Dodelson,
A.Kidambi, M.Yamazaki

Duality Origami

Emergent Ensemble Symmetries in Holography



Part I: Corfu '22
Part II: Today

Jacob M. Leedom

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M.Ashwinkumar, M.Dodelson, A.Kidambi, M.Yamazaki

Two paths to Holography:

From Top-Down: AdS_5 & N 4 SYM

↓

↑

From Bottom-Up: JT & SYK

[M.Berkooz's Talk]

Two paths to Holography:

From Top-Down: AdS$_5$ & N 4 SYM

$\downarrow$

$\uparrow$

From Bottom-Up: Narain Ensembles
& Chern-Simons

Two paths to Holography:

From Top-Down: AdS_5 & N 4 SYM

↓

Quod est superius est sicut quod inferius,

et quod inferius est sicut quod est superius.

The Emerald Tablet

↑

From Bottom-Up: Narain Ensembles
& Chern-Simons

Two paths to Holography:

From Top-Down: AdS_3 & $\mathcal{N} = 4$ SYM

↓

That which is above is like that which is below,

and that which is below is like that which is abo

The Emerald Tablet

↑

From Bottom-Up: Narain Ensembles
& Chern-Simons

Two paths to Holography:

From Top-Down: AdS_5 & N 4 SYM

↓

That which is above is like that which is below,

and that which is below is like that which is abo

The Emerald Tablet

As above, so below

Petr Horava, String Theory I, Fall 2015

↑

From Bottom-Up: Narain Ensembles
& Chern-Simons

$$S_{Narain} \quad » \quad d^2 \quad '' \quad G_{MN} B^a X^M B_a X^N \quad B_{MN} \; {}_{ab} B^a X^M B^b X^N \quad *$$

$$S_{Narain} \quad » \quad d^2 \quad " \quad G_{MN} B^a X^M B_a X^N \quad B_{MN} \;{}_{ab} B^a X^M B^b X^N \quad *$$

Compact Bosons:

$$\begin{matrix} X_L^M \\ X_R^M \end{matrix} \qquad \begin{matrix} X_L^M \\ X_R^M \end{matrix} \qquad EL$$

$L \; P \; Z^{2D}$
E: Narain Vielbein

$$S_{Narain} \quad » \quad d^2 \quad " \quad G_{MN} B^a X^M B_a X^N \quad B_{MN} \;\; {}_{ab} B^a X^M B^b X^N \quad *$$

Compact Bosons:

$$\begin{matrix} X_L^M & & X_L^M & & EL \\ X_R^M & & X_R^M & & \end{matrix}$$

$L \; P \; Z^{2D}$

E: Narain Vielbein

Operators:

$$J^M \quad B \; X^M$$

$$J^M \quad BX^M$$

$$V_{pp_L;p_R q} \quad : e^{ip_L \; X_L \quad ip_R \; X_R} \; :$$

$$S_{\text{Narain}} \quad » \quad d^2 \quad " \quad G_{MN}\, B^a X^M B_a X^N \quad B_{MN} \;{}_{ab} B^a X^M B^b X^N \quad *$$

Compact Bosons:

$$\begin{matrix} X_L^M & & X_L^M \\ X_R^M & & X_R^M \end{matrix} \qquad EL$$

$L \; P \; Z^{2D}$
E: Narain Vielbein

Operators:

$$J^M \quad B \; X^M$$

$$J^M \quad BX^M$$

$$V_{pp_L;p_R q} \quad : e^{ip_L \; X_L \quad ip_R \; X_R} :$$

OPE:

$$V_{pp_L;p_R q} pz q V_{pk_L;k_R q} pw q \quad p \; z \quad wq^{p_L \; k_L} pz \quad wq^{p_R \; k_R} V_{pp_L \quad k_L;p_R \quad k_R q} pw q$$

$$S_{Narain} \quad » \quad d^2 \quad " \quad G_{MN} B^a X^M B_a X^N \quad B_{MN} \; {}_{ab} B^a X^M B^b X^N \quad *$$

Compact Bosons:

$$X_L^M \quad X_L^M \quad EL \qquad\qquad L \; P \; Z^{2D}$$
$$X_R^M \quad X_R^M \qquad\qquad E\text{: Narain Vielbein}$$

Operators:

$$J^M \quad B \; X^M$$

$$J^M \quad BX^M$$

$$V_{pp_L;p_R q} \quad : e^{ip_L \; X_L \quad ip_R \; X_R} :$$

OPE:

$$V_{pp_L;p_R q} pzq V_{pk_L;k_R q} pwq \quad p \; z \quad wq^{p_L \; k_L} pz \quad wq^{p_R \; k_R} V_{pp_L \; k_L;p_R \; k_R q} pwq$$

Closure requires an in nite lattice of operators

OperatorsÑ Lattice

$$_{\text{Narain}} : t \quad p\ p_L ; p_R q \quad E`j` \quad p\ n^i ; w_i q\ PZ^{2D} u$$

Operators ↔ Lattice

$$\Gamma_{\text{Narain}} : \left\{ (p_L; p_R) = E \cdot (n^i; w_i) \in \mathbb{Z}^{2D} \right\}$$

Even: $\ell^2 \in 2\mathbb{Z}$
Self-Dual:

has signature $(D; D)$ with norm:

$$\ell^2 = \ell^T \begin{pmatrix} 1_D & 0 \\ 0 & -1_D \end{pmatrix} \ell = p_L^2 - p_R^2 = 2 n^i w_i$$

OperatorsÑ Lattice Ñ Partition Function

$$_{\text{Narain}} : t \quad p\ p_L ; p_R q \quad E`j` \quad p\ n^i ; w_i q\ PZ^{2D} u$$

$$Z_D p\ ; mq \quad \frac{\#_D p\ ; mq}{j\ p\ qj^{2D}}$$

$$\#_D p\ ; mq \quad \underset{P\ \ \text{Narain}}{¸} \quad q^{\frac{p_L^2}{2}} q^{\frac{p_R^2}{2}}$$

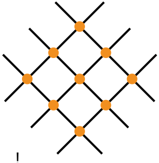
$$q \quad e^{2\ \ i} \quad {}_1 \quad i\ {}_2$$

$$p\ q \quad q^{\frac{1}{24}} \quad \overset{1 8}{\underset{n\ \ 1}{}} \quad p1 \quad q^n q$$

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

$\text{Narain}: t \; p\, p_L; p_R q \quad E`j` \quad p\, n^i; w_i q\, P Z^{2D} u$

Discrete Data:



$: \; ! \; ` P Z^{2D} \mid Q r`s P 2Z)$

$$Z_D p\, ; mq \quad \frac{\#_D p\, ; mq}{j\; p\; q j^{2D}}$$

$$\#_D p\, ; mq \quad \underset{P\ \text{Narain}}{¸} \; q^{\frac{p_L^2}{2}} q^{\frac{p_R^2}{2}}$$

$$Q : \; E^T \begin{matrix} 1_D & 0 \\ 0 & 1_D \end{matrix} \; E \quad \begin{matrix} 0 & 1_D \\ 1_D & 0 \end{matrix}$$

