Ben-Gurion University of the Negev

New duality frames for the swampland distance conjecture

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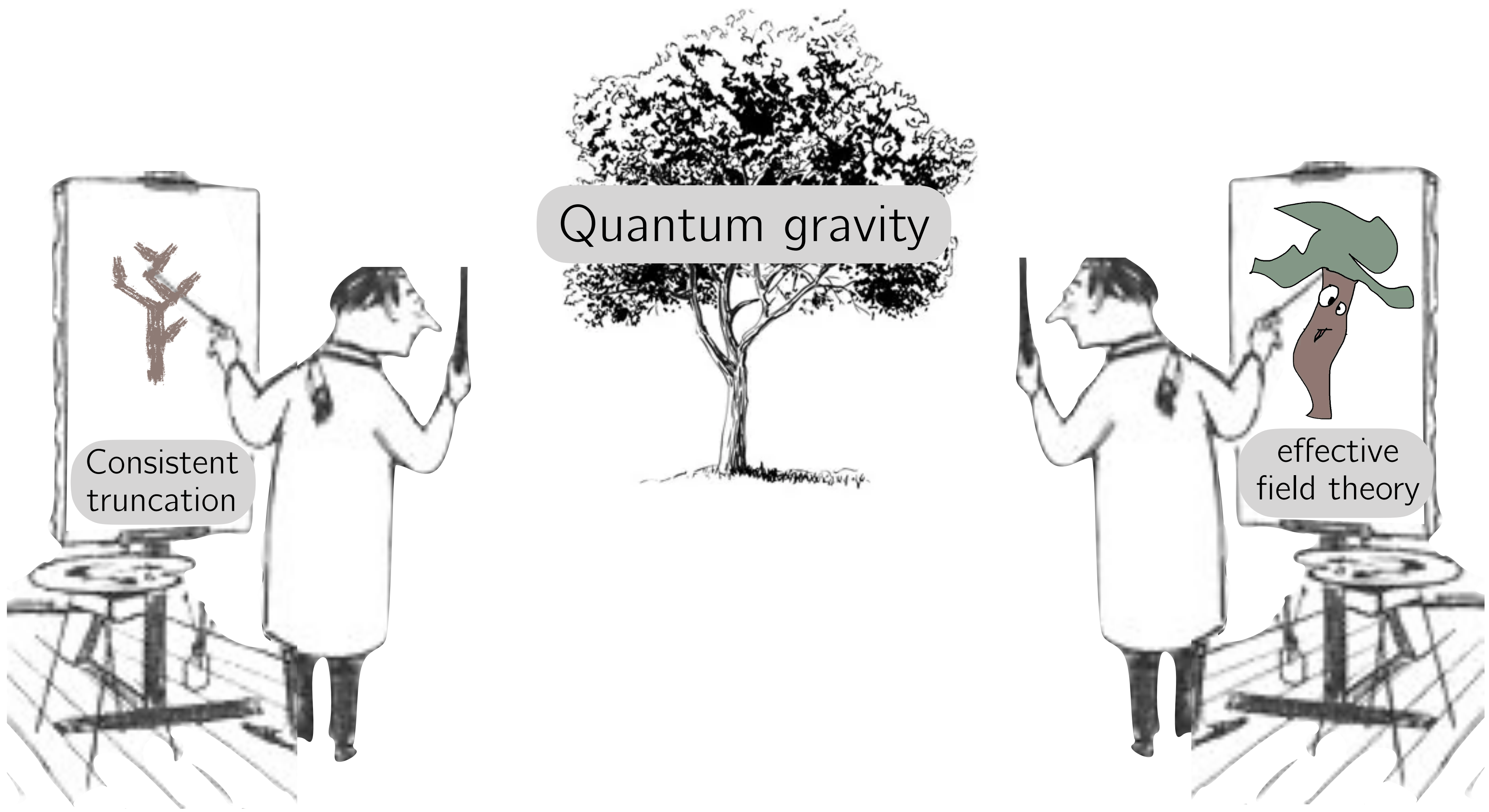
Work in progress in collaboration with
Dieter Lüst and Thomas Raml

DIP collaboration meeting
21st March 2023

Where are we heading to?

- Consistent truncations and generalised T-dualities
- The swampland distance conjecture
- T-duality and winding-momentum exchange
- A basic example

Consistent truncations in the Swampland program



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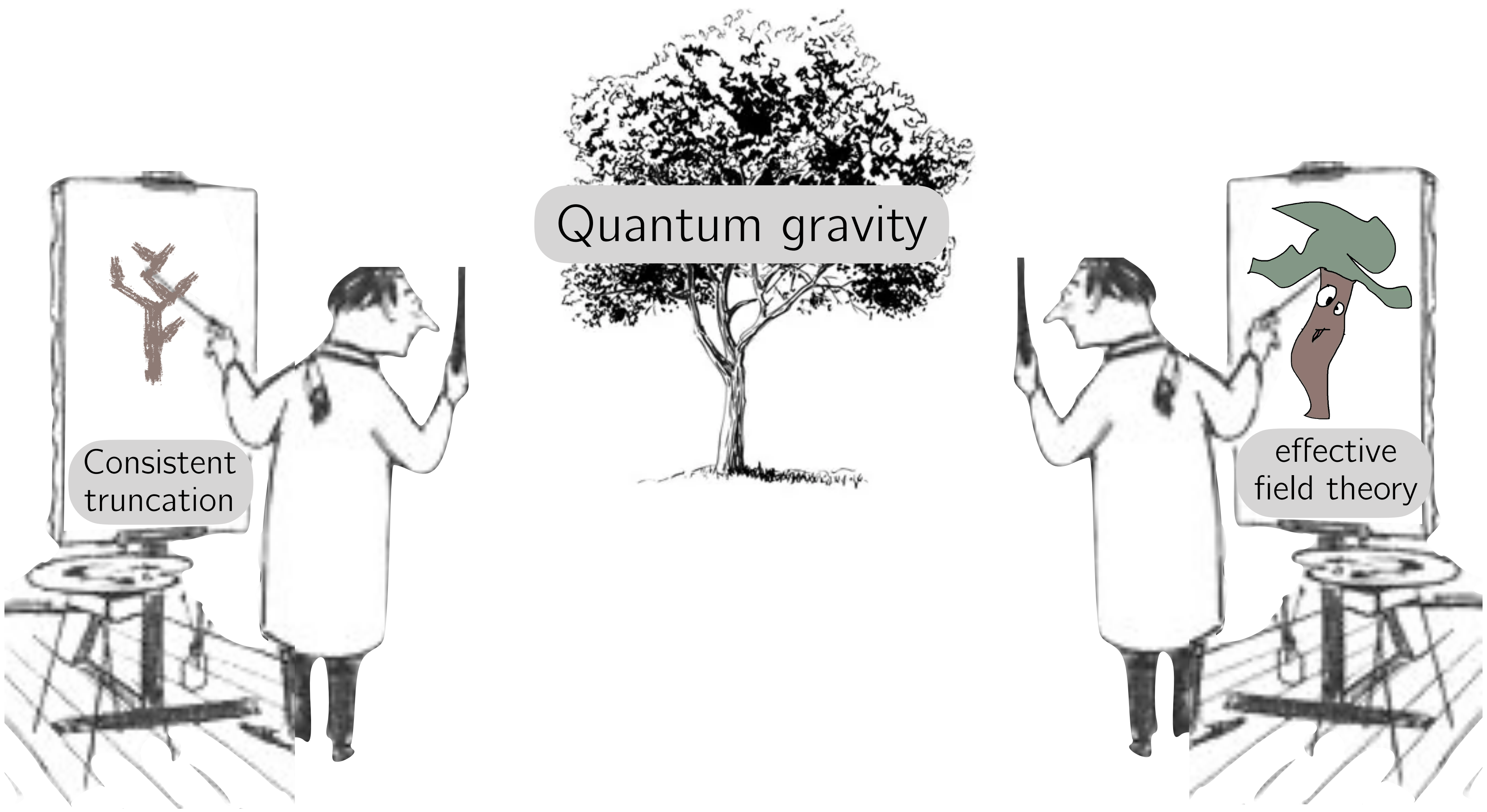
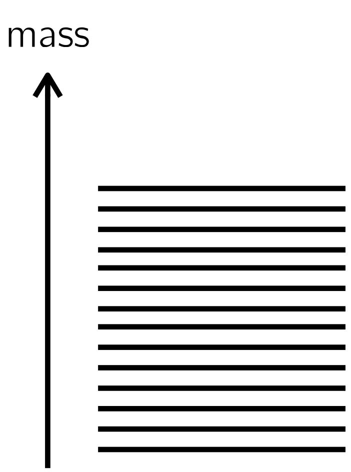


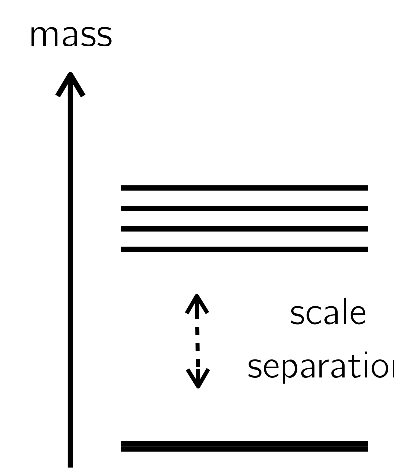


(usually)
<u>no separation of scale</u>
[Gauntlett, Varela]

Keep massive and massless modes

<u>scale separation</u>

Integrate out massive modes



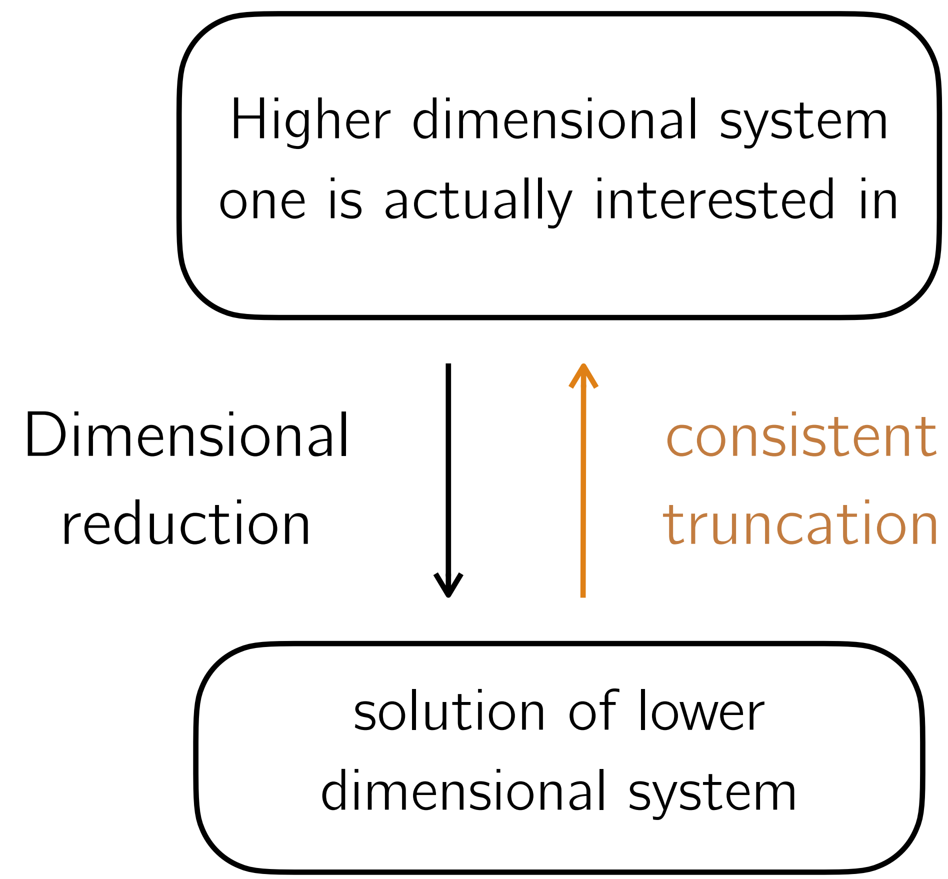
Consistent truncations

Solutions of a low dimensional truncation that
can be **automatically lifted** to the higher dimensional system

