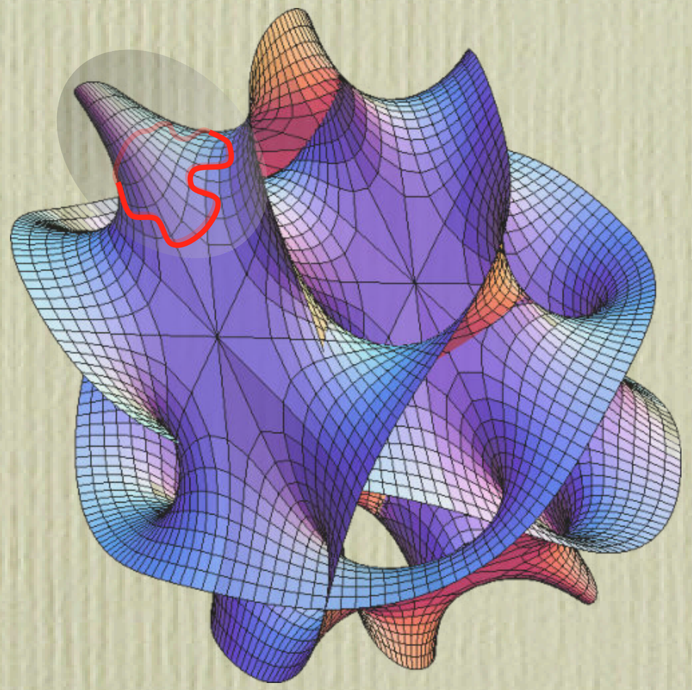
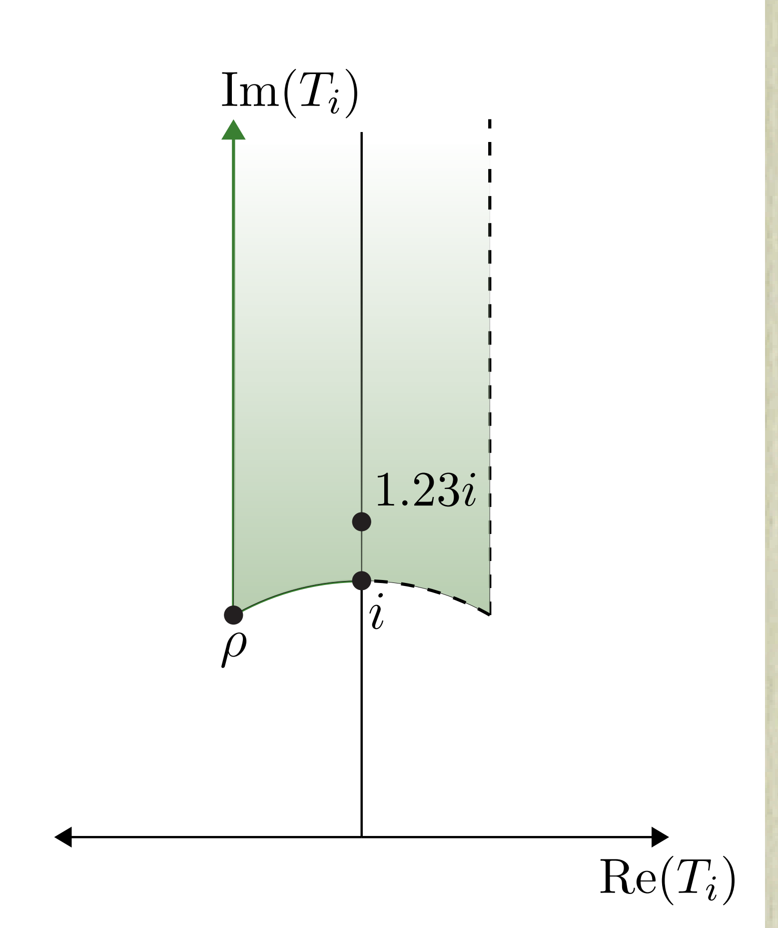
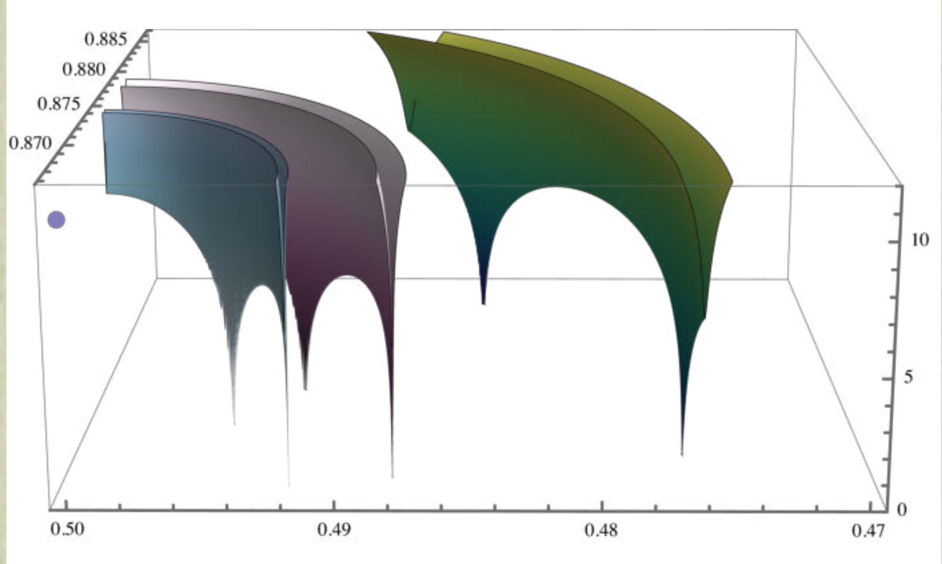
Heterotic de Sitter Beyond Modular Symmetry







Jacob Leedom, Nicole Righi & AW — arXiv:2212.03876

Alexander Westphal
(DESY)

de Sitter vacua in String Theory ...

- observation: $\rho_\Lambda \simeq 10^{-122} > 0$ $\qquad w_\lambda = -1 \pm 0.05$

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constructions of varying degrees of explicitness

[KKLT, LVS, Kähler Uplift, IIB on compact negatively curved spaces, …]

(in the interior of moduli space

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  [KKLT, LVS, Kähler Uplift, IIB on compact negatively curved spaces, …]

  (in the interior of moduli space

- The Swampland:

  EFT constraints from quantum gravity/string theory

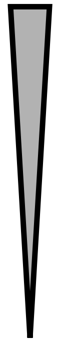
  → no dS vacua in asymptotic regions of moduli space

[Garg,Krishnan,'18]
[Ooguri, Palti, Shiu, Vafa, '18]
[Hebecker, Wrase, '18]

de Sitter vacua in String Theory ...

starting point: partial no-go theorems - here heterotic

[Maldacena-Nunez]



Classical SUGRA?

No dS

AdS OK

de Sitter vacua in String Theory ...

starting point: partial no-go theorems - here heterotic

HE: $E_8 \times E_8$ HO: $Spin(32)/\mathbb{Z}_2$

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\,e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left(\mathrm{Tr}|F|^2 - \mathrm{Tr}|R_+|^2\right)\right]$$

[Maldacena-Nunez] [Maldacena-Nunez]

Classical SUGRA?

No dS

AdS OK

[Green+,'11]

Leading α'?