$Q r`s : \; `^T Q` \quad 2n^i w_i \quad p_L^2 \quad p_R^2$

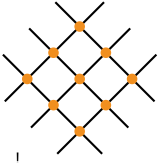
OperatorsÑ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p pL; pR q E`j` p ni; wi q PZ^2D u

Discrete Data:

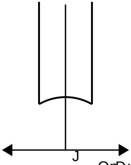
ZD p ; mq   #D p ; mq / j p q|^2D

#D p ; mq   ¸ q^(pL^2/2) q^(pR^2/2)
P Narain



!)
: ` PZ^2D | Qr`s P2Z

Continuous Data: M N



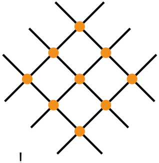
M N : OpD; D; Zq   OpD; D; Rq / OpDq OpDq

H   E^T E P hQ
HQ ^1H   Q
Hr`s:  `^T H`   pL^2   pR^2

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

$_{Narain}$: t ppL; pRq E`j` pn^i; w_iq P Z^{2D}u

Discrete Data:

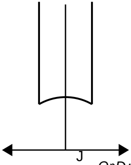


: ! ` P $Z^{2D}$ | Qr`s P 2Z)

Z_Dp ; mq $\frac{\#_D\text{p ; }mq}{j\ \text{p}\ \text{q}j^{2D}}$

$\#_D$p ; mq ¸ P $_{Narain}$ $q^{\frac{p_L^2}{2}} q^{\frac{p_R^2}{2}}$

Continuous Data: M_N



M_N : OpD; D; Zq J $\frac{\text{OpD; D; Rq}}{\text{OpDq} \quad \text{OpDq}}$

H $E^T E$ P h_Q

HQ ^1H Q

Hr`s : $`^T H`$ p_L^2 p_R^2

T-Duality: " *

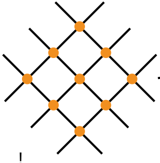
OpD; D; Zq P GLp2D; Zq $^T Q$ Q

[A.Westphal's Talk]
[S.Demulder's Talk]

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t ppL; pRq E`j` pn^i; w$_i$q P Z^{2D}u

Discrete Data:

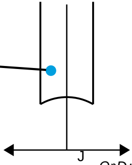


!)
: ` P $Z^{2D}$ | Qr`s P 2Z

Z_Dp ; mq $\dfrac{\#_D\text{p} \; ; m\text{q}}{j \;\text{p}\; \text{q}j^{2D}}$

$\#_D$p ; mq ¸ exp i $_1$Qr`s $_2$Hr`s
`P

Continuous Data: M_N



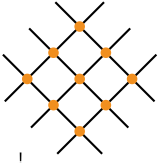
M_N : OpD; D; Zq $\dfrac{\text{O}p D; D; \text{R}q}{\text{O}p D\text{q} \quad \text{O}p D\text{q}}$

H $E^T E$ P h$_Q$

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t ᵽ$p_L; p_R$q E`$j$` ᵽ$n^i; w_i$q P Z^{2D}u

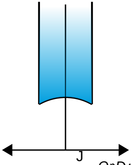
Discrete Data:



: ! ` P $Z^{2D}$ | $Q$r`s P 2Z)

xZ_Dᵽ ; $m$qy » $\frac{\#_D ᵽ ; m q}{j\ ᵽ\ qj^{2D}}$ rdms M_N

Continuous Data: M_N



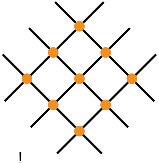
M_N : Oᵽ$D; D$; Zq $\frac{Oᵽ D; D; Rq}{Oᵽ Dq \quad Oᵽ Dq}$ J

H $E^T E$ P h$_Q$

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

$_{Narain}$: t ppL; pRq E`j` pn^i; w$_i$q P Z^{2D}u

Discrete Data:



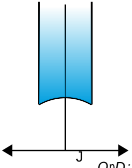
: ! ` P Z$^{2D}$ | Qr`s P 2Z)

xZDp ; mqy » #Dp ; mq / M N j p qj^{2D} rdms

Siegel-Weil Theorem

xZDp ; mqy ED{2p q / D{2 2 j p qj^{2D}

Continuous Data: M N



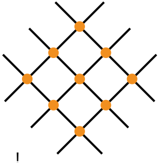
M N : OpD; D; Zq OpD; D; Rq / OpDq OpDq

H E^{T}E P hQ

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p $p_L ; p_R$ q E`j` p $n^i ; w_i$ q PZ^{2D} u

Discrete Data:



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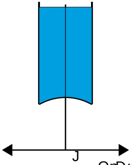
: ` P $Z^{2D}$ | Qr`s P2Z

xZ_D p ; mqy » $\frac{\#_D \text{p ; mq}}{\text{j p q}j^{2D}}$ rdms
M N

Siegel-Weil Theorem

xZ_D p ; mqy $\frac{E_{D\{2} \text{p q}}{2^{D\{2} \text{j p q}j^{2D}}}$

Continuous Data: M N



M N : OpD; D; Zq $\frac{\text{OpD; D; Rq}}{\text{OpDq OpDq}}$

H E^T E P h_Q

E_sp q Real-Analytic Eisenstein Series

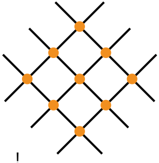
E_sp q E$_s$p q @ P : PSL_2pZq

E_sp q: $\sum_{pc;dq\ 1} \frac{\frac{s}{2}}{\text{jc dj}^{2s}}$

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p $p_L ; p_R$ q E`j` p $n^i ; w_i$ q PZ^{2D} u

Discrete Data:



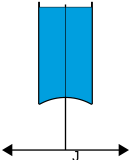
: ! ` P $Z^{2D}$ | Qr`s P2Z)

xZ_D p ; mqy » $\frac{\#_D\text{p ; mq}}{\text{j p q}j^{2D}}$ rdms M N

Siegel-Weil Theorem

xZ_D p ; mqy $\frac{E_{D\{2}\text{p q}}}{2^{D\{2}\text{j p q}j^{2D}}}$

Continuous Data: M N



M N : OpD; D; Zq J $\frac{\text{OpD; D; Rq}}{\text{OpDq OpDq}}$

H E^T E P h_Q

E_sp q: Real-Analytic Eisenstein Series

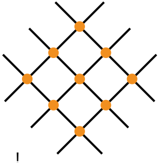
E_sp q: ¸ pc;dq 1 $\frac{\frac{s}{2}}{\text{jc dj}^{2s}}$ ¸ 8 z Imp q^s

E_sp q E_sp q @ P : PSL_2pZq " * 8 P 1 n 0 1

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p pL; pR q E`j` p ni; wi q PZ2D u

Discrete Data:



: ! ` P Z2D | Qr`s P2Z)

xZD p ; mqy » #D p ; mq / M N j p qj2D rdms

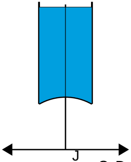
Siegel-Weil Theorem

xZD p ; mqy ED{2p q / 2D{2 j p qj2D

Poincaré Sum

ED{2p q / 2D{2 j p qj2D ¸ 8 z 1 / j p qj2D

Continuous Data: M N



M N : OpD; D; Zq OpD; D; Rq / OpDq OpDq

H ET E P hQ

Esp q: Real-Analytic Eisenstein Series

Esp q: ¸ pc;dq 1 s/2 / jc dj2s ¸ 8 z Imp qs

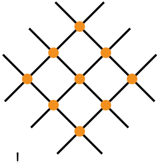
Esp q Esp q @ P : PSL2pZq

" 8 P * 1 n / 0 1

OperatorsÑ Lattice Ñ Partition FunctionÑ Quadratic Forms

Narain : t p pL; pR q E`j` p ni; wi q PZ2D u

Discrete Data:



!)
: ` P Z2D | Qr`s P2Z

xZD p ; mqy » #D p ; mq / j p qj2D rdms
M N

Siegel-Weil Theorem

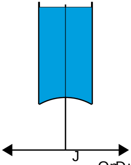
xZD p ; mqy ED{2p q / 2D{2j p qj2D

Poincaré Sum

ED{2p q / 2D{2j p qj2D ¸ 8 z 1 / j p qj2D ñ Gravitational Interpretation?

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Continuous Data: M N



M N : OpD; D; Zq OpD; D; Rq / OpDq OpDq

H ET E P hQ

Esp q Real-Analytic Eisenstein Series

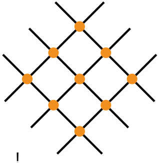
Esp q: ¸ pc;dq 1 s/2 / jc dj2s ¸ 8 z Imp qs

Esp q Esp q @ P : PSL2pZq
" *
8 P 1 n
0 1

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p $p_L ; p_R$ q E`j` p $n^i ; w_i$ q PZ^{2D} u

Discrete Data:



!)
: ` P $Z^{2D}$ | Qr`s P2Z

xZ_D p ; mqy » $\frac{\#_D \text{p ; mq}}{\text{j p q}^{2D}}$ rdms
M N

Siegel-Weil Theorem

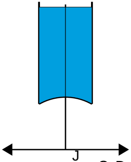
xZ_D p ; mqy $\frac{E_{D\{2}\text{p q}}{2^{D\{2}\text{j p q}^{2D}}}$

Poincaré Sum

$\frac{E_{D\{2}\text{p q}}{2^{D\{2}\text{j p q}^{2D}}}$ ¸ $\frac{1}{\text{j p q}^{2D}}$ ñ Gravitational Interpretation?
8 Z

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Continuous Data: M N



M N : OpD; D; Zq $\frac{\text{OpD; D; Rq}}{\text{OpDq OpDq}}$

H E^T E P h_Q

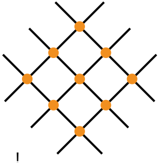
3D Abelian Chern-Simons:

S_{CS} » $Q_{IJ}\ A^I dA^J d^3x$

OperatorsÑ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p p_L ; p_R q E`j` p n^i ; w_i q PZ^{2D} u

Discrete Data:



!)
: ` P $Z^{2D}$ | Qr`s P2Z

xZ_D p ; mqy » $\frac{\#_D \text{p ; mq}}{\text{j p q}j^{2D}}$ rdms
M N

Siegel-Weil Theorem

xZ_D p ; mqy $\frac{E_{D\{2} \text{p q}}{{}^{D\{2}_{2}\text{j p q}j^{2D}}}$

Poincaré Sum

$\frac{E_{D\{2} \text{p q}}{{}^{D\{2}_{2}\text{j p q}j^{2D}}}$ ¸ $\frac{1}{\text{j p q}j^{2D}}$ ñ Gravitational Interpretation?
8 z

[Witten,Maloney,'20]
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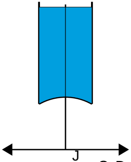
Continuous Data: M N



M N : OpD ; D ; Zq $\frac{\text{OpD ; D ; Rq}}{\text{OpDq OpDq}}$

H E^T E P h_Q

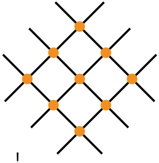
3D Abelian Chern-Simons:

S_{CS} » Q_{IJ} $A^I dA^J d^3x$ —On Thermal AdS_3→ Z_{ThAdS_3} $\frac{1}{\text{j p q}j^{2D}}$

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p pL; pR q E`j` p ni; wi q PZ2D u

Discrete Data:



: ! ` PZ2D | Qr`s P2Z)

xZD p ; mqy » #D p ; mq / j p qj2D rdms M N

Siegel-Weil Theorem

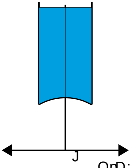
xZD p ; mqy ED{2p q / 2D{2 j p qj2D

Poincaré Sum

ED{2p q / 2D{2 j p qj2D ¸ 8 z 1 / j p qj2D ñ Gravitational Interpretation?

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Continuous Data: M N



M N : OpD; D; Zq OpD; D; Rq / OpDq OpDq

H ET E P hQ

3D Abelian Chern-Simons:

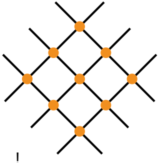
SCS » QIJ AI dAJ d3x On Thermal AdS3 → ZThAdS3 1 / j p qj2D

Fill with ThAdS3

OperatorsÑ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p p_L ; p_R q E`j` p n^i ; w_i q PZ^{2D} u

Discrete Data:



: ! ` P$Z^{2D}$ | Qr`s P2Z)

xZ_D p ; mqy » M N $\#_D$p ; mq / j p qj^{2D} rdms

Siegel-Weil Theorem

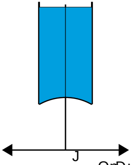
xZ_D p ; mqy $E_{D\{2}$p q / $_2^{D\{2}$j p qj^{2D}

Poincaré Sum

$E_{D\{2}$p q / $_2^{D\{2}$j p qj^{2D} ¸ $_{8\ Z}$ 1 / j p qj^{2D} ñ Gravitational Interpretation?

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Continuous Data: M N



M N : OpD ; D ; Zq OpD ; D ; Rq / OpDq OpDq

H E^T E P h_Q

3D Abelian Chern-Simons on PS^2_Zq Black Hole Background:

S_{CS} » Q_{IJ} $A^I dA^J d^3x$ —On $M_{pc;dq}$→ $Z_{M_{pc;dq}}$ 1 / j p qj^{2D}

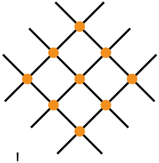
Fill with $M_{pc;dq}$

P $_8$ Z

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p $p_L ; p_R$ q E`j` p $n^i ; w_i$ q PZ^{2D} u

Discrete Data:



: ! ` $PZ^{2D}$ | Qr`s P2Z)

xZ_D p ; mqy » $\frac{\#_D \text{ p ; mq}}{\text{j p q}j^{2D}}$ rdms M N

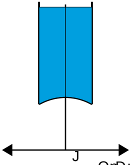
Siegel-Weil Theorem

xZ_D p ; mqy $\frac{E_{D\{2}\text{p q}}{\frac{D\{2}{2}\text{j p q}j^{2D}}$

Poincaré Sum

$\frac{E_{D\{2}\text{p q}}{\frac{D\{2}{2}\text{j p q}j^{2D}}$ ¸ $_{8\ z}$ $\frac{1}{\text{j p q}j^{2D}}$