Total manifold = external x internal

$$\mathcal{M} = M_D \times M_{D-n}$$

external internal



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external internal

smart Ansatz:
single out dofs that **decouple from the rest**

$$\mathcal{A}(x, Y) = U(Y) A(x) U(Y)^T$$

internal coordinates (Y) — external coordinates (x)

Higher dimensional system one is actually interested in

Dimensional reduction ↓ ↑ consistent truncation

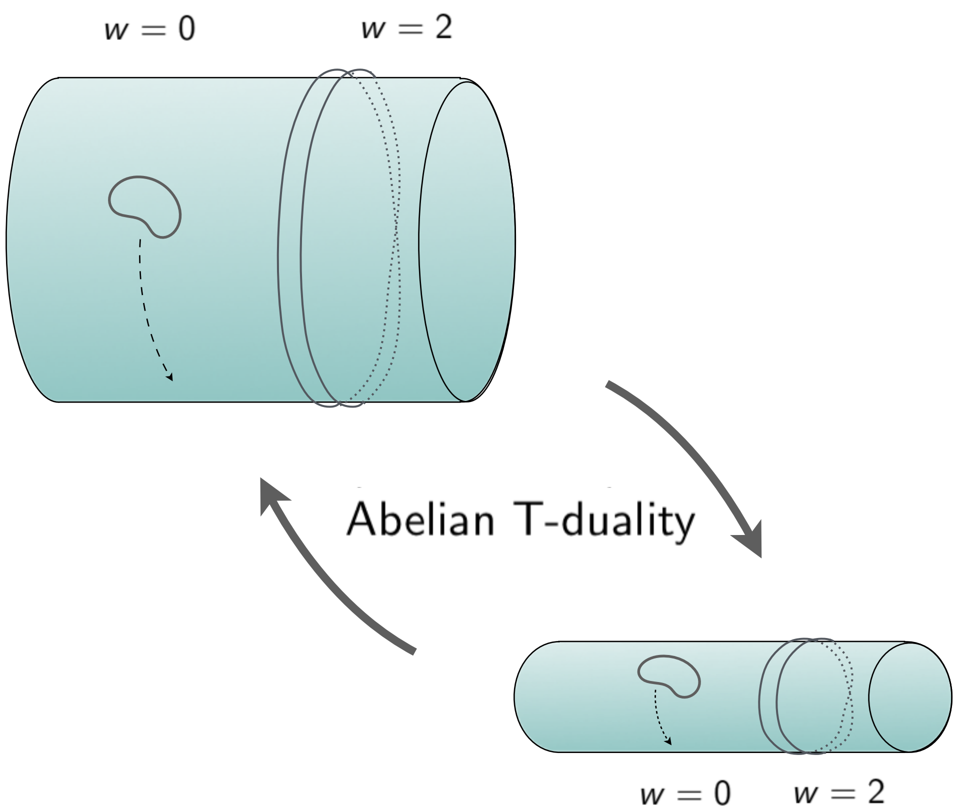
solution of lower dimensional system

In general highly non-trivial! **Guiding tool**: exceptional and generalised geometry

… and **generalised T-duality** ?

Generalised T-duality in under 2 mins

Abelian T-duality



- Compactification on a circle
- Apply T-duality

$$1/R \leftrightarrow R$$

$$\text{momentum modes} \leftrightarrow \text{winding modes}$$

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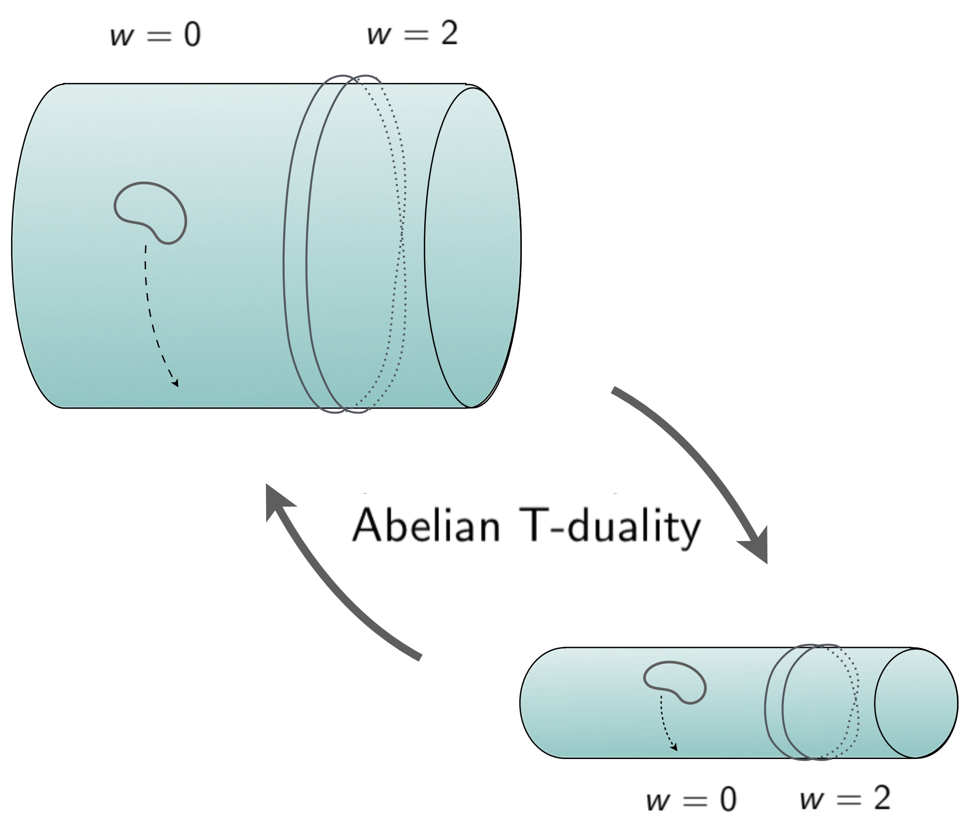

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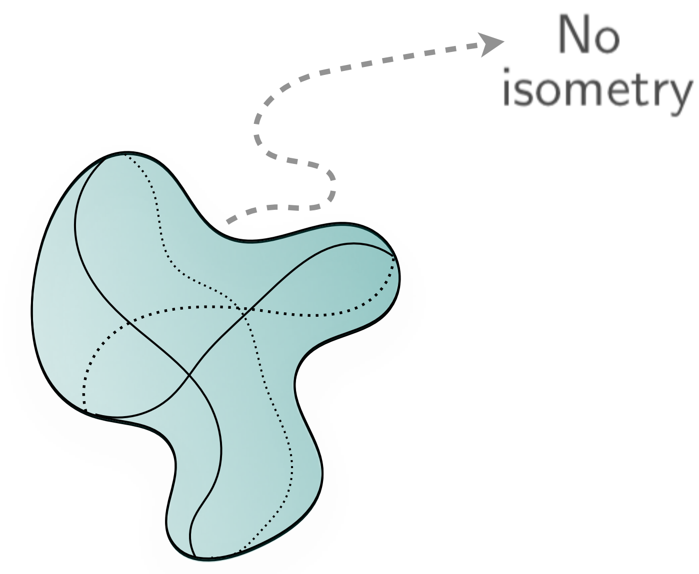

- Can we do something similar for a manifold with a non-Abelian structure ?

Generalised T-duality in under 2 mins

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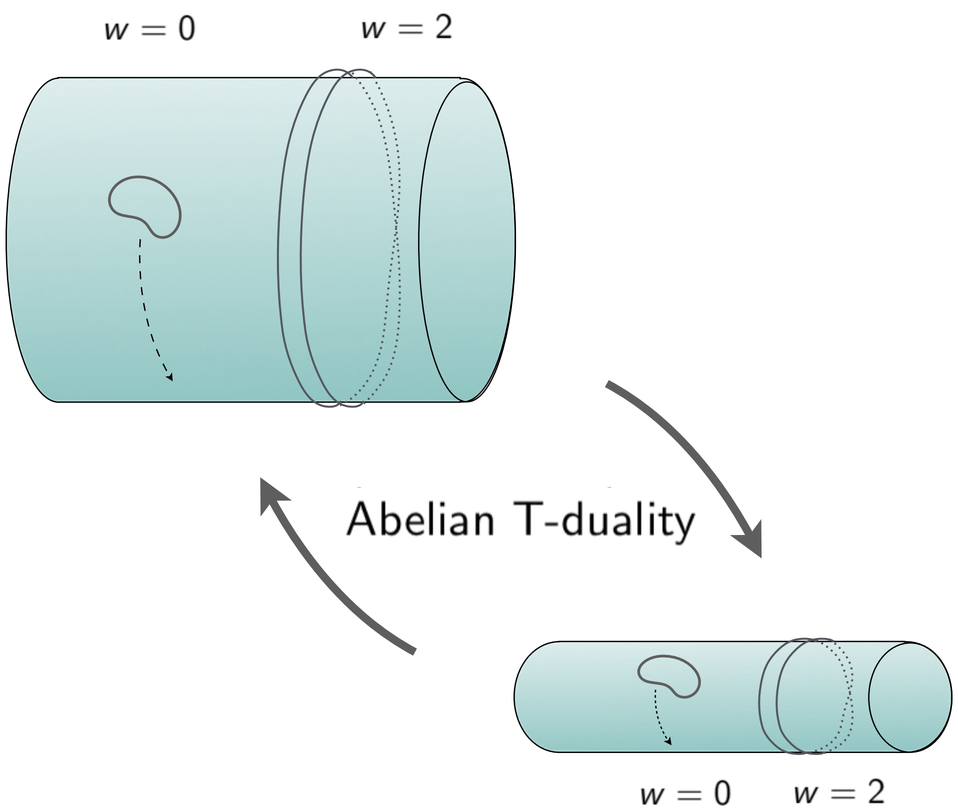