No dS

AdS OK

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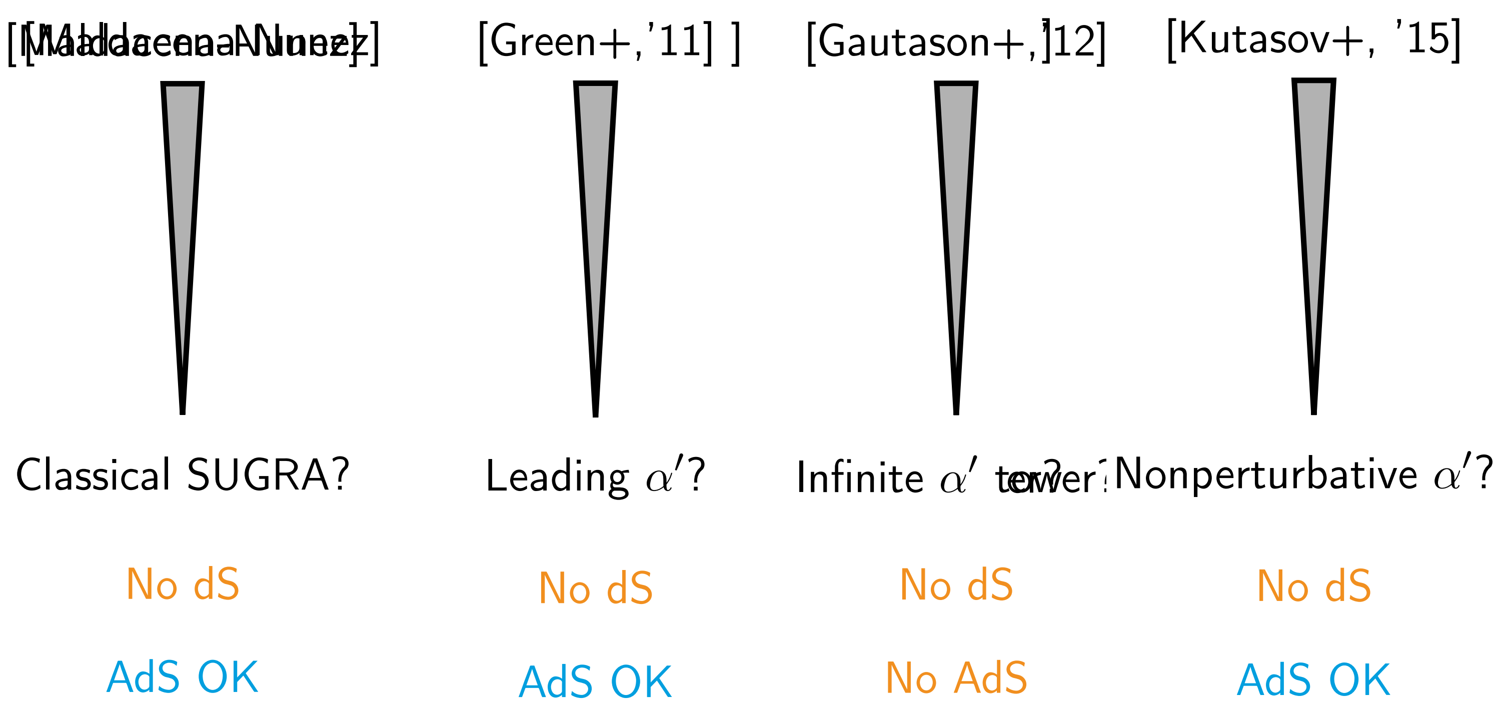
[Maldacena-Nunez]	[Green+,'11]]	[Gautason+,'12]
Classical SUGRA?	Leading α'?	Infinite α' tower?
No dS	No dS	No dS
AdS OK	AdS OK	No AdS

de Sitter vacua in String Theory …

starting point: partial no-go theorems - here heterotic

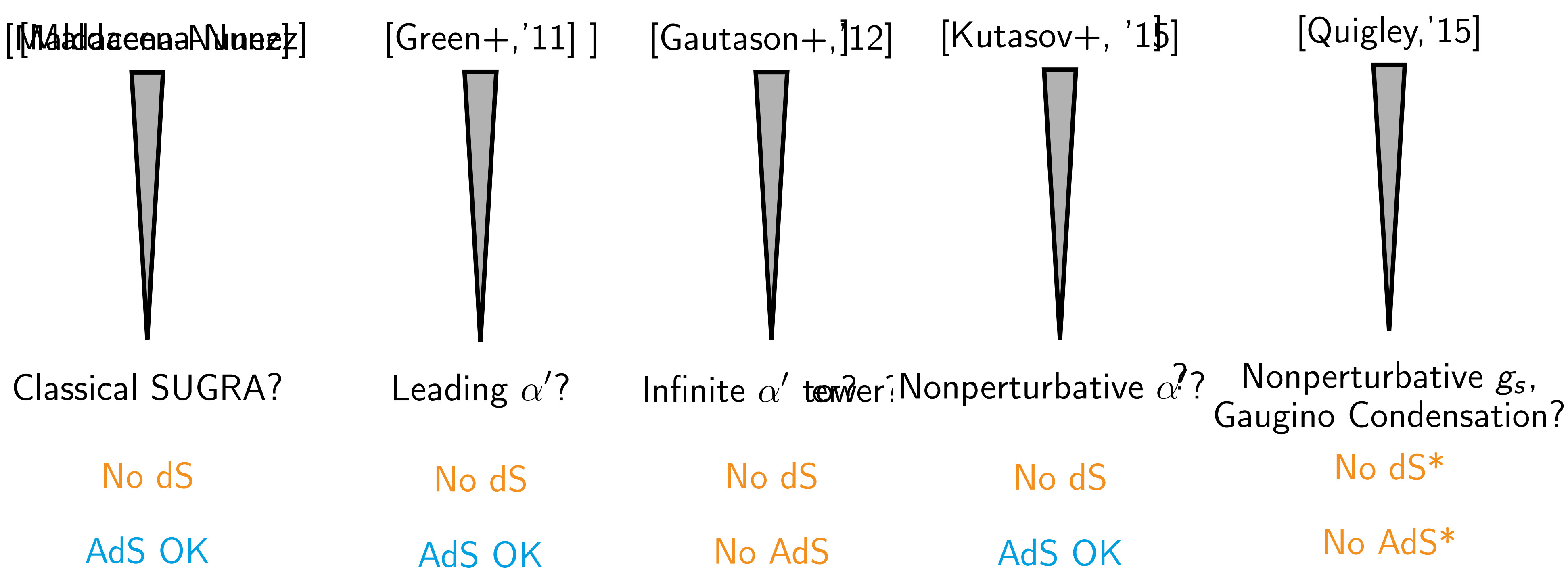
Includes worldsheet instantons & high curvature solutions



de Sitter vacua in String Theory ...

starting point: partial no-go theorems - here heterotic

$$W(S) \sim e^{-S} \rightarrow \delta\mathcal{L} \sim \exp[-1/g_s^2]$$



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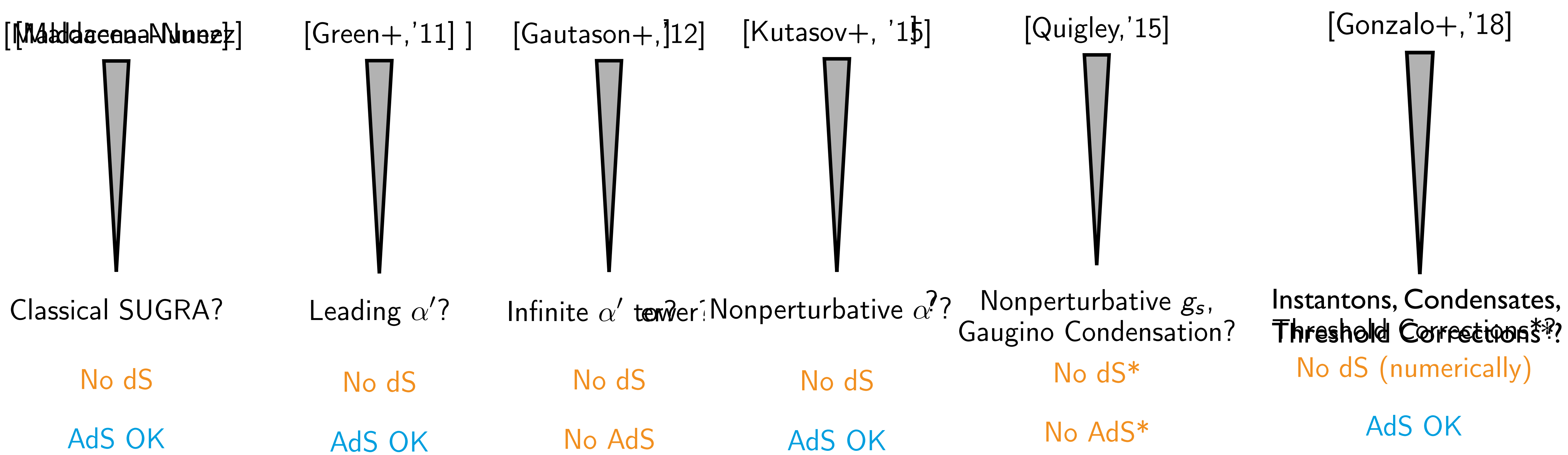