Continuous Data: M N



M N : OpD; D; Zq $\frac{\text{OpD; D; Rq}}{\text{OpDq OpDq}}$

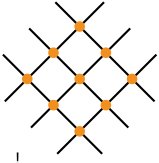
H E^T E P h_Q

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

OperatorsÑ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p $p_L; p_R$ q E`j` p $n^i; w_i$ q PZ^{2D} u

Discrete Data:



!)
: ` $PZ^{2D}$ | Q r`s P2Z

Ensemble of 2D CFTs

Holographic Correspondence

Sum of 3D Geometries

xZ_D p ; m qy » $\frac{\#_D p ; mq}{j\ p\ q j^{2D}}$ r dm s
M N

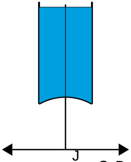
Siegel-Weil Theorem

xZ_D p ; m qy $\frac{E_{D\{2} p\ q}{2^{D\{2} j\ p\ q j^{2D}}$

Poincaré Sum

$\frac{E_{D\{2} p\ q}{2^{D\{2} j\ p\ q j^{2D}}$ ¸ $\frac{1}{j\ p\ \ q j^{2D}}$
8 z

Continuous Data: M N



M N : OpD; D; Zq $\frac{OpD; D; Rq}{OpDq\ \ OpDq}$

H E^T E P h_Q

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Operators Ñ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p pL; pR q E`j` p n^i; w_i q PZ^{2D} u

Discrete Data: Ensemble of 2D CFTs

xZ_D p ; mqy » #_D p ; mq / j p q^{2D} rdms
M N

Siegel-Weil Theorem

xZ_D p ; mqy E_{D{2} p q / 2^{D{2} j p q^{2D}

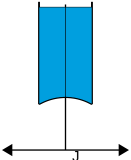
Holographic Correspondence

Poincaré Sum

!)
: ` P Z^{2D} | Qr`s P2Z

Sum of 3D Geometries E_{D{2} p q / 2^{D{2} j p q^{2D} ¸ 1 / j p q^{2D}
8 z

Continuous Data: M N



M N : OpD; D; Zq OpD; D; Rq / OpDq OpDq

H E^T E P h_Q

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

OperatorsÑ Lattice Ñ Partition Function Ñ Quadratic Forms

Narain : t p p_L; p_R q E`j` p n^i; w_i q PZ^{2D} u

Discrete Data: Ensemble of 2D CFTs

xZ_D p ; mqy » M N $\frac{\#_D \text{p ; mq}}{\text{j p q}|^{2D}}$ rdms

Siegel-Weil Theorem

xZ_D p ; mqy $\frac{E_{D\{2} \text{p q}}{{}_2^{D\{2}\text{j p q}|^{2D}}$

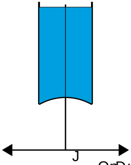
Holographic Correspondence

Poincaré Sum

: ! ` P $Z^{2D}$ | Qr`s P2Z)

Sum of 3D Geometries $\frac{E_{D\{2} \text{p q}}{{}_2^{D\{2}\text{j p q}|^{2D}}$ ¸ $\frac{1}{\text{j p q}|^{2D}}$ 8 z

Continuous Data: M N



M N : OpD; D; Zq $\frac{\text{OpD; D; Rq}}{\text{OpDq OpDq}}$

H E^T E P h_Q

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

Holography

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

¸ Z_{Bulk} r s » rdms Z_{CFT} rm; s

AdS_3 geometries

moduli space

Number Theory

Quantum Gravity

Holography

[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]

¸ Z_{Bulk} r s » rdms Z_{CFT} rm; s

AdS$_3$ geometries

moduli space

Number Theory

Quantum Gravity

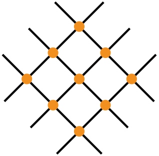
How far can we push this correspondence?

OperatorsÑ Lattice Ñ Partition FunctionÑ Quadratic Forms

OperatorsÐ Lattice Ð Partition FunctionÐ Quadratic Forms

OperatorsÐ Lattice Ð Partition FunctionÐ Quadratic Forms

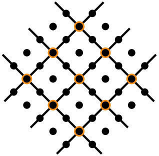
Discrete Data:



: ` P Z$^{p}$ $^{q}$ | Qr`s P2Z$^{($

Signaturepp; qq ñ p q
Even
Not Self-Dual

Discrete Data:



: ` P Z$^{p\ q}$ | Qr`s P 2Z$^{(}$

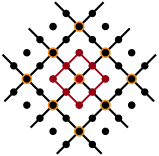
: t x P R$^{p\ q}$ | Qrx; `s P Z u

Signaturepp; qq ñ p q

Even

Not Self-Dual

Discrete Data:



: ` P Z$^{p\ \ q}$ | Qr`s P2Z(

: t x P R$^{p\ \ q}$ | Qrx; `s P Z u

D : {

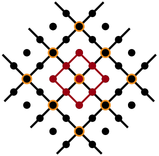
Signaturepp; qq ñ p q

Even

Not Self-Dual

OperatorsÐ Lattice Ð Partition FunctionÐ Quadratic Forms

Discrete Data:

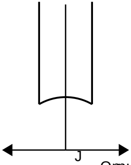


: ` P Z^p q | Qr`s P2Z^(

: t x P R^p q | Qrx; `s PZ u

D : {

Continuous Data: M

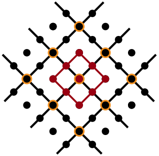


M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

h : t H P GLpp q; Rq | HQ ^1H Qu

OperatorsÐ Lattice Ð Partition Function Ð Quadratic Forms

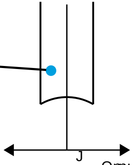
Discrete Data:



Z p ; mq   # p ; mq / pp q qp q

p ; mq ¸ `P exp i 1Qr` s 2Hr` s

Continuous Data: M



: ` P Zp q | Qr`s P2Z(

: t x P Rp q | Qrx; `s PZ u

D : {

M : O pp; q; Zq   Opp; q; Rq / Oppq Opqq

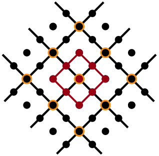
h : t H P GLpp q; Rq | HQ 1H Qu

T : # p 1; mq ei Qr s# p ; mq

S : # 1; m   e i/4 a / jD j   p/2 — q/2 ¸ ! PD   e 2 iQr ;! s#! p ; mq

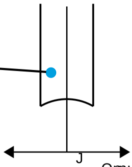
OperatorsÐ Lattice Ð Partition Function Ð Quadratic Forms

Discrete Data:



Z p ; mq   # p ; mq / ᵖp q ᵠp q

Continuous Data: M



p ; mq ¸ exp i ₁Qr` s ₂Hr` s
`P

: ` P Zᵖ ᵠ | Qr`s P2Z (
: t x P Rᵖ ᵠ | Qrx; `s PZ u
D : {

M : O pp; q; Zq Opp; q; Rq / Oppq Opqq
h : t H P GLpp q; Rq | HQ ¹H Qu

pg ; mq ¸ U; pg; q# p ; mq
PD

g a b / c d P

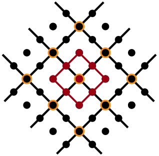
U; pg; q: p c dq^(p/2) pc— dq^(q/2) ; pgq