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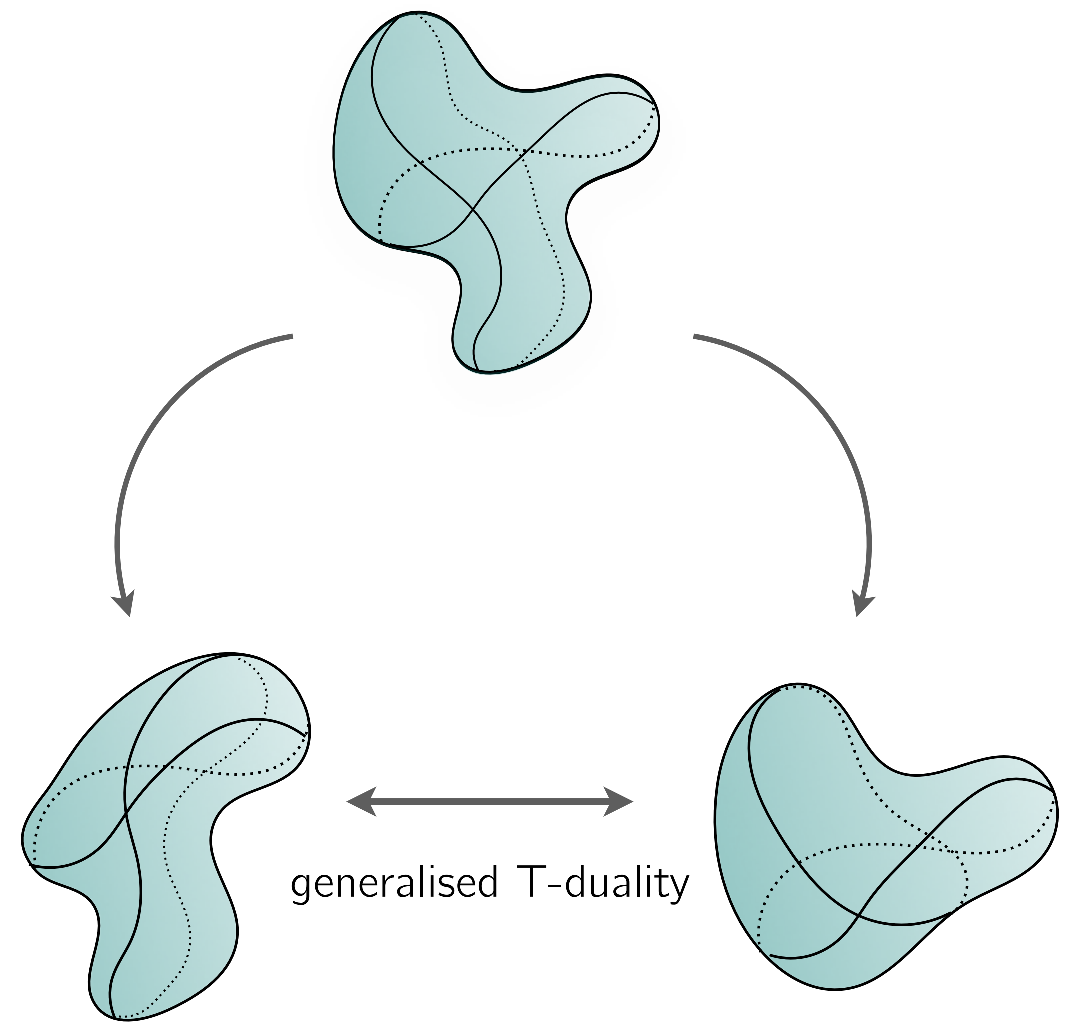
non-Abelian T-duality



Consistent truncations $\longleftarrow ? \longrightarrow$ generalised T-duality

[Lee, Strickland-Constable, Waldram] , [Cassani, Josse, de Felice, Malek, Petrini, Waldram], [Butter, Hassler, Pope, Zhang], ...

In many examples the **truncation Ansätze** are **related by generalised T-dualities**

$$\text{truncation Ansatz}_1 \longleftrightarrow \text{truncation Ansatz}_2 \longleftrightarrow \dots \longleftrightarrow \text{truncation Ansatz}_n$$

$$\text{Generalised T-duality} \underset{\Leftarrow}{\Rightarrow} \text{consistent truncations}$$

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Prompts the question:

What can we learn by applying generalised T-duality
in Swampland scenario examples ?

Here: apply in the context of the **distance conjecture**

The next slides are a basic intro swampland Distance Conjecture

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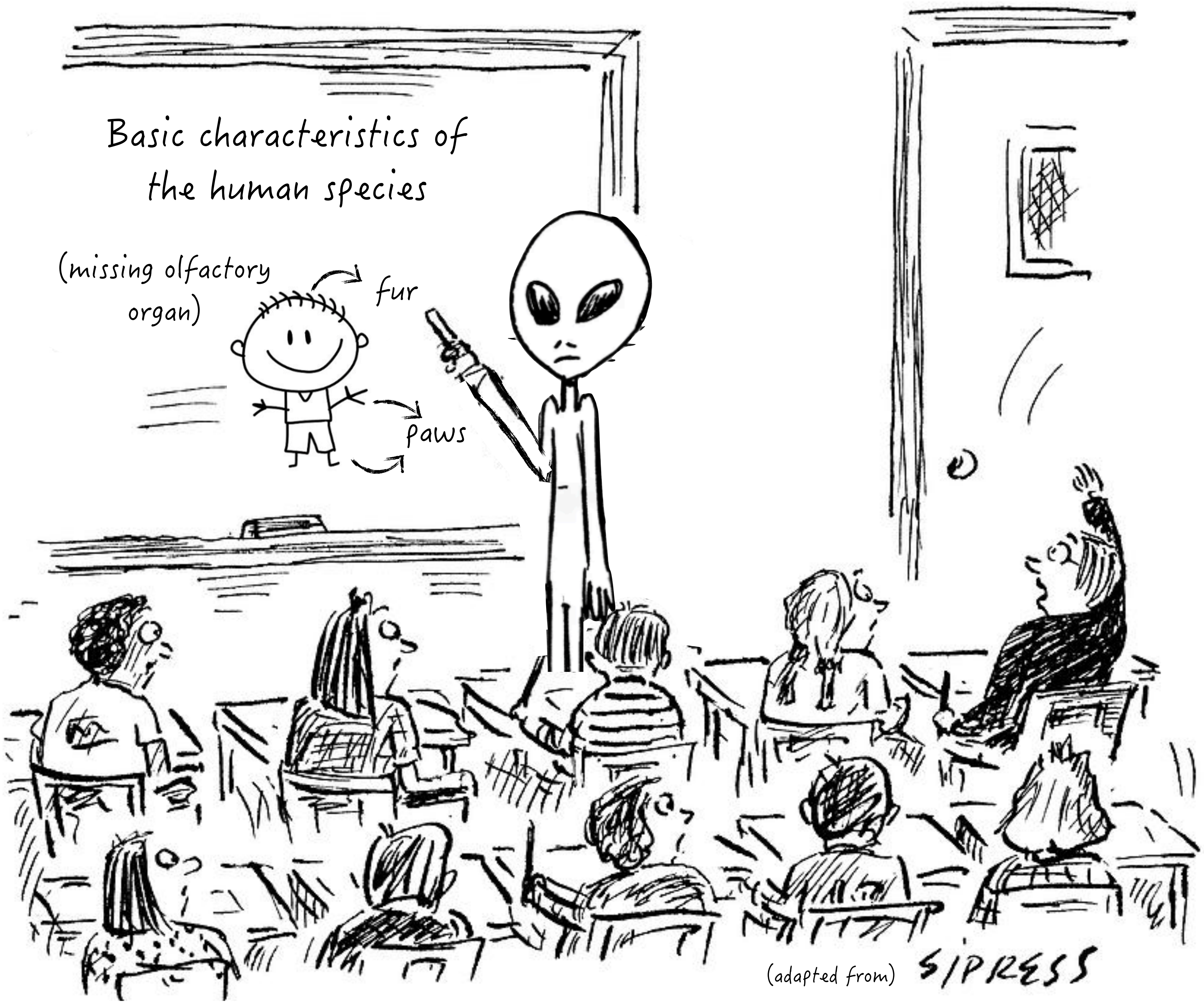
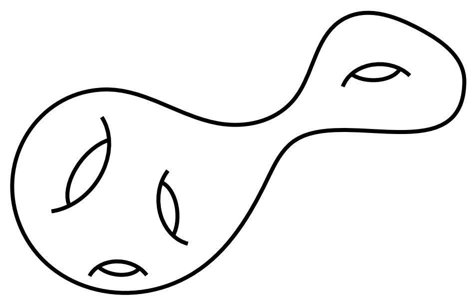
— the experts may want to consider the opportunity of taking a nap

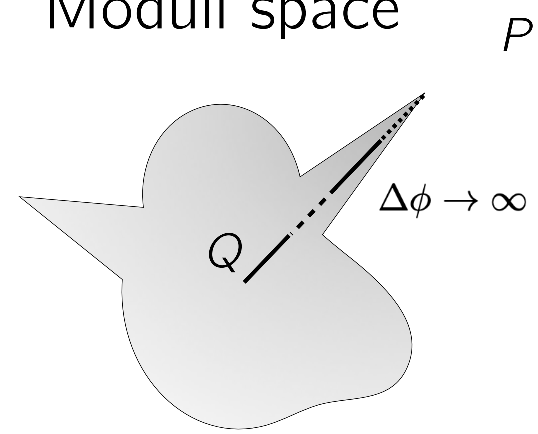


Distance conjecture

Target space manifold



Moduli space



In any consistent theory of quantum gravity:
When going to large distances in its moduli space,
encounter an infinite tower of particles which become light exponentially

[Ooguri, Vafa] ....

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty \quad \text{and} \quad \Delta\phi \equiv d(P,Q)$$

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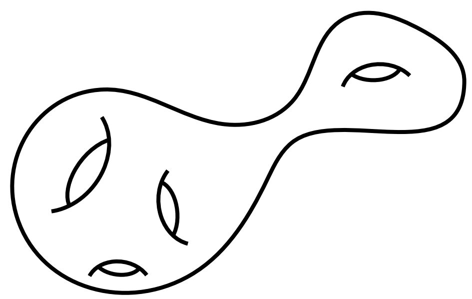

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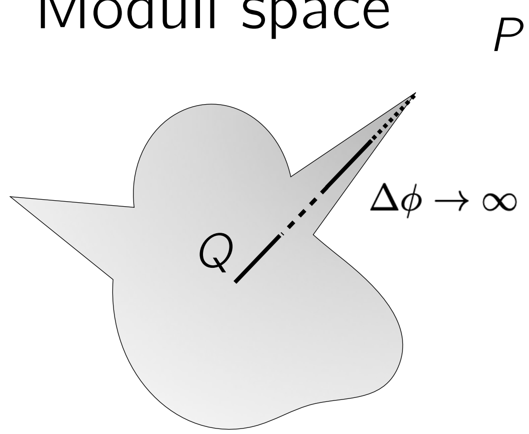

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$\rightarrow$ T-duality is closely related to the distance conjecture

Distance conjecture & non-Abelian T-duality

free boson on a circle $\mathcal{M}_D = M_{D-1} \times S^1$

Moduli space:

$R = 0$ infinite distance point

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At $R \to \infty$
Infinite tower of massless KK-modes

$$M_n^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \to 0$$

Infinite tower of massless states ✓

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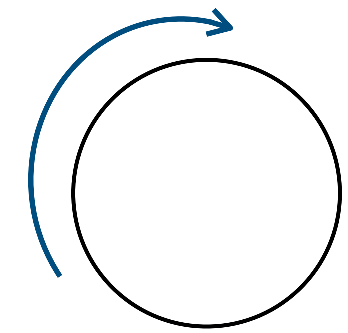