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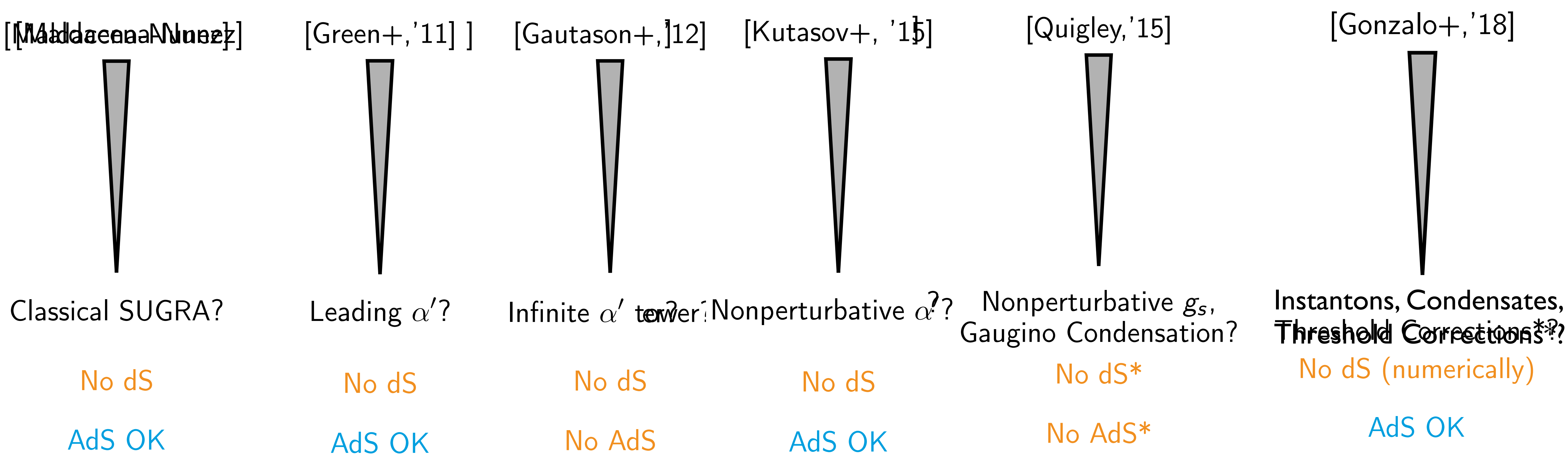


Our goal is to extend the above results: ⟶ effects stronger than gaugino condensation

heterotic strings on torus orbifolds …

[Font, Ibanez, Lust & Quevedo '90]
[Cvetic, Font, Ibanez, Lust & Quevedo '91]
[Gonzalo, Ibanez & Uranga '18]

- Overall Kähler Modulus T and Dilaton S
  - T has a $PSL(2,\mathbb{Z})$ symmetry from T-Duality:

$$T \to \gamma \cdot T = \frac{aT+b}{cT+d} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

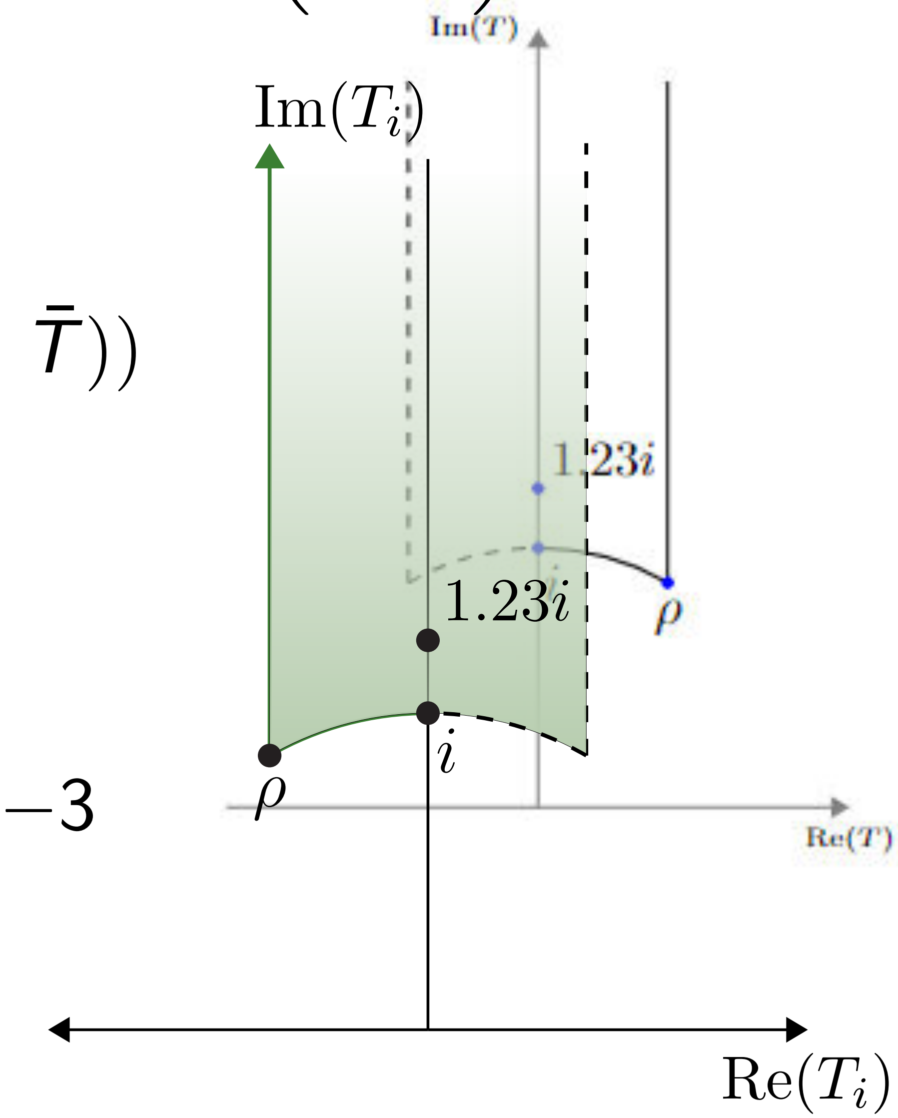
  - Kähler potential

$$\mathcal{K} = -\ln(S+\bar{S}) - 3\ln(-i(T-\bar{T}))$$

  - For action to be invariant under $PSL(2,\mathbb{Z})$,

$$\mathcal{G} = \mathcal{K} + \ln|W|^2$$

must be invariant $\Rightarrow$ W has modular weight -3



heterotic strings on torus orbifolds ...

- gaugino condensation:

$$\langle \lambda\lambda \rangle \sim \Lambda^3 \sim e^{-f_a/b_a} \sim e^{-S/b_a}$$

- modular invariance from thresholds:

$$\delta f_a \simeq b_a \ \ln[\eta^6(T)] + \cdots$$

$\Rightarrow$ Here be moonshine [Wrase,'14]

- $\rightarrow$ W:

$$W = \frac{H(T)e^{-S/b_a}}{\eta^6(T)}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathcal{P}(j(T))$$

[Rademacher,Zuckerman,'38]
[Lehner]

infinite sum of $e^{2\pi i T}$ - terms — like ws instantons

heterotic strings on torus orbifolds ...