; pgq: e^(i/4) a / |D | c^((p q)/2) ¸ e^(i/c paQr`c s 2Qr`c ; s dQr sq)
`c P {pc q

OperatorsÐ Lattice Ð Partition Function Ð Quadratic Forms

G:N: : t p pL; pR q E`j` P Z p q u

Discrete Data:



: ` P Z p q | Qr`s P 2Z (

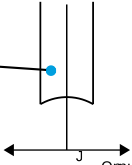
: t x P R p q | Qrx; `s P Z u

D : {

Z p ; mq = # p ; mq / p p q q p q

p ; mq ¸ `P exp i 1Qr` s 2Hr` s

Continuous Data: M



M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

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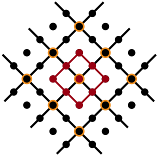
Operators Ð Lattice Ð Partition Function Ð Quadratic Forms

G:N: : t p pL; pR q E`j` P Zp q u

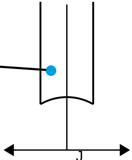
Z p ; mq # p ; mq / pp q qp q

Discrete Data:



p ; mq ¸ `P exp i 1Qr` s 2Hr` s

Continuous Data: M



: ` P Zp q | Qr`s P2Z(

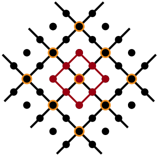
: t x P Rp q | Qrx; `s PZ u

D : {

M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

h : t H P GLpp q; Rq | HQ 1H Qu

Discrete Data:



: ` P Z p q | Q r`s P2Z (

: t x P R p q | Qrx; `s PZ u

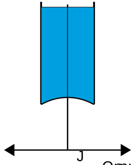
D : {

xZ p ; mqy » M # p ; mq / pp q qp q rdms

Siegel-Weil Theorem

xZ p ; mqy E p q / pp q qp q

Continuous Data: M



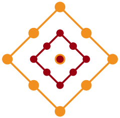
M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

E p q: P ¸ pc;dq 1, c¡ 0 pc; dq / pc dq^{p/2} pc dq^{q/2}

pc; dq: e^i {4jD j ½ c ^{(p q)/2} ¸ `1P {c exp i d/c Qr`1 s

Operators Ð Lattice Ð Partition Function Ð Quadratic Forms

Discrete Data:



: ` P Z p q | Qr`s P2Z (

: t x P R p q | Qrx; `s PZ u

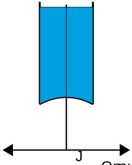
D : {

xZ p ; mqy » M # p ; mq / pp q qp q rdms

Siegel-Weil Theorem

xZ p ; mqy E p q / pp q qp q

Continuous Data: M



M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

E p q : P ¸ pc;dq 1 c¡ 0 pc; dq / pc dq^(p/2) pc dq^(q/2)

pc; dq : e^i {4jD j ^(1/2) c ^((p q)/2) ¸ `1P {c exp i (d/c) Qr`1 s

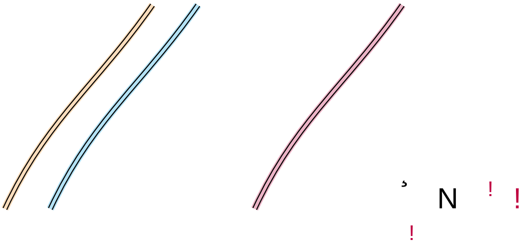
Can this be written as a Poincaré sum?

TQFT = nite collection of anyons A with data:

- Fusion Rules: Commutative, Associative product $A \;\; A \to A$
- Topological Spin: Map $: A \to U(1)$
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms

TQFT = nite collection of anyons A with data:

- Fusion Rules: Commutative, Associative product : A A Ñ A
- Topological Spin: Map : A Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



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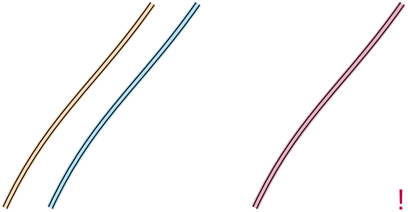
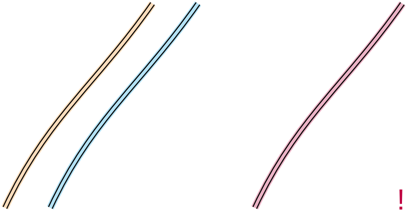


or Abelian

Abelian TQFT = nite collection of Abelian anyons A with data:

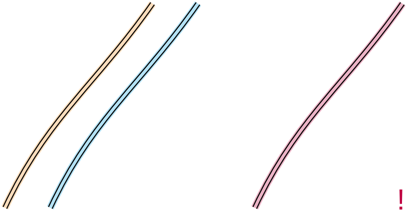
- Fusion Rules ñ anyons form a nite Abelian group
- Topological Spin: Map : A Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



$$S_{CS} \quad Q_{IJ}\, A^I dA^J\, d^3x$$

Abelian TQFT = nite collection of Abelian anyons A with data:

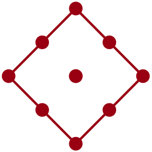
- Fusion Rules ñ anyons form a nite Abelian group
- Topological Spin: Map : A Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



$$S_{CS} \quad Q_{IJ}\, A^I dA^J\, d^3x$$

Abelian TQFT = nite collection of Abelian anyons D with data:

- Fusion Rules ñ anyons form a nite Abelian group
- Topological Spin: Map : D Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



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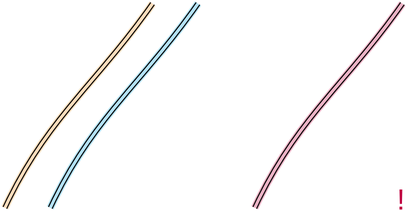
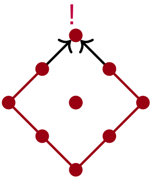


!

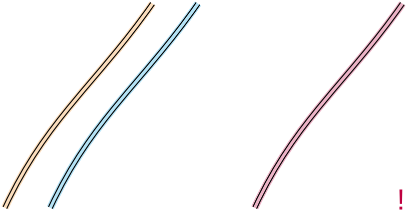
$$S_{CS} \quad Q_{IJ}\, A^I dA^J\, d^3x$$

Abelian TQFT = nite collection of Abelian anyons D with data:

- Fusion Rules ñ anyons form a nite Abelian group
- Topological Spin: Map : D Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



D {

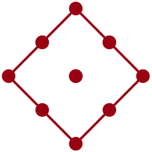


! mod

$$S_{CS} \quad Q_{IJ}\, A^I dA^J\, d^3x$$

Abelian TQFT = nite collection of Abelian anyons D with data:

- Fusion Rules ñ anyons form a nite Abelian group
- Topological Spin : Map : D Ñ Up1q
- Rep. of Modular Group: Generated by S & T
- Associator & Braiding Isomorphisms