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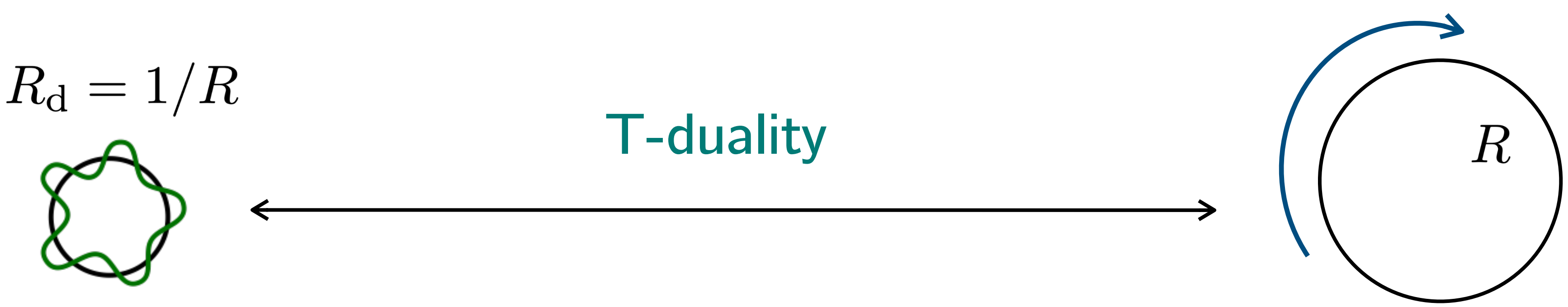

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Distance conjecture and (extensions of) T-duality

Crucially, works because $\mathbb{Z} \ni w \longleftrightarrow m \in \mathbb{Z}$

“perfect” winding-momentum exchange

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- an Abelian T-duality in presence of a non-trivial H-flux
  [Bouwknegt, Evslin, Mathai]
- a non-Abelian and Poisson-Lie T-duality
  [Klimčík, Ševera]
- the space is non-geometric
  [Hellerman, McGreevy, Williams], [Hull], [Dasgupta, Rajesh, Sethi],...

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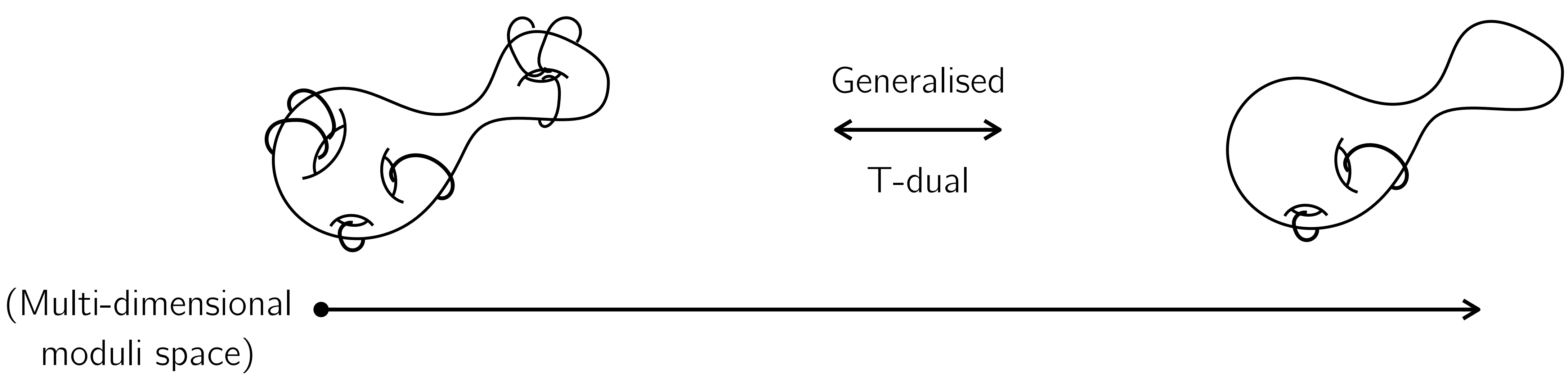

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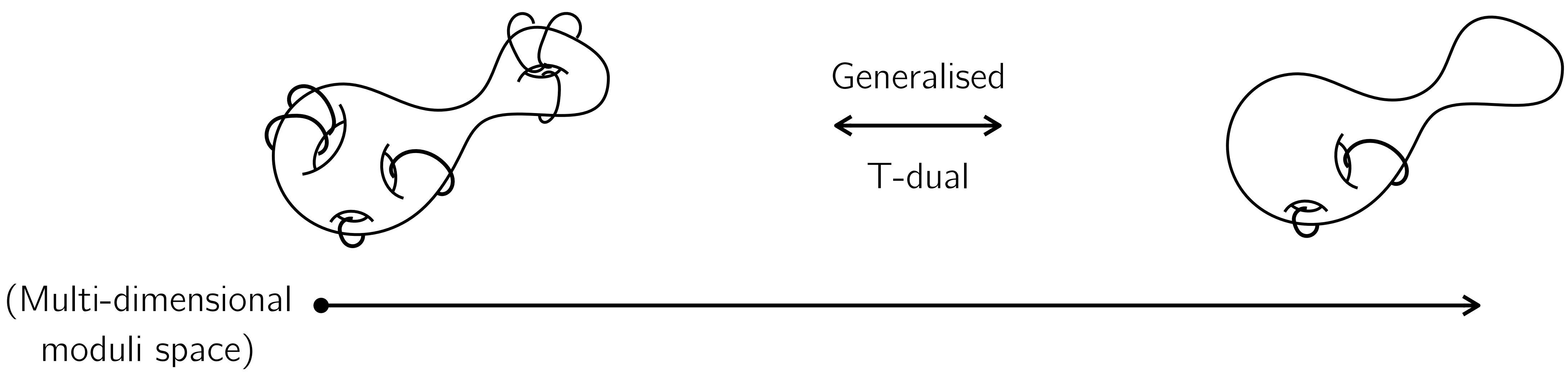
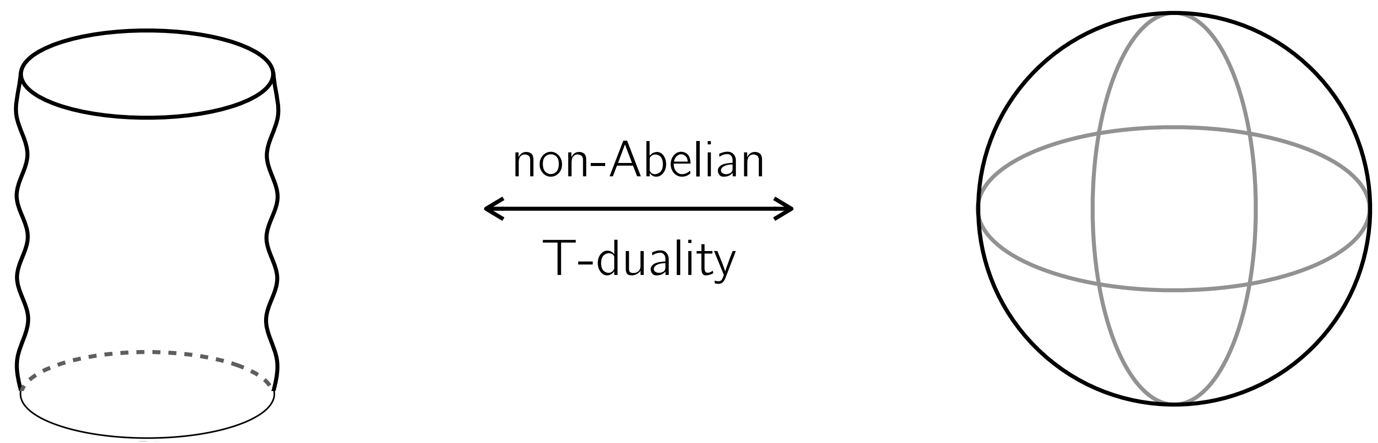


What happens to the tower of states in these new *generalised* T-duality frames ?

Distance conjecture & non-Abelian T-duality

$$\mathcal{M}_D = M_d \times N_n$$

We take to be a three-sphere $SU(2) \cong S^3$ and its non-abelian T-dual "T^3"



"T^3" $SU(2)$

a very "deformed" three-torus
no Abelian isometries left, still cycles though

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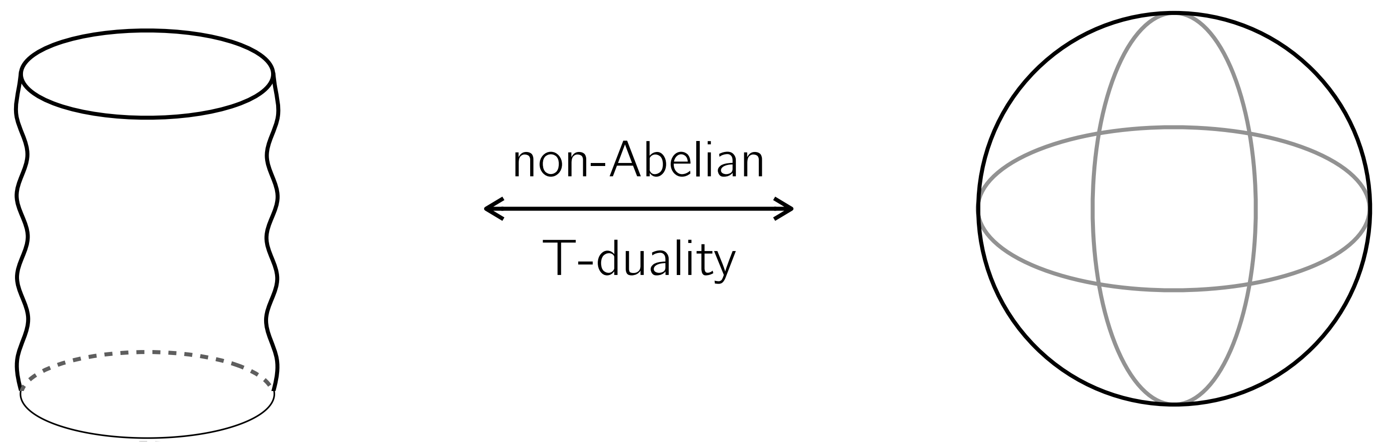

"T^3" $SU(2)$

$$U(1)^3 : w = \mathbb{Z}^{\oplus 3} \text{ and } m = 0 \quad \longleftrightarrow \quad SU(2) : w = 0 \text{ and } m = \mathbb{Z}^{\oplus 3}$$

[Klimčík, Ševera]