- scalar potential:

$$V(S,\bar{S},T,\bar{T}) = e^{\mathcal{K}}\left(\mathcal{K}^{S\bar{S}}F_S\bar{F}_{\bar{S}} + \mathcal{K}^{T\bar{T}}F_T\bar{F}_{\bar{T}} - 3|W|^2\right)$$

$$= e^{k(S,\bar{S})}Z(T,\bar{T})|\Omega(S)|^2\left\{|H(T)|^2\left(A(S,\bar{S})-3\right)+\hat{V}(T,\bar{T})\right\}$$

Conjectures [Gonzalo,Ibanez,Uranga,'18 - GIU]: no dS for tree-level $k(S,\bar{S})$

- 2 classes of S extrema:

Class A: $\Omega_S(S) + K_S\Omega(S) = 0 \rightarrow F_S = 0$

Class B: $F_S \neq 0$

class A no-go

[Leedom, Righi & AW '22]

- establish 2 no-go theorems — proving the GIU conjecture & extending it:

Class A

- **Theorem 1:** At a point (T_0, S_0), the scalar potential $V(T, S)$ can not simultaneously satisfy:
  1. $V(T_0, S_0) > 0$
  2. $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$
  3. $(\Omega_S + k_S \Omega)|_{S=S_0} = 0$
  4. Eigenvalues of the Hessian of $V(T, S)$ at (T_0, S_0) are all $\geqslant 0$.

class A no-go

[Leedom, Righi & AW '22]

- **Corollary 1.1:** Class A extrema in the two-modulus model with $k(S, \bar{S}) = -\ln(S + \bar{S})$ can never be dS vacua.
- **Corollary 1.2:** The one-modulus model with $W(T) = H(T)/\eta^6(T)$ and $\mathcal{K} = -3\ln(-i(T - \bar{T}))$ can not have dS vacua.

Proves several conjectures from GIU

class B no-go ?

[Leedom, Righi & AW '22]

- What about Class B extrema?
  In general Hessian doesn't factorize – much more complicated
  Enter the power of modular symmetry
- $V(T, S)$ is a non-holomorphic modular function in T, so $\partial_T V$ is a weight 2 modular form and vanishes at $T = i, \rho$
- All mixed derivatives of T & S are weight 2 modular forms
  $\Rightarrow$ Hessian is block diagonal
- Self dual points always extremum - when are they minima in T-sector?

class B no-go

[Leedom, Righi & AW '22]

Class B

- even SUSY-breaking S extrema cannot give dS minima if S has tree-level Kähler potential ...

- **Theorem 2:** At a point (T_0, S_0), the scalar potential $V(T, S)$ with $k(S, \bar{S}) = -\ln(S + \bar{S})$ can not simultaneously satisfy:
  1. $V(T_0, S_0) > 0$
  2. $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$
  3. $\widetilde{F}_T(T_0) = 0$
  4. Eigenvalues of the Hessian of $V(T, S)$ at (T_0, S_0) are all $\geqslant 0$

a look into the modular landscape ...

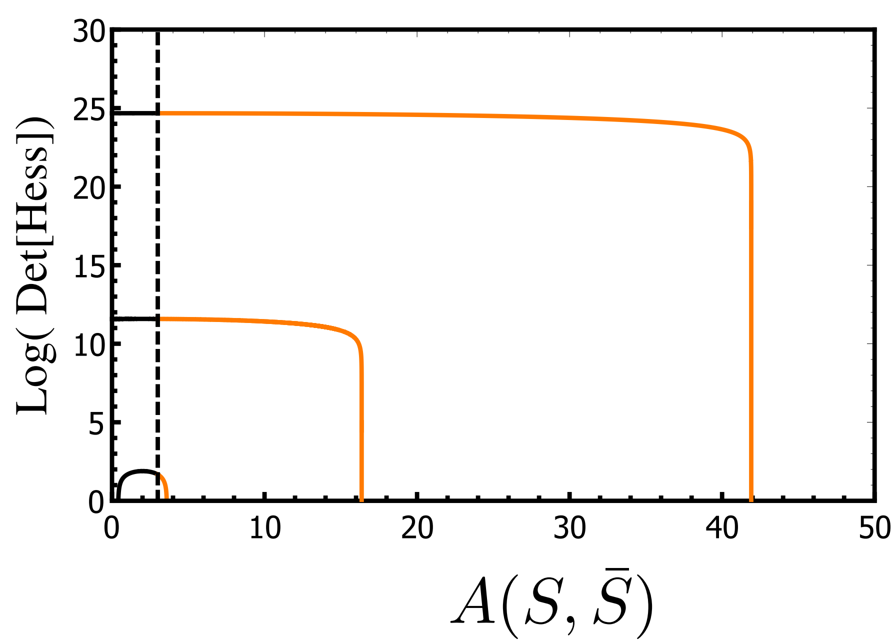
- $T = i$:

[Leedom, Righi & AW ’22]

$$V(S,\bar{S},i,-i) = \frac{2^{4n+9}\pi^{8n+9}}{\Gamma^{12}(1/4)}|\Omega(S)|^2|\mathcal{P}(1728)|^2 e^{k(S,\bar{S})}\left(A(S,\bar{S})-3\right)$$

- Set $m = 0$ or else extremum is Minkowski
- dS extremum at $T = i$ if dilaton is stabilized with $\langle A(S,\bar{S})\rangle > 3$
- If we set $\mathcal{P}(j(T)) = 1$, then this point is stable in T sector if

$$2 - \frac{(1+8n)\Gamma^8(1/4)}{192\pi^4} < A(S,\bar{S}) < 2 + \frac{(1+8n)\Gamma^8(1/4)}{192\pi^4}$$