D {

$$p \; q : \quad \exp p i \quad Q r \quad s q$$

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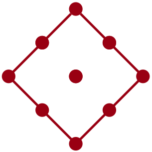
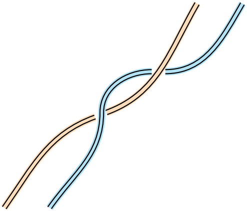


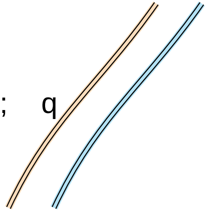
D {

$$p\ q: \quad \exp pi \quad Qr \quad sq$$

$$T \qquad p\ qe^{\frac{i}{12}} \quad \& \quad S \qquad \frac{Bp; \quad q}{a\ \overline{jD\ j}}$$



$$Bp; \quad q$$



$$Bp; \quad q \quad \frac{p \qquad q}{p\ q\ p\ q}$$

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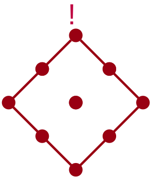

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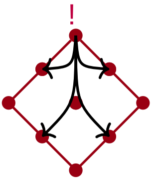

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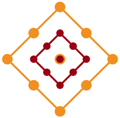
$$|!\ y\ \tilde{N}\ \ T_! \quad |\ y \quad \& \quad |!\ y\ \tilde{N}\ \ S_! \quad |\ y$$

$$|!\ y\ \tilde{N}\ \ U_! \quad pgq\ |\ y \qquad\qquad g\ P$$

Modular Group Representation identical to that of & E

Operators Ð Lattice Ð Partition Function Ð Quadratic Forms

Discrete Data:



: ` P Z p q | Qr`s P2Z (

: t x P R p q | Qrx; `s PZ u

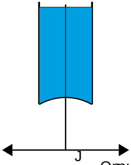
D : {

xZ p ; mqy M » # p ; mq / p p q q p q rdms

Siegel-Weil Theorem

xZ p ; mqy E p q / p p q q p q

Continuous Data: M



M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

[ADKLY,'20]

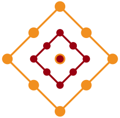
E p q : P ¸ pc; dq 1 / c¡ 0 — pc; dq / pc dq^{p/2} pc dq^{q/2}

pc; dq : e^i {4 jD j ½ c^{(p q)/2} ¸ `1P {c exp i (d/c) Qr` 1 s

Can this be written as a Poincaré sum?

Operators Đ Lattice Đ Partition Function Đ Quadratic Forms

Discrete Data:



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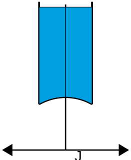

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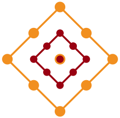
[ADKLY,'20]

pc; dq ; 0pg 1q e^{i pMq / 12} i/4 x | Upg q 1 |0y

CS partition function on Lens Spaces

Operators Ð Lattice Ð Partition Function Ð Quadratic Forms

Discrete Data:



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Siegel-Weil Theorem

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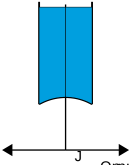
Poincaré Sum

E p q / ^pp q ^qp q ¸ 8 z x | Upgq ^1 |0y / ^ppg q ^qpg q

Continuous Data: M



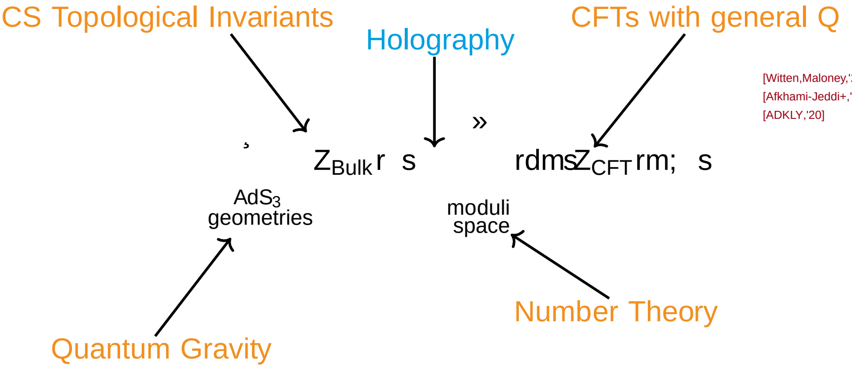
M : O pp; q; Zq Opp; q; Rq / Oppq Opqq

[ADKLY,'20]

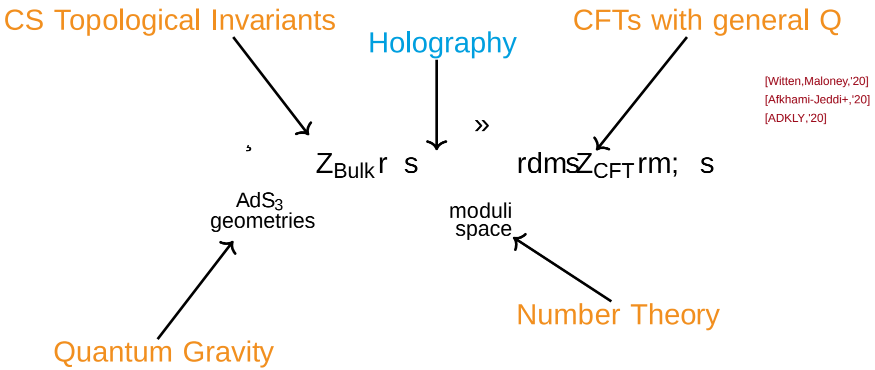
Holographic Interpretation:

Bulk theory looks like Abelian CS Theory with level matrix Q

Partition functions arise from di erent anyons encircling non-contractible cycle of M pc;dq

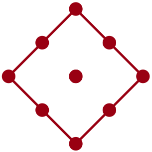


[Witten,Maloney,'20]
[Afkhami-Jeddi+,'20]
[ADKLY,'20]



What else is hiding in this correspondence?

S_{CS} » $Q_{IJ}\, A^I dA^J\, d^3x$



D {

Symmetries of a TQFT are automorphisms of its data
ùñ permutations of the anyonD that leave
TQFT data invariant modulo gauge transformations
(and maybe complex conjugation)

TQFT data for Abelian TQFTs is completely determined by nite Abelian groupD and topological spins

[Delmastro, Gomis, ,'19]

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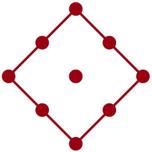

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0-form Global SymmetriesY : D Ñ D

Unitary	Anti-Unitary
Y p q Y p q Y p q pY p qq p q BpY p q, Y p qq Bp; q	Y p q Y p q Y p q pY p qq p q BpY p q, Y p qq Bp; q
Classical Y S_{CS} S_{CS}	Quantum Y S_{CS} S_{CS} but obey Ward Identities

G_0 : AutpD ; q t Y P Z^n ^{n}jQPQ $Y^T QY$ Qu{

If P 0 @representatives ofY s, then the symmetry is quantum

[Delmastro, Gomis, ,'19]

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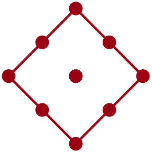

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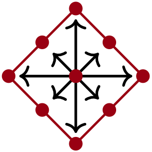

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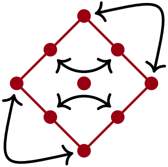

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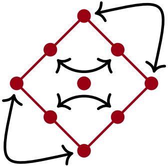

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If P 0 @representatives of Y s, then the symmetry is quantum

Are these present in ensemble averaged theories?