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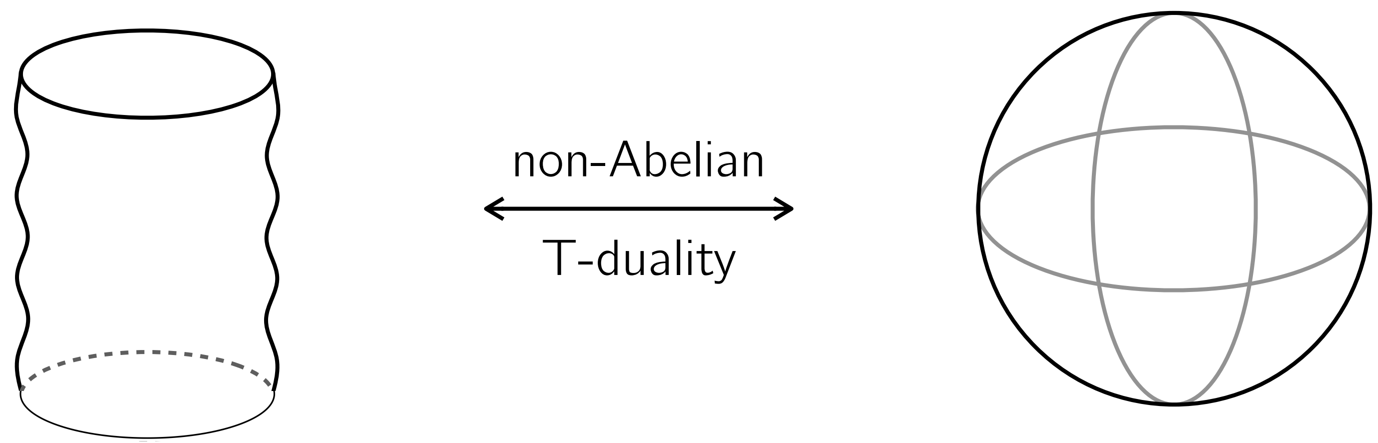

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[Klimčík, Ševera]

- ▷ **New phenomenon**: winding/(certain) momentum modes are forbidden *!*
- ▷ What does that imply for the validity of these theories within the SDC ?

Distance conjecture & non-Abelian T-duality

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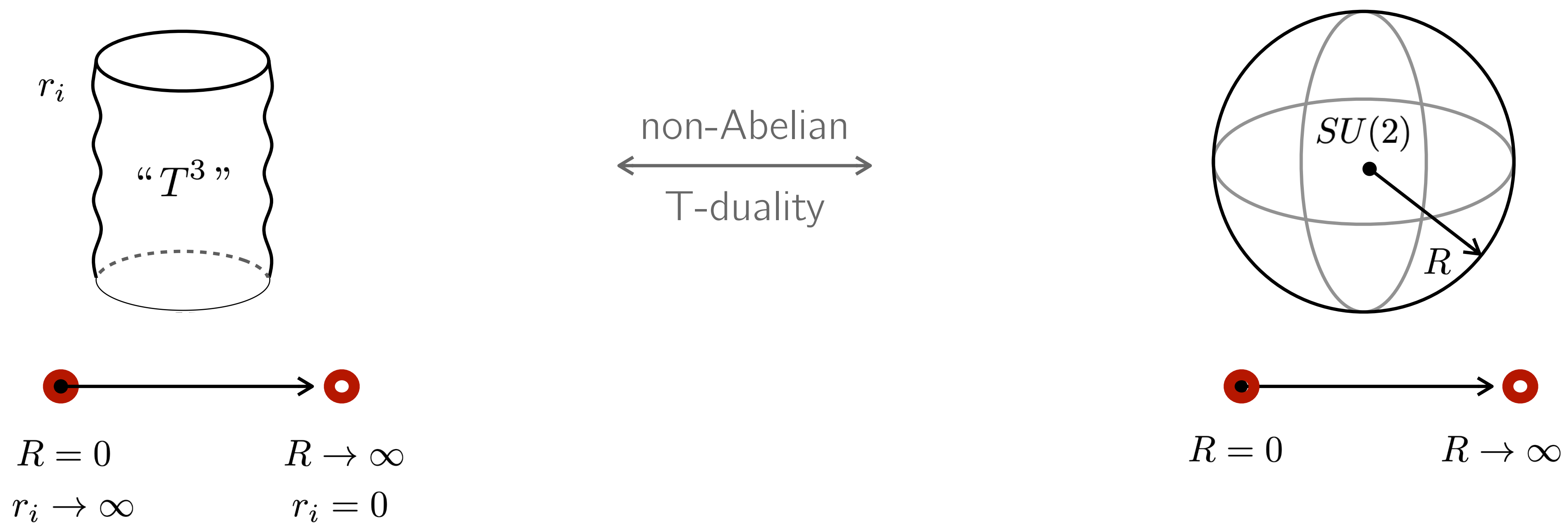

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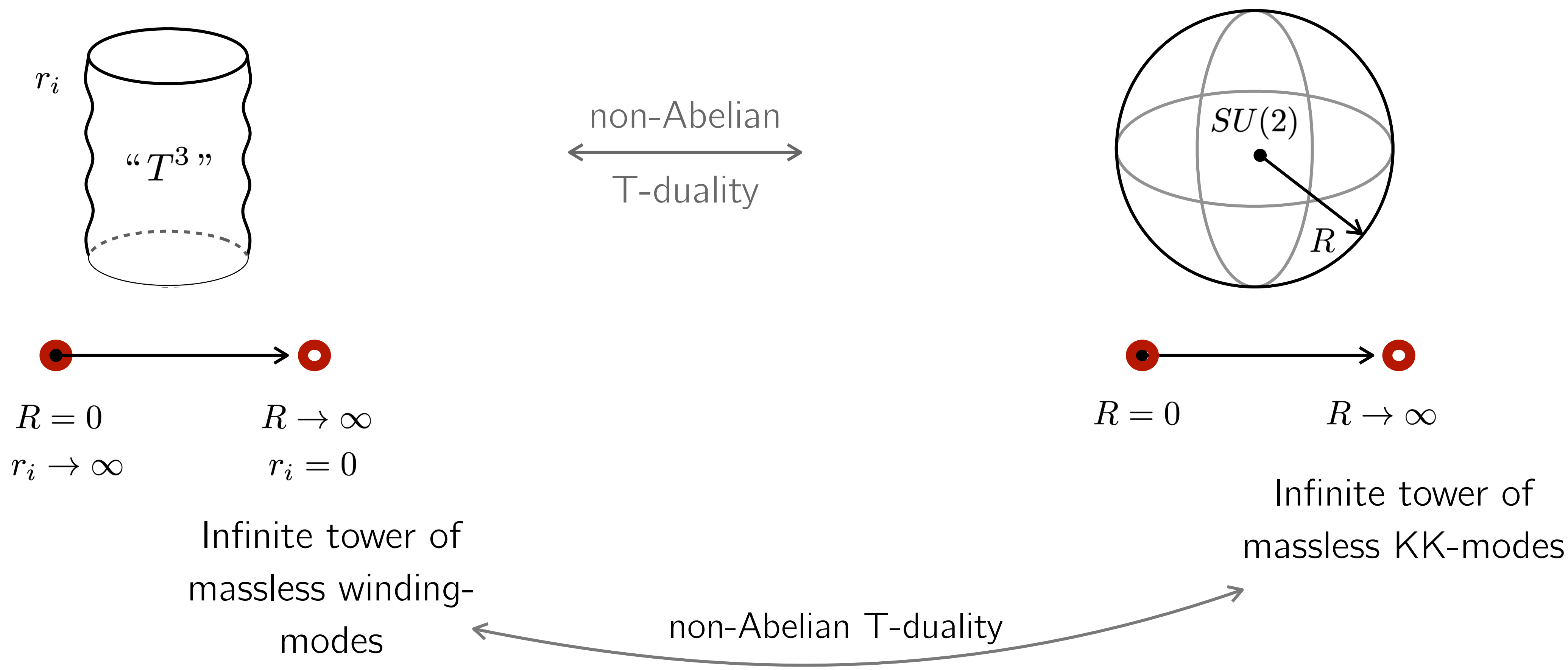

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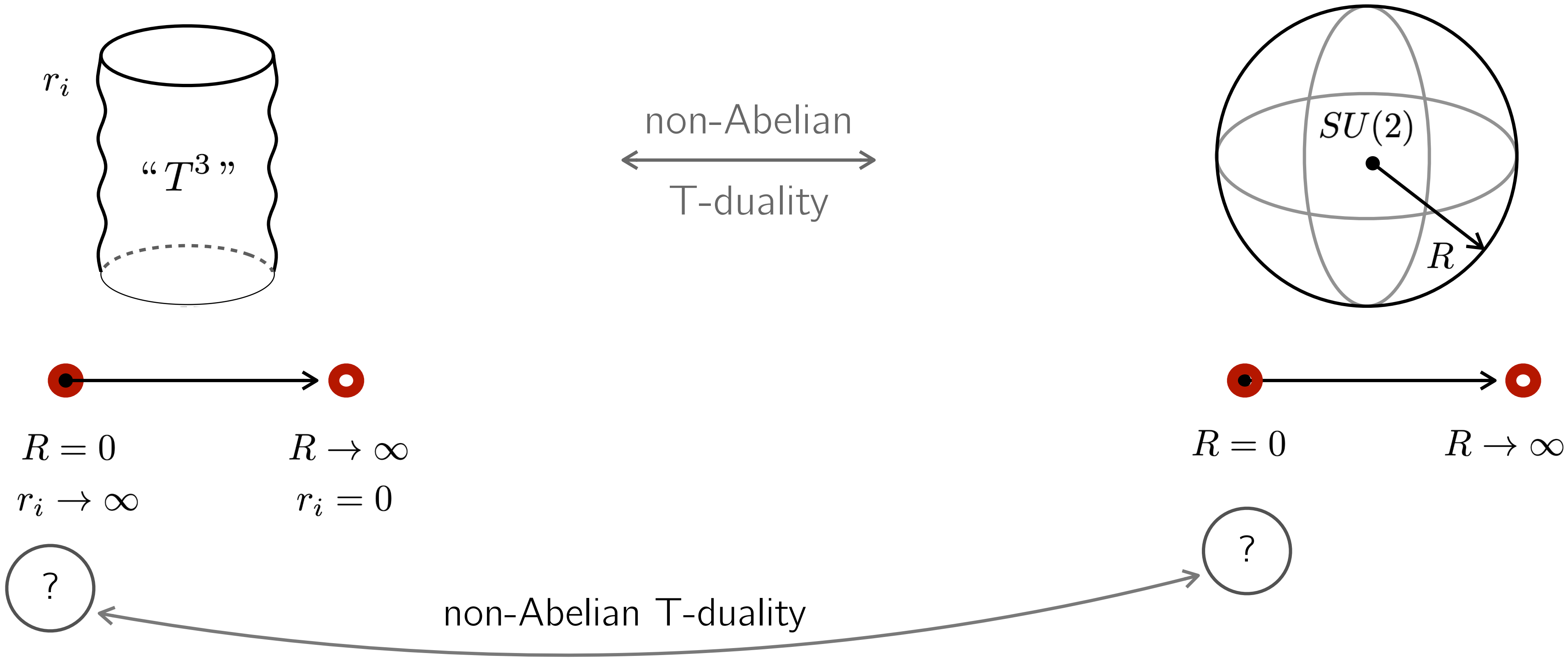