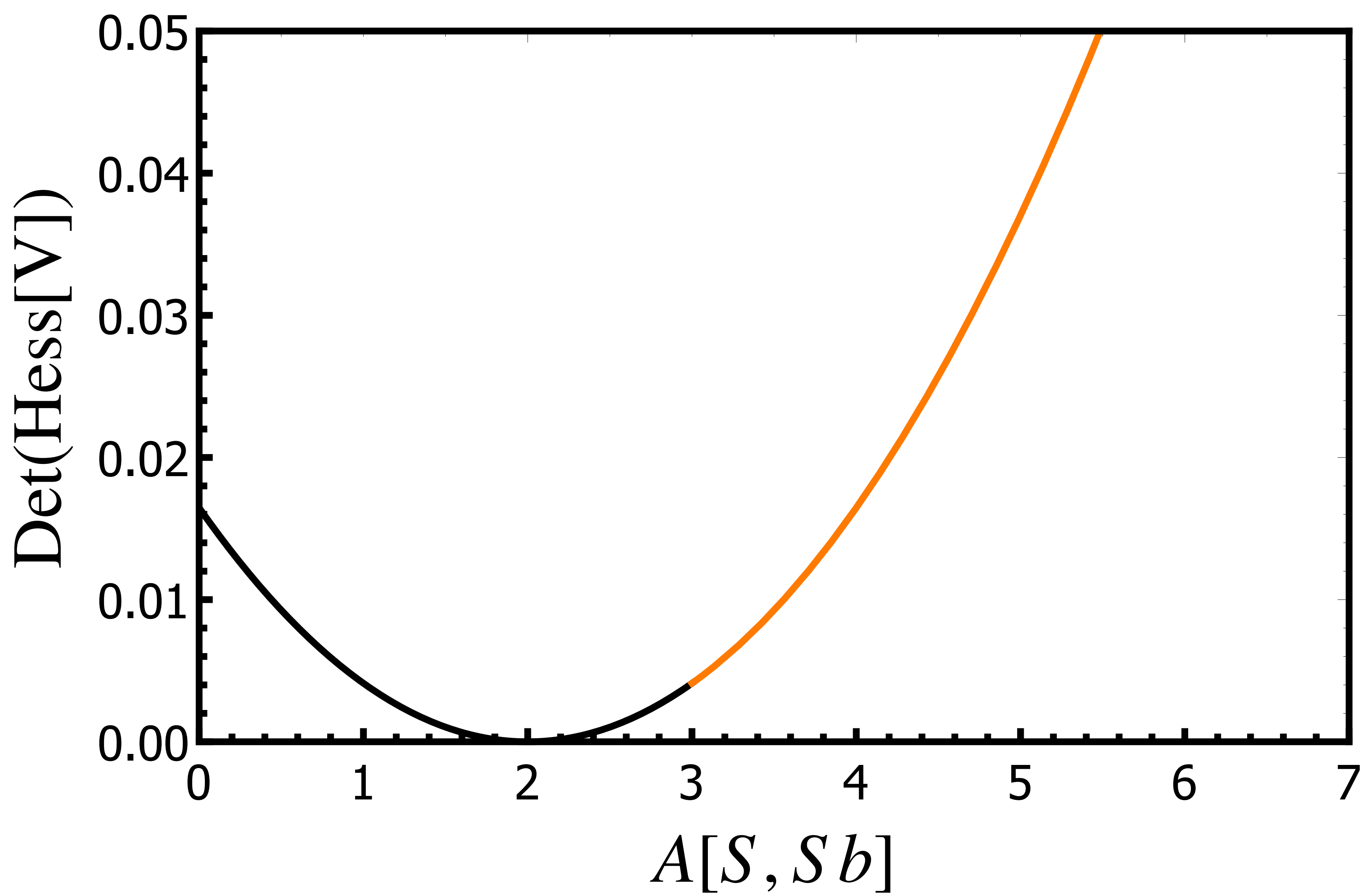


a look into the modular landscape ...

- $T = \rho$ is remarkable:

[Leedom, Righi & AW '22]

$$\partial_t^2 V|_{T=\rho} = \partial_a^2 V|_{T=\rho} = \frac{2^{8m+13}\pi^{12m+12}}{3^{3(m+1)} \times 1225^m} |\Omega(S)|^2 |\mathcal{P}(0)|^2 e^{k(S,\bar{S})} \left(A(S,\bar{S}) - 2\right)$$



into the bulk ...

[Leedom, Righi & AW ’22]

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- [Novichkov+ ’22]: counter-examples

for certain (n,m) extrema near $T = \rho$ **off boundary**

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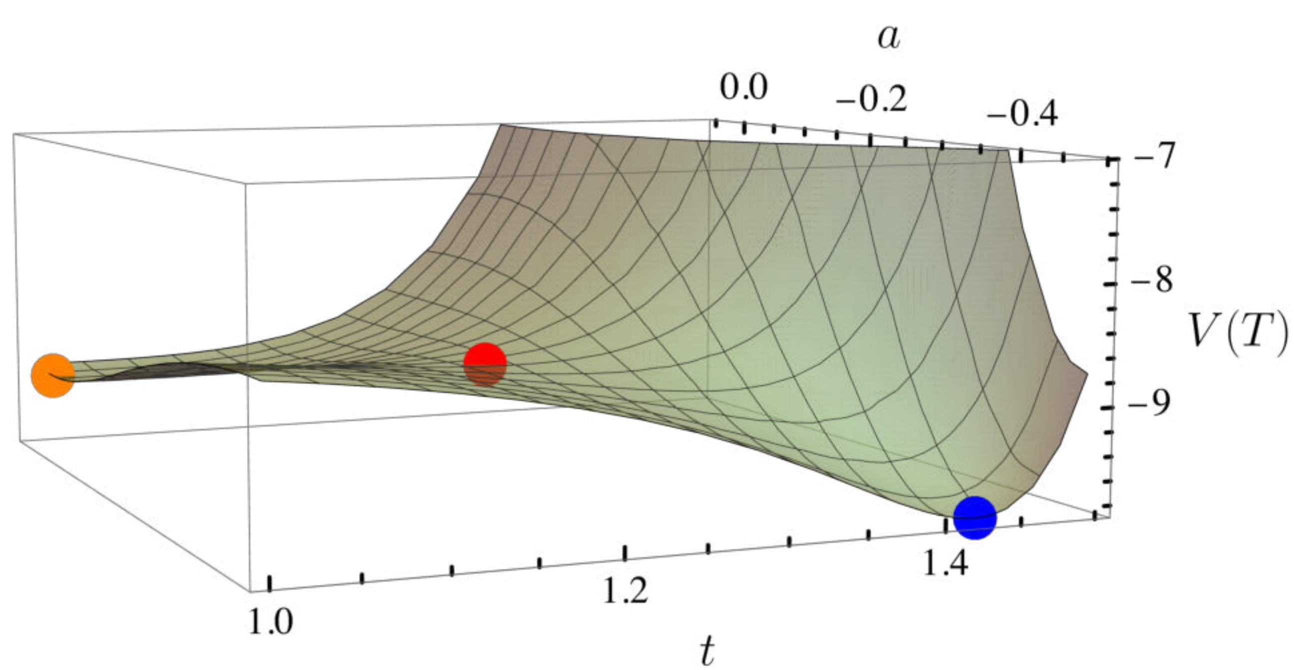
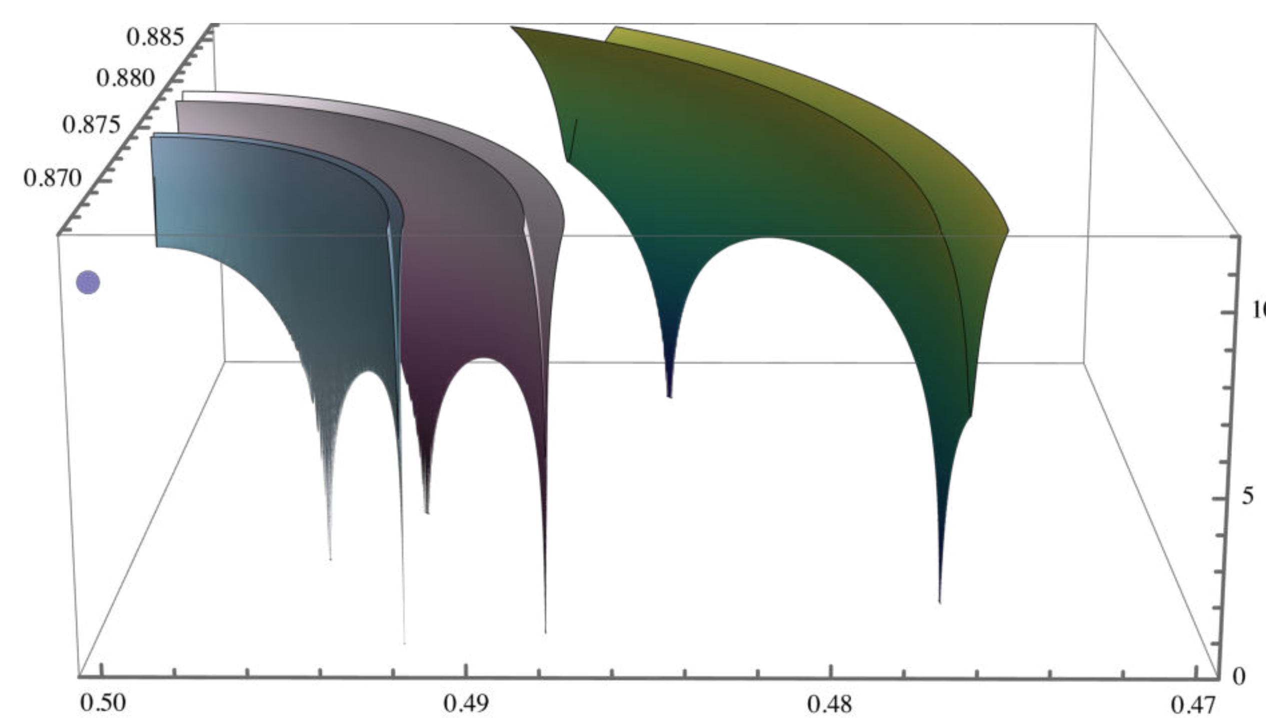
  for certain (n,m) extrema near $T = \rho$ **off boundary**