For classical & quantum unitary symmetries, a proof in a few lines:

$$E\ p\ q: \quad P \quad \underset{\substack{pc;dq\ 1 \\ c_i\ 0}}{,} \quad \frac{pc;dq}{pc \quad dq^{\frac{p}{2}}\, pc \quad dq^{\frac{q}{2}}}$$

$$pc;dq \quad {}_{;\,0}pg\ \ {}^{1}q \quad e^{\frac{i}{12}\ \ pMq}\ \ \frac{i}{4}\ x\ \ |\,Upgq\ {}^{1}|0y$$

For classical & quantum unitary symmetries, a proof in a few lines:

$$E(p,q): \quad \sum_{\substack{(c;d)=1 \\ c>0}} \frac{\rho(c;d)}{(c\tau+d)^{\frac{p}{2}}(c\bar{\tau}+d)^{\frac{q}{2}}}$$

$$\rho(c;d) \quad \rho(g^{-1}) \quad e^{\frac{i}{12}\Phi(M)} \quad \frac{i}{4} \quad x\,|U(g)^{-1}|0\rangle$$

A modular transformation on the Hilbert space can be decomposed as

$$U \quad S T^{m_\ell} S \ldots T^{m_1}$$ [Jeffrey,'92]

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$$U \quad ST^{m_{`}} S ::: T^{m_{1}}$$

[Je rey,'92]

But the generators

$$S \quad \frac{Bp; \quad q}{jD \quad j} \quad \& \quad T \quad p \quad qe^{\frac{i}{12}}$$

are invariant under unitary symmetry transformations. Then

$$Y \quad pc;dq \quad pc;dq$$

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These are Emergent Ensemble Symmetries

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$$E_Y\ \ p\ q\quad E\ p\ q$$

These are Emergent Ensemble Symmetries

Why?

Average Observable Op ; ; mq over m:

$$\Theta p ; \; q : x \, Op ; ; mqy \; \int_M^{\gg} Op ; ; mq \, dms$$

If there exists a transformation g such that

$$Op ; g \; ; g \; mq \quad Op ; ; mq$$

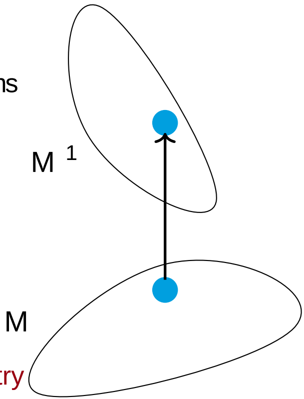
$$r \, dpg \; mqs \quad r \, dms$$

$$M^{1} \quad M$$

then

$$\Theta pg \quad q \quad \Theta p \; q$$

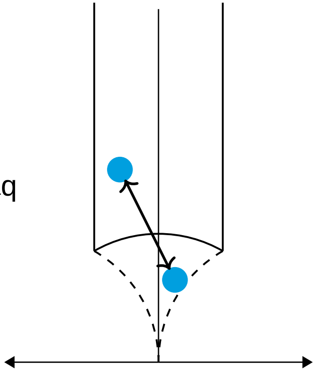
Basic notion of emergent ensemble symmetry



$\#_D p \; ; g \quad mq \qquad \#_D p \; ; mq$

$\quad \text{if } g \; P \; O p D ; D ; Z q$

$E_{D\{2} p \; q \qquad \frac{1}{2} p x \#_D p \; ; mq y_M \qquad x \; \#_D p \; ; mq y_{M \; 1} q$



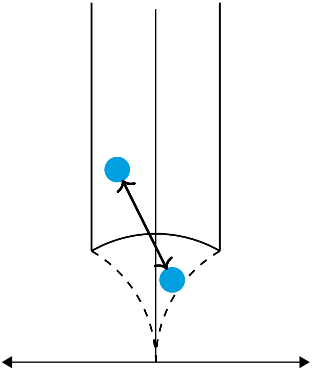
#Y p ;Y mq # p ;mq

if Y P O pp; q; Zq

"

*

O pp; q; Zq PGLpp q; Zq T Q Q



#$_Y$ p ;Y mq # p ;mq

if Y P O pp; q; Zq

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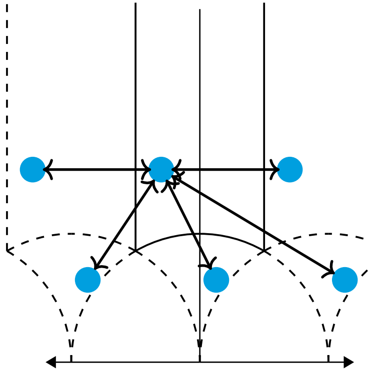
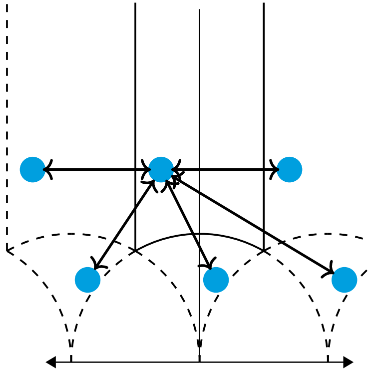
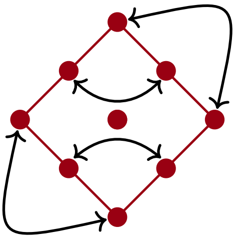
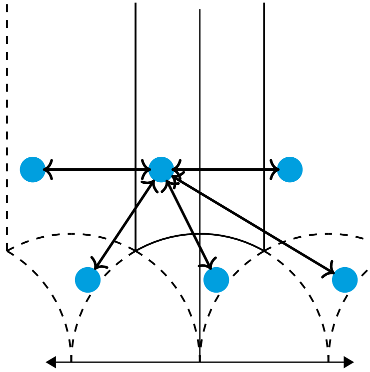






Siegel-Weil Theorem
transmutes
T-Duality
into
CS Global Symmetries



One problem: $Y \in PO(p,q;\mathbb{Z})$ implies that Y implements a classical symmetry

Can Quantum Symmetries be understood via origami?

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Can Quantum Symmetries be understood via origami?

Maybe

End of Part 2: Duality Origami

Part 1: Corfu '22

End of Part 2: Duality Origami

Part 1: Corfu '22

End of Part 2: Duality Origami

Preview for Part 3: The Fate of Emergent Ensemble Symmetries

What does all of this mean?

Swampland: Quantum Gravity should have no global symmetries

Are emergent ensemble symmetries in con ict with this notion?

Not quite: the bulk theory is not Einstein gravity, so no-gos do not apply

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Option A: Gauge the Global Symmetries

Option B: Introduce defects/corrections to break the symmetries

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Similar idea was pursued in [Benini+, '22] for global 1-form symmetries of Abelian CS

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How?