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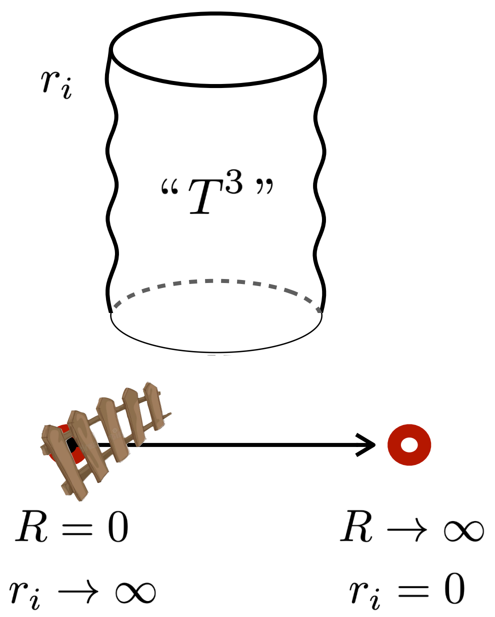
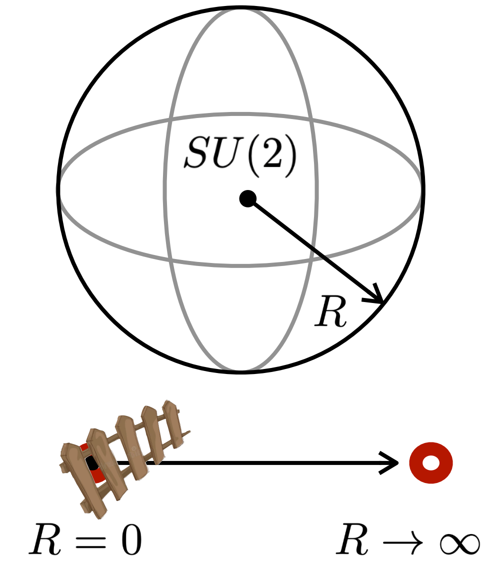


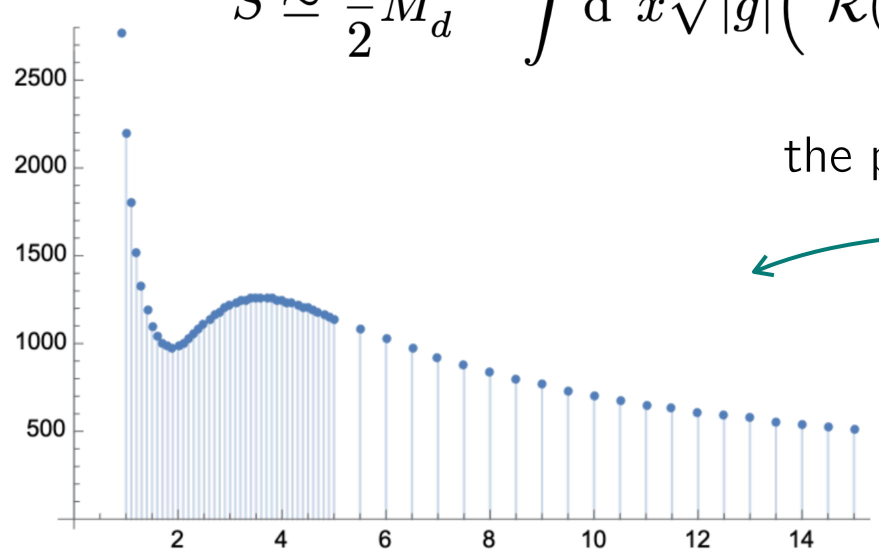
non-Abelian ⟷ T-duality



$$S \simeq \frac{1}{2} M_d^{d-2} \int \mathrm{d}^d x \sqrt{|g|} \left(\mathcal{R}(g) - \frac{\alpha}{R^2} (\partial R)^2 + V(R) \right)$$

the potential is non-vanishing

$R = 0$
is not accessible !



$\rightarrow$ No need for a tower of light states !

Summary

- ▷ Compactifications beyond the simple circle
- ▷ Strange things happen when looking at more general manifolds and their properties under generalised T-duality
- ▷ New types of winding-momentum exchange
- ▷ What does that imply or tell us about (generalised) T-duality and the Distance Conjecture ?


Thank you for
your attention !

Appendix

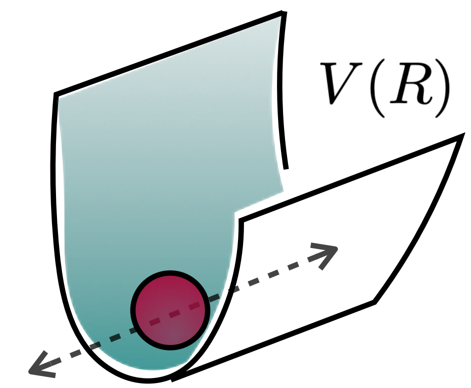
Distance conjecture and potentials

Distance conjecture and potentials

$$S = \frac{1}{2} M_D^{D-2} \int \mathrm{d}^d x \mathrm{d}^3 y \sqrt{|G|} \mathcal{R}(G)$$

$$\downarrow$$

$$S = \frac{1}{2} M_d^{d-2} \int \mathrm{d}^d x \sqrt{|g|} \Big(\mathcal{R}(g) - \frac{\alpha}{R^2} (\partial R)^2 + V(R) \Big)$$



No longer a true moduli space: preferred flat directions

Distance conjecture appears to also apply to fields with non-vanishing potentials

[Baume, Palti] [Klaewer, Palti] [Lüst, Palti, Vafa]

“Reversed reasoning” $\rightarrow$ puts constraints on allowed potentials

For the SDC to hold, should be impossible to generate a potential with trajectories is sufficiently non-geodesic so that the exponential behaviour of the tower is violated

[Calderón-Infante, Uranga, Valenzuela]

Consistent truncations in the Swampland program

Conjecture: all supersymmetric AdS supergravity vacua feature no scale separation but however admit a consistent truncation

[Lüst, Patti, Vafa],[Buratti, Calderon, Mininno, Uranga], [Cribiori, Dall'Agata], ...

Conjecture: gauged supergravities with AdS vacua which is not constructed from a consistent truncations
$\rightarrow$ must live in the "swampland" [Josse, Malek, Petrini, Waldram]

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Conjecture: gauged supergravities with AdS vacua which is not constructed from a consistent truncations
$\rightarrow$ must live in the "swampland" [Josse, Malek, Petrini, Waldram]

$\rightarrow$ explore all possible consistent truncations
and their relations to the swampland conjectures

Details HE-action to E-frame

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + h(x,y)_{ij}\mathrm{d}y^i\mathrm{d}y^j$$

$$h(x,y) = \frac{1}{R^2(R^4+\phi^2+\psi^2+\theta^2)}\begin{pmatrix} R^4+\phi^2 & \phi\psi & \theta\phi \\ \phi\psi & R^4+\psi^2 & \theta\psi \\ \theta\phi & \theta\psi & R^4+\theta^2 \end{pmatrix}$$

$$\begin{aligned} S &= \frac{1}{2}M_D^{D-2}\int \mathrm{d}^d x \mathrm{d}^3 y \sqrt{|G|}\mathcal{R}(G) \\ &= \frac{1}{2}M_D^{D-2}\int \mathrm{d}^d x \mathrm{d}^3 y \sqrt{|g|}\Big(\mathcal{R}(g) - \hat{A}(\partial R)^2 + \delta^{-2/(d-2)}\mathcal{R}(h)\Big) \end{aligned}$$

$$\begin{aligned} \text{at } R = 0 :& \hat{A} \simeq \frac{1}{R^2}\frac{9d-17}{d-2} + R^2 \ldots \\ \text{at } R = \infty :& \hat{A} \simeq \frac{1}{R^2}\frac{9(d-1)}{d-2} + \frac{1}{R^6} \ldots \end{aligned}$$

$$S = \frac{1}{2}M_d^{d-2}\int \mathrm{d}^d x \sqrt{|g|}\Big(\mathcal{R}(g) - \frac{\alpha}{R^2}(\partial R)^2 + V(R)\Big)$$

$$S \simeq \frac{1}{2}M_d^{d-2}\int \mathrm{d}^d x \sqrt{|g|}\Big(\mathcal{R}(g) - \frac{1}{2}(\partial \Psi)^2 + V(\Psi)\Big)$$

Topological T-duality

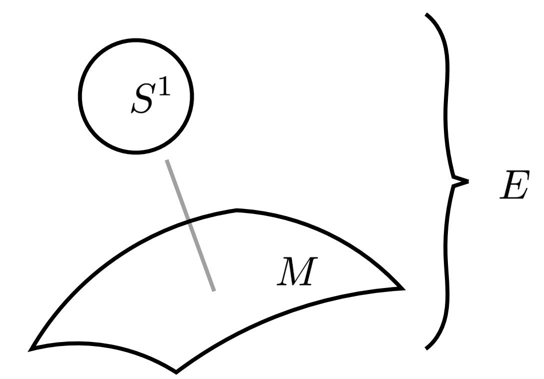
Topological T-duality

→ version of T-duality that only keeps track of the topological properties

Starting point:
circle fibered over a base manifold

$$M \cong \hat{M}, \quad F = \int_{\hat{S}^1} \hat{H}, \quad \hat{F} = \int_{S^1} H$$

$$\begin{array}{ccc} S^1 & \longrightarrow & E \\ & & \downarrow \\ & & M \end{array}$$



$$\begin{array}{ccc} & (E \times_M \widehat{E}, [H] = [\widehat{H}]) & \\ \swarrow & & \searrow \\ (E, H) & & (\widehat{E}, \widehat{H}) \\ \searrow & & \swarrow \\ & M & \end{array}$$

[Bouwknegt, Evslin, Mathai]

In general, total space admits non-trivial H-flux

Under T-duality, the topology is changed

$$\imath_v H \quad \longleftrightarrow \quad c_1(E_v)$$

background H-flux ⟷ Chern-class

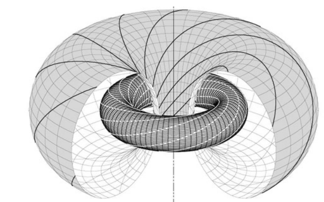
(type of fibration)

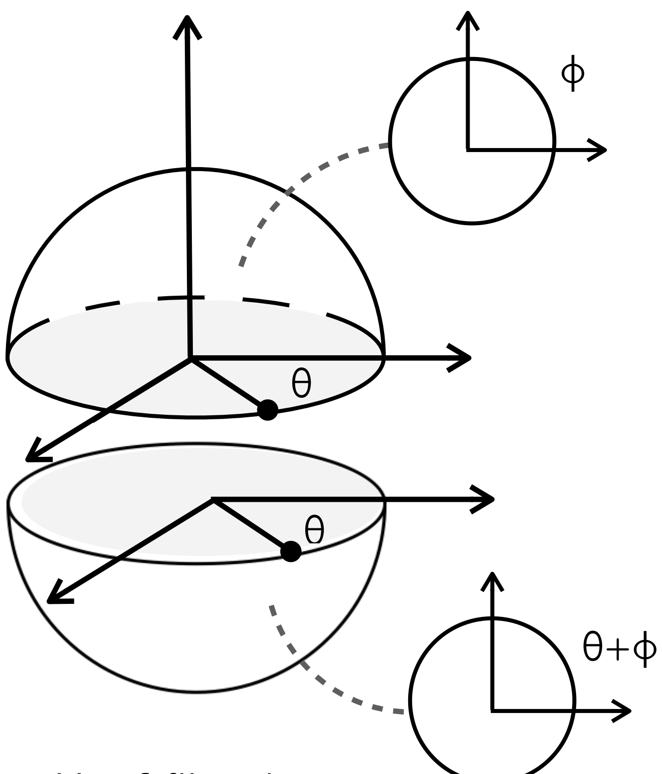
→ change in topology !