- verify & find more:

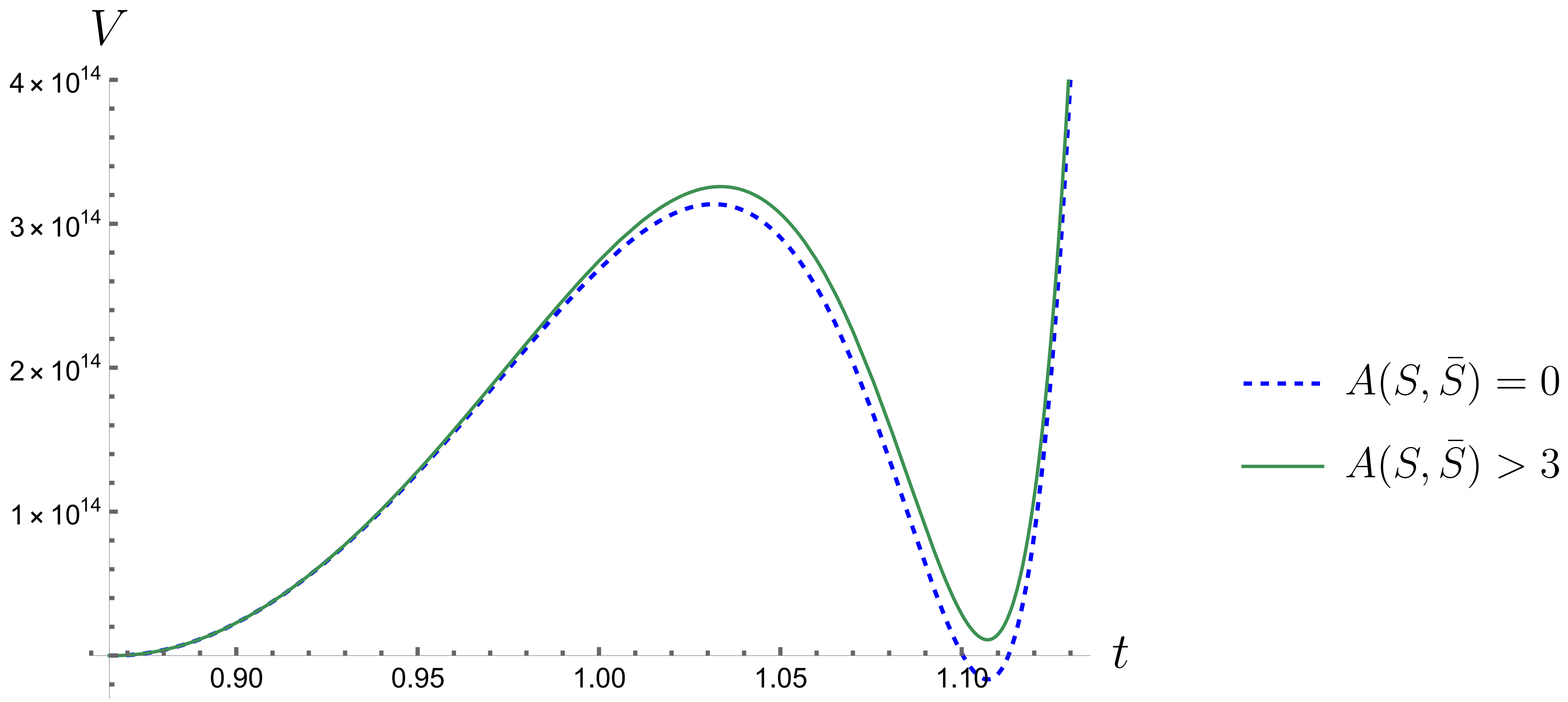




is there dS ?

[Leedom, Righi & AW '22]

- outcome: dS must come from class B …



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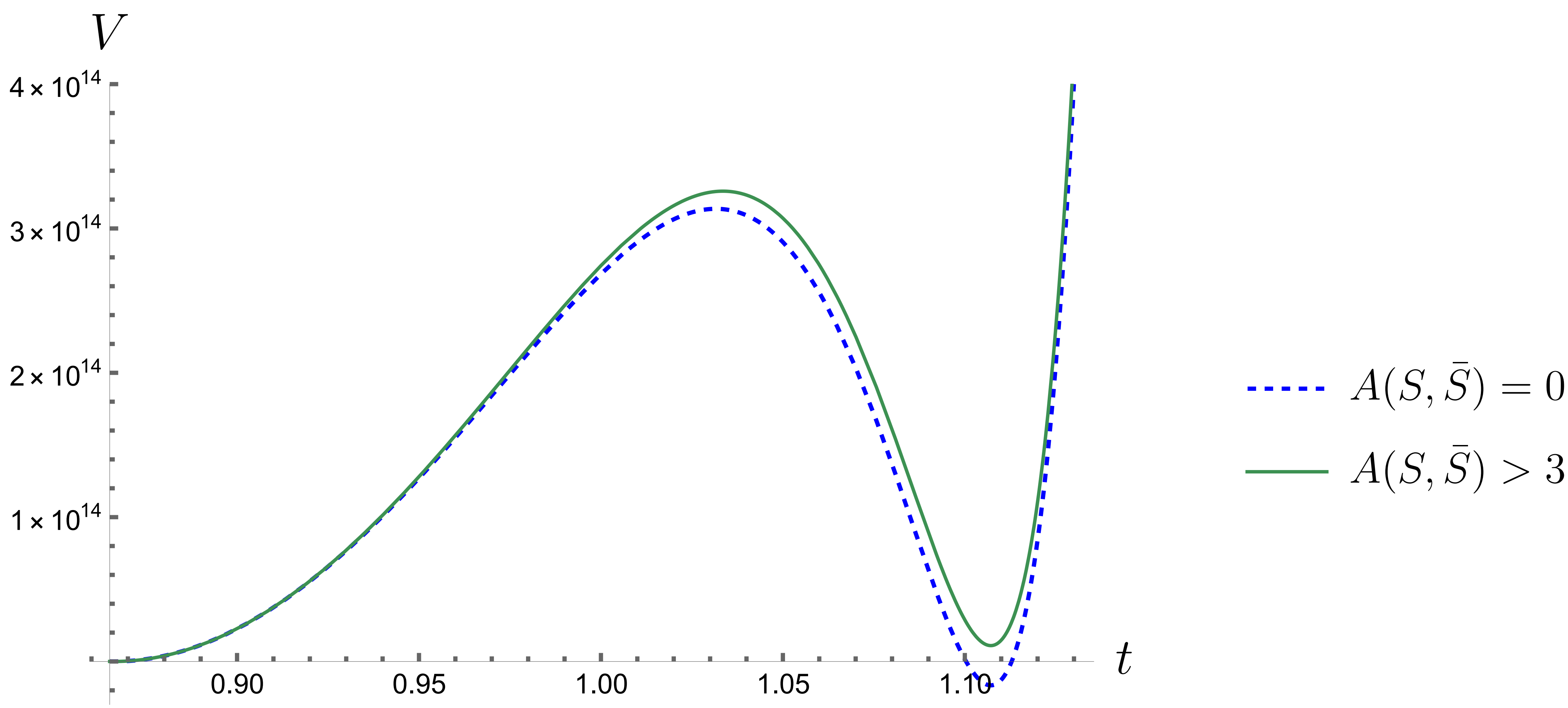

But impossible with tree level dilaton Kähler potential!

beyond the no-go ...

[Leedom, Righi & AW '22]

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We want to utilize Shenker-like effects in Heterotic vacua.

We should be sure that they exist

evade the no-go ...

[Leedom, Righi & AW '22]

- Linear Multiplet Formalism: $L \supset \{\ell, \psi, B_2\}$

$$\mathcal{L}_{KE} = \int d^4\theta E\Big(-2 + f(L) \Big)$$

$$\left\langle \frac{\ell}{1+f(\ell)} \right\rangle = \frac{g_s^2}{2}$$

$$k(L) = \ln(L) + g(L)$$

evade the no-go ...