Potential answer: Go back to the theta functions

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Roelcke-Selberg Spectral Decomposition:

$$f(z) = \frac{1}{4\pi i}\int_{\mathrm{Re}(s)=\frac{1}{2}} \langle f ; E_s\rangle E_s(z)\,ds + \sum_{n \geq 0}^{\infty} \langle f ; \phi_n\rangle \phi_n(z)$$

[Terras]

Potential answer: Go back to the theta functions

Roelcke-Selberg Spectral Decomposition:

[Terras]

f p q 1 / 4 i » Repsq 1/2 pf ; E_s qE_s p qds ¸8 n 0 pf ; _n q _n p q

[Zagier, '81]
[Benjamin+, '22]

2 D{2 D p ; mq E D{2 p q 1 / 4 i » Repsq 1/2 ds s c/2 p s D{2qE pDq s D{2 pmqE_s p q

3 — 1 D{2 p 1 D{2qE pDq 1 D{2 pmq ¸8 n 1 p 2 D{2 D ; n q / p n ; n q n p q

Potential answer: Go back to the theta functions

Roelcke-Selberg Spectral Decomposition:

[Terras]

$$f p q \quad \frac{1}{4\ i} \text{»}_{\text{Repsq } \frac{1}{2}} \text{pf ; } E_s \text{q} E_s \text{p qds} \quad \text{¸}^{8}_{n \ 0} \text{pf ; } {}_n \text{q} \, {}_n \text{p q}$$

[Zagier, '81]
[Benjamin+, '22]

$$ {}^{D\{2}_{2} \, {}_{D} \text{p ; mq} \quad E_{D\{2} \text{p q} \quad \frac{1}{4\ i} \text{»}_{\text{Repsq } \frac{1}{2}} \text{ds} \ {}^{s \ \frac{c}{2}} \text{ p s} \quad D\{2 \text{q} E^{\text{p}D\text{q}}_{s} \ {}_{D\{2} \text{pmq} E_s \text{p q}$$

$$\frac{3}{} \ {}_{1} \ {}^{D\{2} \text{ p 1} \quad D\{2 \text{q} E^{\text{p}D\text{q}}_{1} \ {}_{D\{2} \text{pmq} \quad \text{¸}^{8}_{n \ 1} \frac{\text{p} \ {}^{D\{2}_{2} \ {}_{D} \text{; } {}_n \text{q}}{\text{p } {}_n \text{; } {}_n \text{q}} \ {}_n \text{p q}$$

Potential answer: Go back to the theta functions

Roelcke-Selberg Spectral Decomposition:

[Terras]

$$f\,\text{p}\;\;\text{q} \quad \frac{1}{4\;\; i} \text{»}_{\text{Re}\text{p}s\text{q}\;\;\frac{1}{2}} \text{p}f\,;E_s\text{q}E_s\text{p}\;\;\text{q}ds \quad \overset{8}{\text{¸}}_{n\;\;0} \text{p}f\,;\;\;{}_n\text{q}\;\;{}_n\text{p}\;\;\text{q}$$

[Zagier, '81]
[Benjamin+, '22]

$$\frac{D\{2}{2}\;{}_D\text{p}\;\;;m\text{q} \quad E_{D\{2}\text{p}\;\;\text{q} \quad \frac{1}{4\;\;i}\text{»}_{\text{Re}\text{p}s\text{q}\;\;\frac{1}{2}} ds\;\;{}^{s}\;\;{}^{\frac{c}{2}}\;\;\text{p}\;\;s\quad D\{2\text{q}E^{\text{p}D\text{q}}_{s}\;{}_{D\{2}\text{p}m\text{q}E_s\text{p}\;\;\text{q}$$

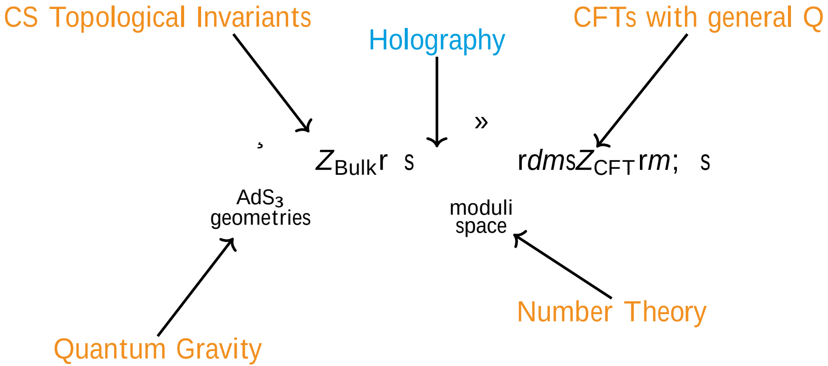
$$\frac{3}{}\;\;{}_1\;\;{}^{D\{2}\;\;\text{p}\;\;1\quad D\{2\text{q}E^{\text{p}D\text{q}}_{1}\;{}_{D\{2}\text{p}m\text{q}\quad \overset{8}{\text{¸}}_{n\;\;1}\frac{\text{p}\;\;{}^{D\{2}_{2}\;\;\;D;\;\;{}_n\text{q}}{\text{p}\;\;{}_{n;}\;\;{}_n\text{q}}\;\;{}_n\text{p}\;\;\text{q}$$

First term is ensemble average – interpret remainder as corrections? **1/2-Wormholes?**

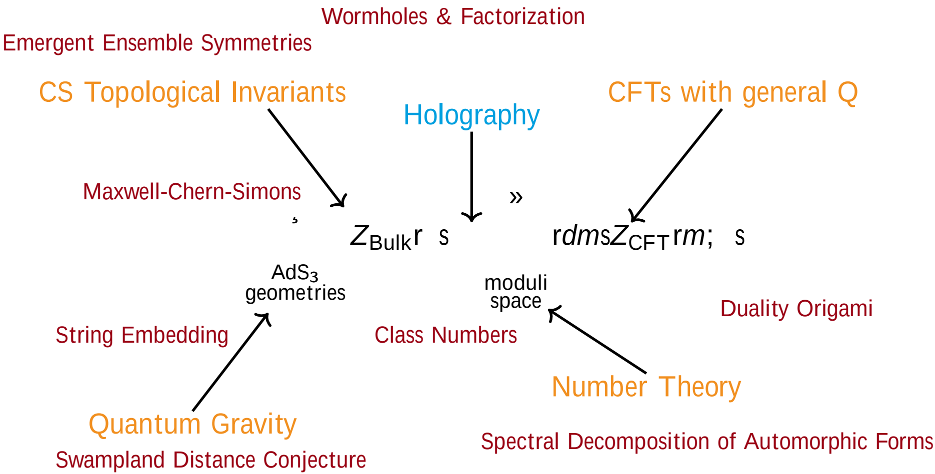
Does a decomposition exist for generalized Narain CFTs? **Partially**

Is the ensemble a particular limit, like N 4 SYM story of [Collier,Perlmutter, '22]?**Maxwell-CS?**

As above, so below



What else is hiding in this correspondence?



What else is hiding in this correspondence?

Google for donuts

Nicole Righi for original fundamental domain code

[arXiv:1904.12884] for original anyon braiding code