An example: Hopf fibration and H-flux

Take Type II compactified on a 3-sphere crossed with an (irrelevant) 7-manifold

Instead of trivially fibbing the circle S^1 over S^2, choose the Hopf fibration





Hopf fibration

Taking initially no H-flux we have

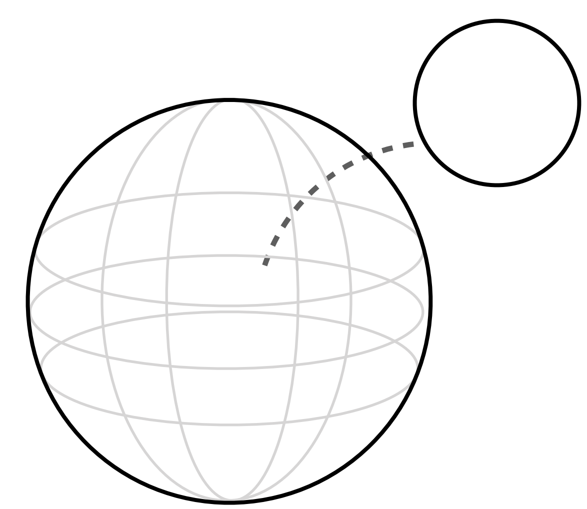
$$E = S^3_{\mathrm{Hopf}} \qquad H = 0 \qquad F = [1] \qquad H^2(S^2) = \mathbb{Z}$$

T-duality

$$\imath_v H \longleftrightarrow c_1(E_v)$$

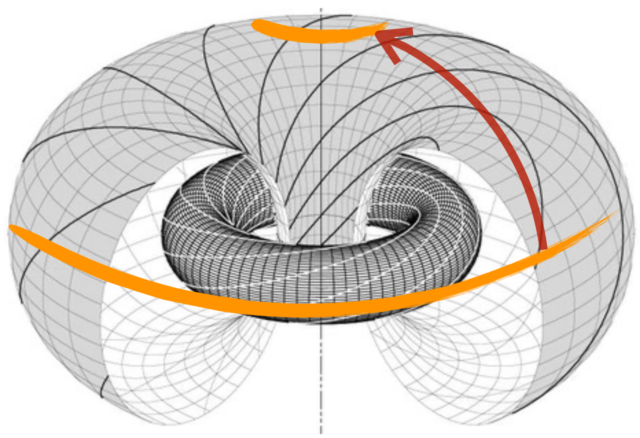
leads to the trivial fibration with non-trivial H-flux

$$\widehat{E} = S^2 \times S^1 \qquad H = [1] \qquad F = [0] \qquad H^3(S^2 \times S^1) = \mathbb{Z}$$



Trivial fibration $S^2 \times S^1$

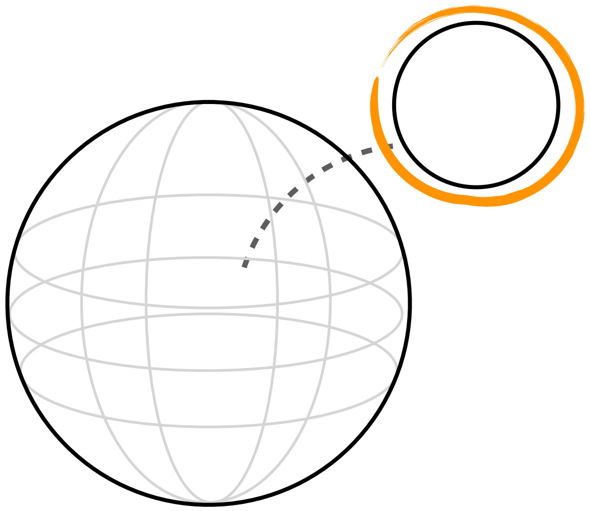
Winding and momentum exchange ?



Hopf fibration

T-duals

↔



Trivial fibration $S^2 \times S^1$

No winding
(all closed loops are contractible)

A whole $\mathbb{Z}$-worth of winding

Option 1: even in the Abelian case, when there is funky fibration or non-trivial H-flux, the winding-momentum exchange is flawed

Option 2: these cases are not valid string theory backgrounds

Option 3: there are no momentum modes to be exchanged with

Poisson-Lie T-duality details

T-duality and its generalisations

The sigma-model characterisation of T-duality

$$S = \int \mathrm{d}^2\sigma (G_{ij} + B_{ij}) \partial_\mu X^i \partial^\mu X^j = \int \mathrm{d}^2\sigma E_{ij} \partial_\mu X^i \partial^\mu X^j \,, \qquad J_{a,\pm} = k_a{}^i E_{ij} \partial_\pm X^j$$

Abelian T-duality	Abelian isometry	*exact symmetry of string theory*
$[k_a, k_b] = 0$	$L_{k_a} E_{ij} = 0$	$\mathrm{d} \star J_a = 0$
non-Abelian T-duality	non-Abelian isometry	*solution generating technique*
$[k_a, k_b] = f_{ab}{}^c k_c$	$L_{k_a} E_{ij} = 0$	$\mathrm{d} \star J_a = 0$
Poisson-Lie T-duality	non-Abelian ~~isometry~~	*? (and rest of the talk)*
$[k_a, k_b] = f_{ab}{}^c k_c$	$L_{k_a} E_{ij} = \tilde{f}^{bc}{}_a k_b{}^m E_{mi} E_{jn} k_c{}^n$	$\mathrm{d} \star J_a = \tilde{f}^{bc}{}_a J_b \wedge J_c$

Has a **natural algebraic interpretation** $\rightarrow$ G fits into a **Drinfel'd double** $D = G \cdot \widetilde{G}$

Jargon: G and $\widetilde{G}$ are called Poisson-Lie groups

Poisson-Lie group lingo

Drinfel'd double

$$\mathrm{alg}(D) = \mathfrak{d} = \tilde{\mathfrak{g}} \oplus \mathfrak{g} \qquad [T_A, T_B] = F_{AB}{}^C T_C \qquad \text{where} \qquad T_A = (\widetilde{T}^a, T_a)$$

- with an ad-invariant inner-product $\langle \bullet , \bullet \rangle$, with respect to which G and $\widetilde{G}$ are isotropics

$$\langle T_A, T_B \rangle = \eta_{AB} \qquad \langle T_a, T_b \rangle = 0 \quad \text{and} \quad \langle \widetilde{T}^a, \widetilde{T}^b \rangle = 0$$

- defined by the commutation relations

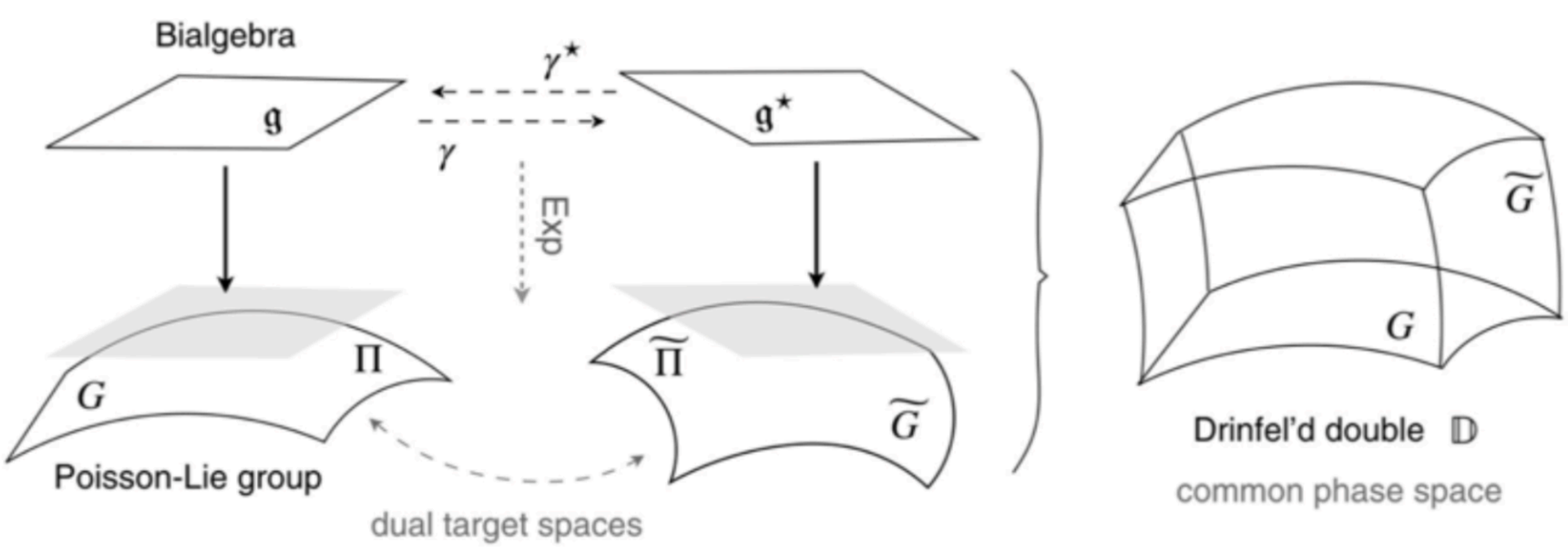
$$\tilde{\mathfrak{g}} : \; [\widetilde{T}^a, \widetilde{T}^b] = \tilde{f}^{ab}{}_c \widetilde{T}^c \qquad \mathfrak{g} : \; [T_a, T_b] = f_{ab}{}^c T_c$$

$$\text{mixed relations:} \quad [\widetilde{T}^a, T^b] = \tilde{f}^{ac}{}_b T^c + f_{bc}{}^a \widetilde{T}^c$$

Poisson-Lie groups

- at the level of the groups: G and $\widetilde{G}$ admit natural Poisson bi-vectors

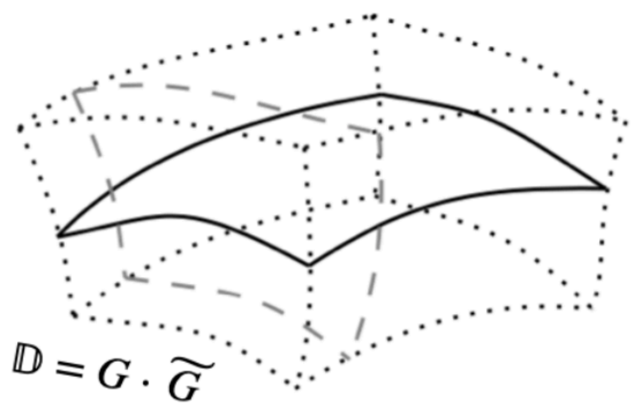
$$d_e \Pi = [\,,\,]^T_{\tilde{\mathfrak{g}}} \,, \qquad d_e \widetilde{\Pi} = [\,,\,]^T_{\mathfrak{g}}$$