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- Parametrize Shenker-like effects [Gaillard & Nelson,'07]+:

$$f(\ell) = \sum_{n=0} A_n \ell^{-q_n} e^{-B/\sqrt{\ell}}$$

$$L\frac{df}{dL} = -L\frac{dg}{dL} + f$$

evade the no-go ...

[Leedom, Righi & AW '22]

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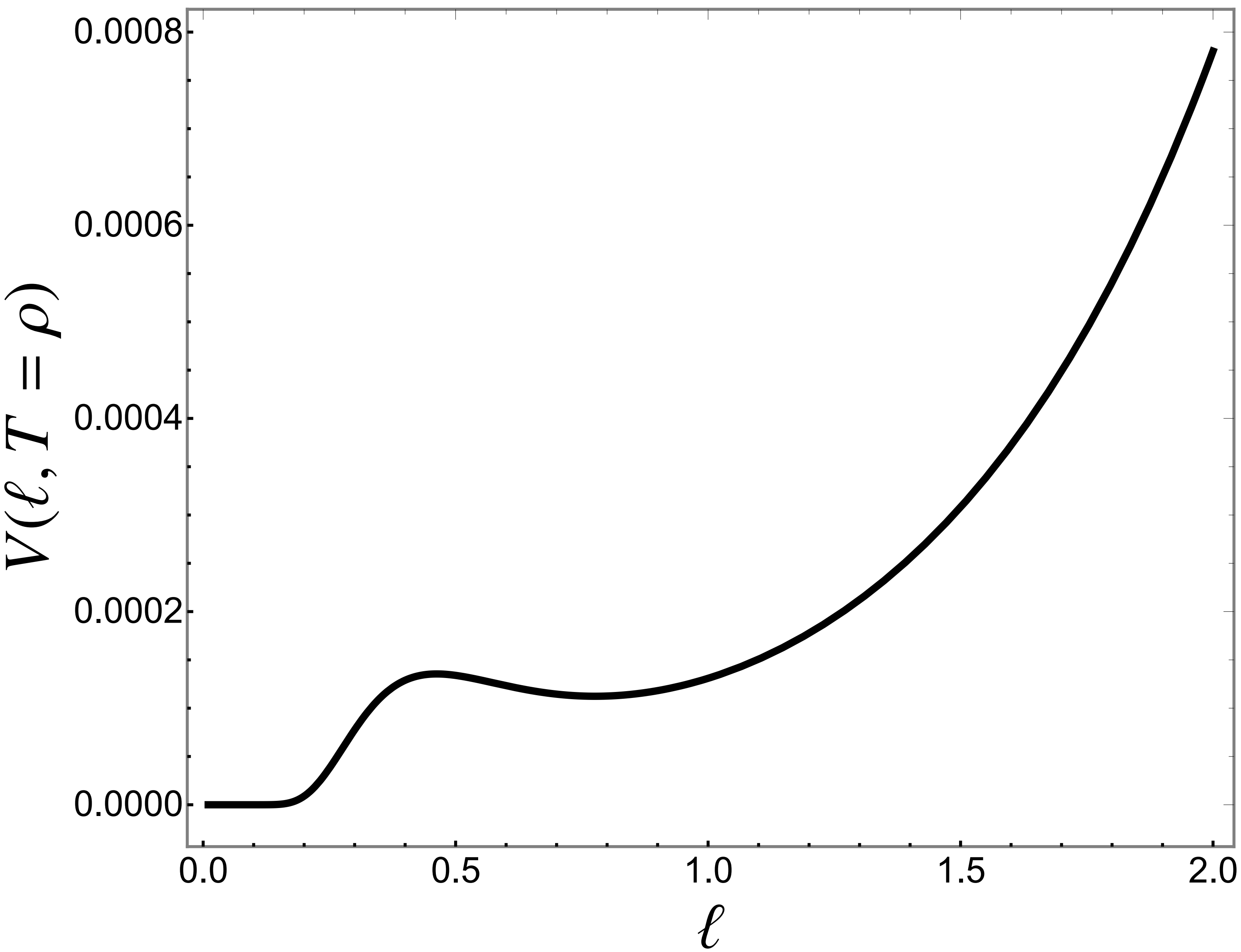
$$f(\ell) = \sum_{n=0} A_n \ell^{-q_n} e^{-B/\sqrt{\ell}} \qquad L\frac{df}{dL} = -L\frac{dg}{dL} + f$$

- Scalar potential for single gaugino condensate:

$$V(\ell) = \frac{\mathcal{T}}{\ell}\left[(1+\ell g')(1+b\ell)^2 - 3b^2\ell^2\right]e^{g-(f+1)/b\ell} \qquad \langle T\rangle = \rho$$

a road to heterotic dS - an example

[Leedom, Righi & AW '22]



$$f(\ell) = (A_0 + A_1 \ell^{\frac{1}{2}}) e^{-B/\sqrt{\ell}}$$

$$A_0 = 10 \qquad A_1 = 9$$

$$B = 0.6\pi \qquad b_{E_8} = \frac{30}{8\pi^2}$$

$$g_4 \simeq 0.70$$

Metastable dS

$$\langle e^{-B/\sqrt{\ell}} \rangle \simeq 0.11$$

evidence for heterotic Shenker-like effects

- [Silverstein,'96]: Can find Heterotic Shenker-like effects via duality arguments. They correct the Kähler potential
- Type I-Heterotic: $g^H_{MN} = \lambda_H g^I_{MN}$ & $\lambda_H = \lambda_I^{-1}$

Type I Worldsheet Instantons: $\delta\mathcal{L}_I \sim e^{-A^I/\alpha'} \leftrightarrow \delta\mathcal{L}_H \sim e^{-\frac{A^H}{\alpha'\lambda}}$

- Type IIA-Heterotic: **If** $S_H \leftrightarrow T_{IIA}$ in 4d and **if** there is a non-trivial π_1:

Type IIA Worldline Instantons : $\delta\mathcal{L}_{IIA} \sim \sum_m e^{-mR^{IIA}} \leftrightarrow \delta\mathcal{L}_H \sim \sum_m e^{-m/\lambda}$

- Does not explain the fundamental origins of these effects within the Heterotic frame
- Very schematic – no explicit calculations

Can do a bit better in M-Theory

Summary

[Maldacena-Nunez]	[Green+,'11]	[Gautason+,'12]	[Kutasov+, '15]	[Quigley,'15]	[Gonzalo+,'18]
Classical SUGRA?	Leading α'?	Infinite α' tower?	Nonperturbative α'?	Nonperturbative g_s, Gaugino Condensation?	Instantons,Condesates, Threshold Corrections*?
No dS	No dS	No dS	No dS	No dS*	No dS (numerically)
AdS OK	AdS OK	No AdS	AdS OK	No AdS*	AdS OK

Summary

[Maldacena-Nunez]	[Green+,'11]	[Gautason+,'12]	[Kutasov+, '15]	[Quigley,'15]	**[This Work]**
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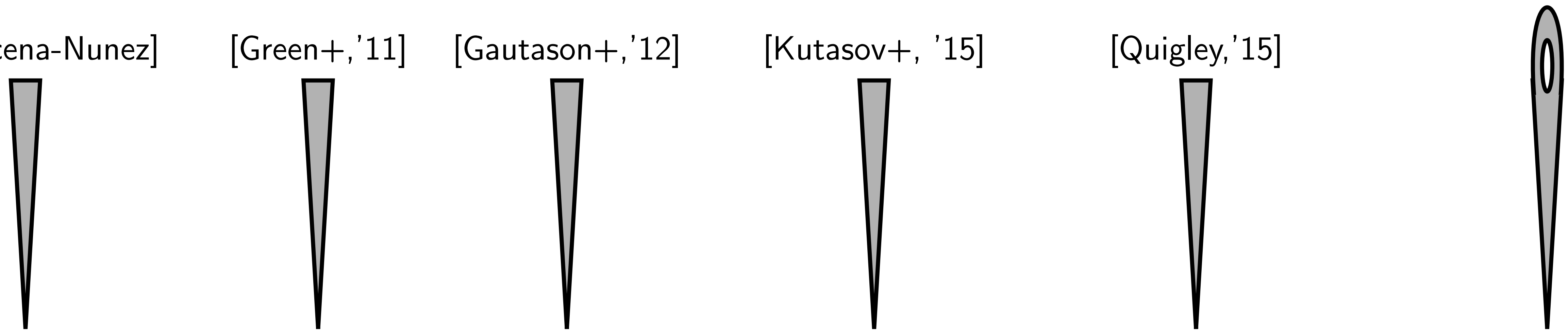