$\mathscr{E}$-models: PL dual pair 'factory'

First order formalism on the double $\quad \mathbb{D} = G \cdot \widetilde{G}$

with an idempotent and ad-invariant operator $\mathscr{E} : \mathfrak{d} \to \mathfrak{d}$

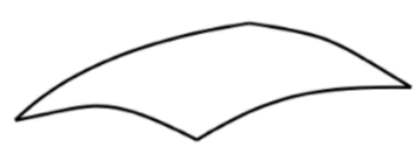


$$S_{\mathscr{E}} = \int \mathrm{d}\alpha - \mathrm{d}t \, \mathrm{Ham}_{\mathscr{E}}$$

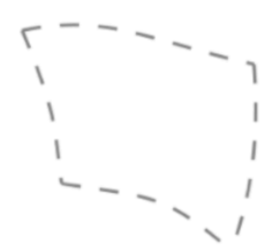
$$\mathrm{Ham}_{\mathscr{E}} = \oint \mathrm{d}\sigma \langle J(\sigma), \mathscr{E} J(\sigma) \rangle \, , \qquad J(\sigma) \in L\mathbb{D}$$

$$\{J^A(\sigma), J^B(\sigma)\} = F^{AB}{}_C J^C(\sigma) \delta(\sigma - \sigma') + \eta^{AB} \partial_\sigma \delta(\sigma - \sigma')$$

σ-model on $\mathbb{D}/\widetilde{G}$



σ-model on $\mathbb{D}/G$



$$E = (G + B) = (E_0^{-1} - \Pi)^{-1}$$

Poisson-Lie* ⟷ *T-duals

$$\widetilde{E} = (\widetilde{G} + \widetilde{B}) = (E_0 - \widetilde{\Pi})^{-1}$$

→ The dual sigma models are related by a ***canonical transformation*** *[Sfetsos, Klimcik,Severa]*

→ Backgrounds are often quite (unsurprisingly) unwieldy and complicated

Examples of Poisson-Lie T-duals

Different choices of Drinfel'd doubles $D = G \cdot \widetilde{G}$

✓ *Abelian T-duality*

$$D = U(1)^N \times U(1)^N$$

$$\Pi = \widetilde{\Pi} = 0$$

$$G_0 \longleftrightarrow G_0^{-1}$$

✓ *non-Abelian T-duality*

$$D = G \times U(1)^N$$

$$\Pi = 0\,, \quad \widetilde{\Pi}_{ab} = f_{ab}{}^c \tilde{x}_c$$

$$E_0 \longleftrightarrow [(E_0)_{ab} - f_{ab}{}^c \tilde{x}_c]^{-1}$$

¡η-deformation!

$$D \equiv G^{\mathbb{C}} = G \times AN$$

$$\Pi = R - R_g$$

$$S_\eta \longleftrightarrow S_{\lambda^\star}$$

All known ***integrable deformations*** are ***examples of Poisson-Lie T-dualisable models***

Winding-momentum exchange

Winding-momentum exchange in generalised T-duality

Generalised T-duality Narain-lattice

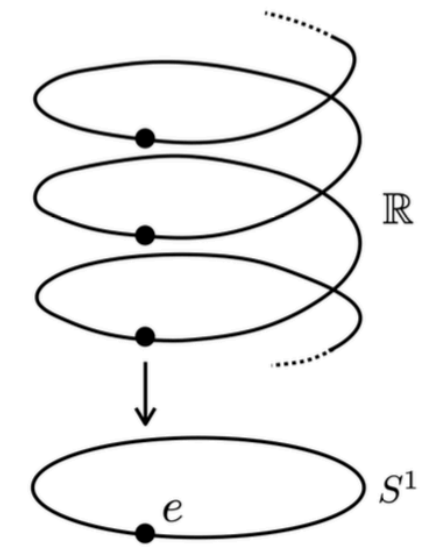
=

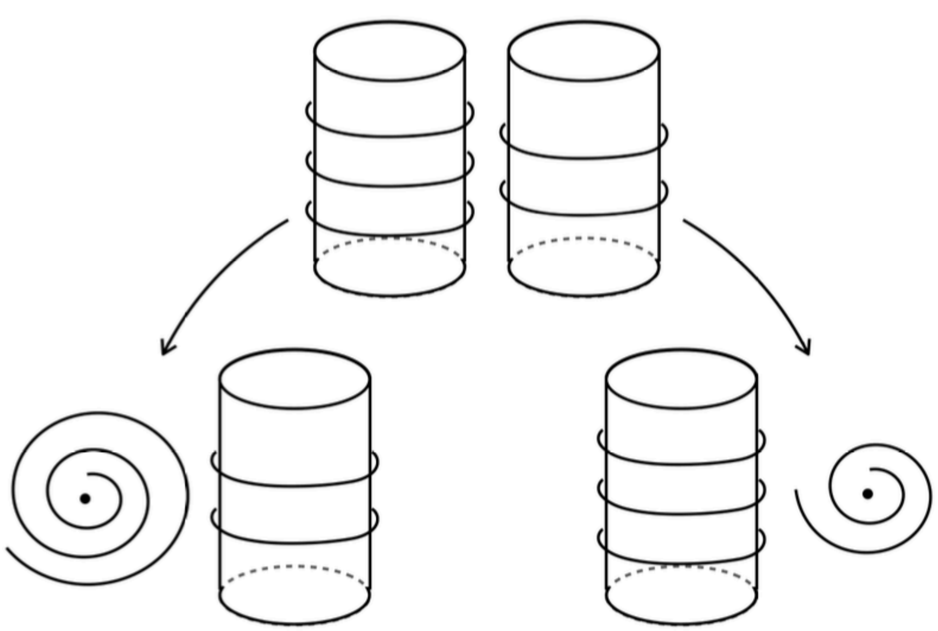
fundamental group of the Drinfel'd double

[Klimčík, Ševera]

Keeps track of non-Abelian momentum and winding exchange modulo unit-monodromy constraint

$$P \exp \oint \mathcal{J} = \tilde{e} \in \tilde{G}$$



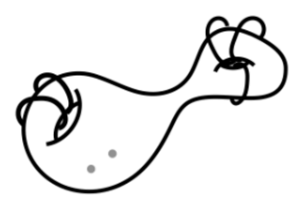
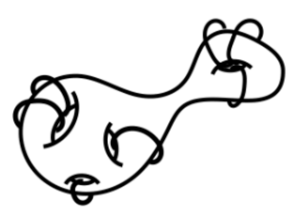
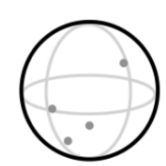
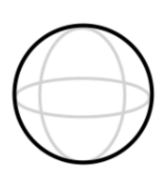


$$0 \to \pi_1(G)/\pi_2(D/G) \to \pi_1(D) \to \pi_1(D/G) \to 0$$

$$\underbrace{\pi_1(D)}_{\text{winding in } D} \longrightarrow \overbrace{\pi_1(D/G)}^{\text{winding in } D/G} \oplus \underbrace{(\pi_1(G)/\pi_2(D/G))}_{\text{non-comm. momentum in } D/G}$$

Duality frames for generalised T-duality

$$\underbrace{\pi_1(D)}_{\text{winding in } D} \longrightarrow \overbrace{\pi_1(D/G)}^{\text{winding in } D/G} \oplus \underbrace{(\pi_1(G)/\pi_2(D/G))}_{\text{non-comm. momentum in } D/G}$$

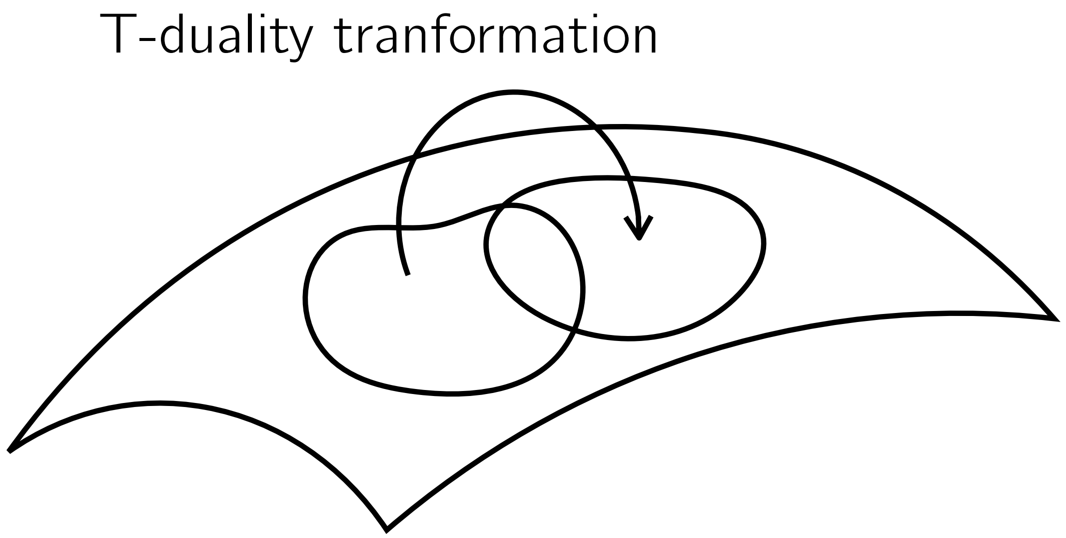
	$\pi_1(D)$	$\pi_1(D/G)$	$\pi_1(G)/\pi_2(D/G)$	example
case i				$SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ $SL(2,\mathbb{R})_{\text{diag}}$
case ii			nihil	$T^\star SU(2)$ $SU(2)$
case iii		 nihil		$T^\star SU(2)$ $U(1)^3$
case iv	 nihil	nihil	nihil	$SL(2,\mathbb{C})$ Iwasawa

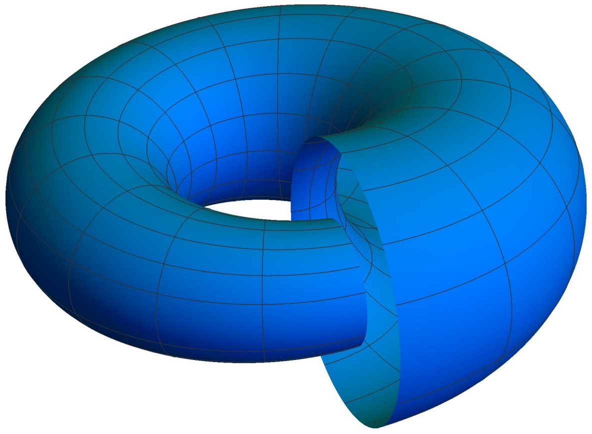
Non-geometry

Non-geometric spaces

Space is not only sewn together my diffeomorphisms
One has to include T-duality transformations !

[Hellerman, McGreevy, Williams], [Hull], [Dasgupta, Rajesh, Sethi],...





Constructed by applying consecutive T-duality transformations: valid string backgrounds (?)

Challenge
Unclear how to even define winding modes

Winding-momentum exchange by invoking exotic differential forms?

[Fan, Mathai]