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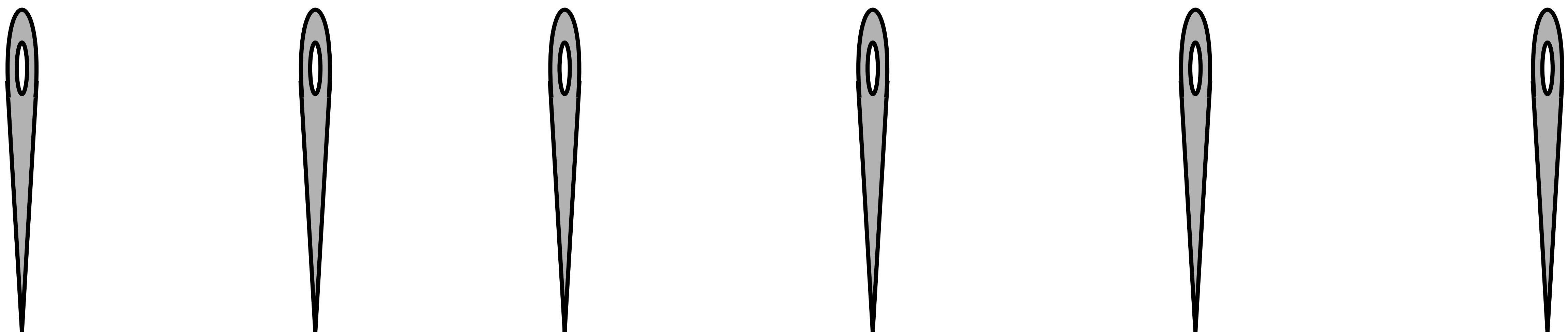

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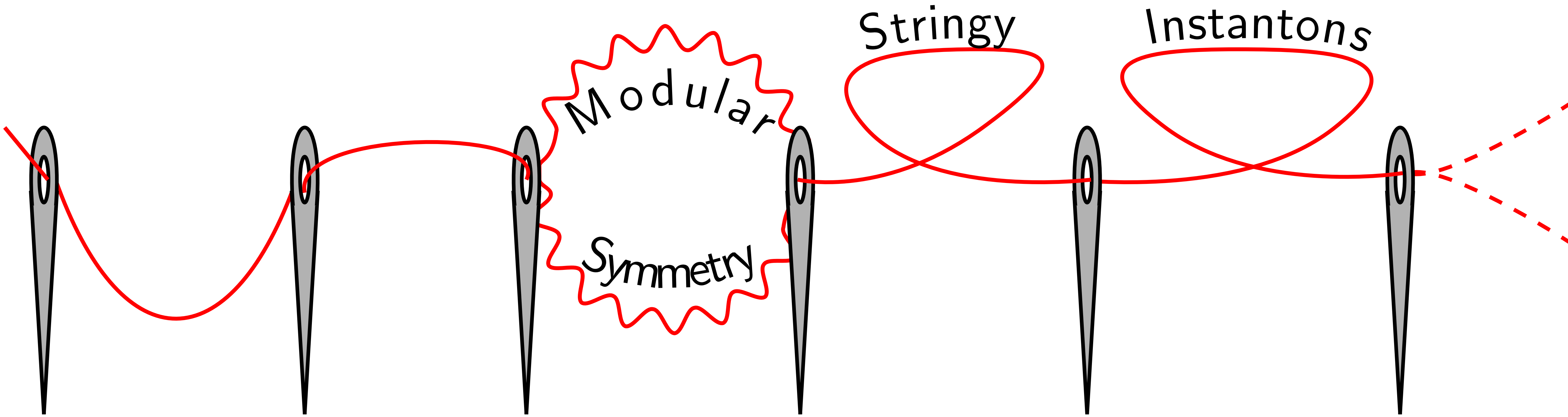

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Number Theory:**[CFC,JML,NR,AW]** & **[AK,JML,NR,AW]**,...

Shenker Effects:**[RAG,CFC,JML,NR]**

backup slides

evidence for heterotic Shenker-like effects

Low-Energy Limit: 11D Supergravity

$$S_{11D} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|G_4|^2\right) - \frac{1}{6}\int C_3 \wedge G_4 \wedge G_4$$

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Type IIA ← on S^1, $g_{IIA} \sim R^{\frac{3}{2}}$ — **M-Theory** — on T^2, $U = \tau$ → Type IIB

M-Theory ↓ on $S^1/\mathbb{Z}_2$

HE

$g_{HE} \sim R^{\frac{3}{2}}$

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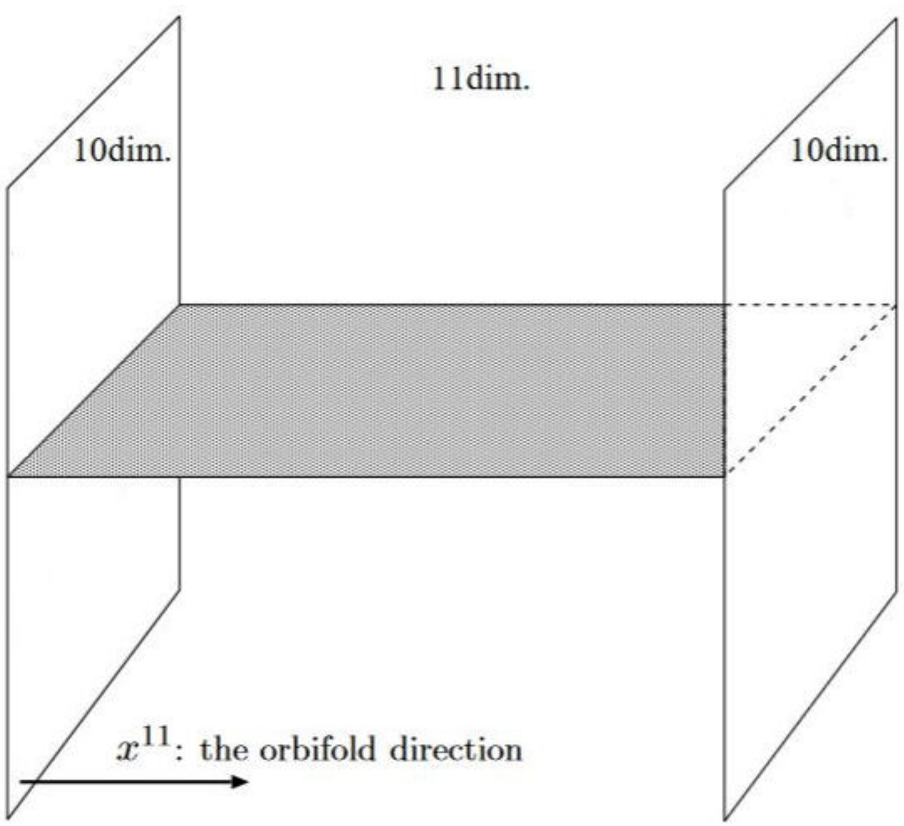
Type IIA $\xleftarrow[g_{IIA} \sim R^{\frac{3}{2}}]{\text{on } S^1}$ M-Theory $\xrightarrow[U = \tau]{\text{on } T^2}$ Type IIB

M-Theory $\xrightarrow{\text{on } S^1/\mathbb{Z}_2}$ HE

$g_{HE} \sim R^{\frac{3}{2}}$

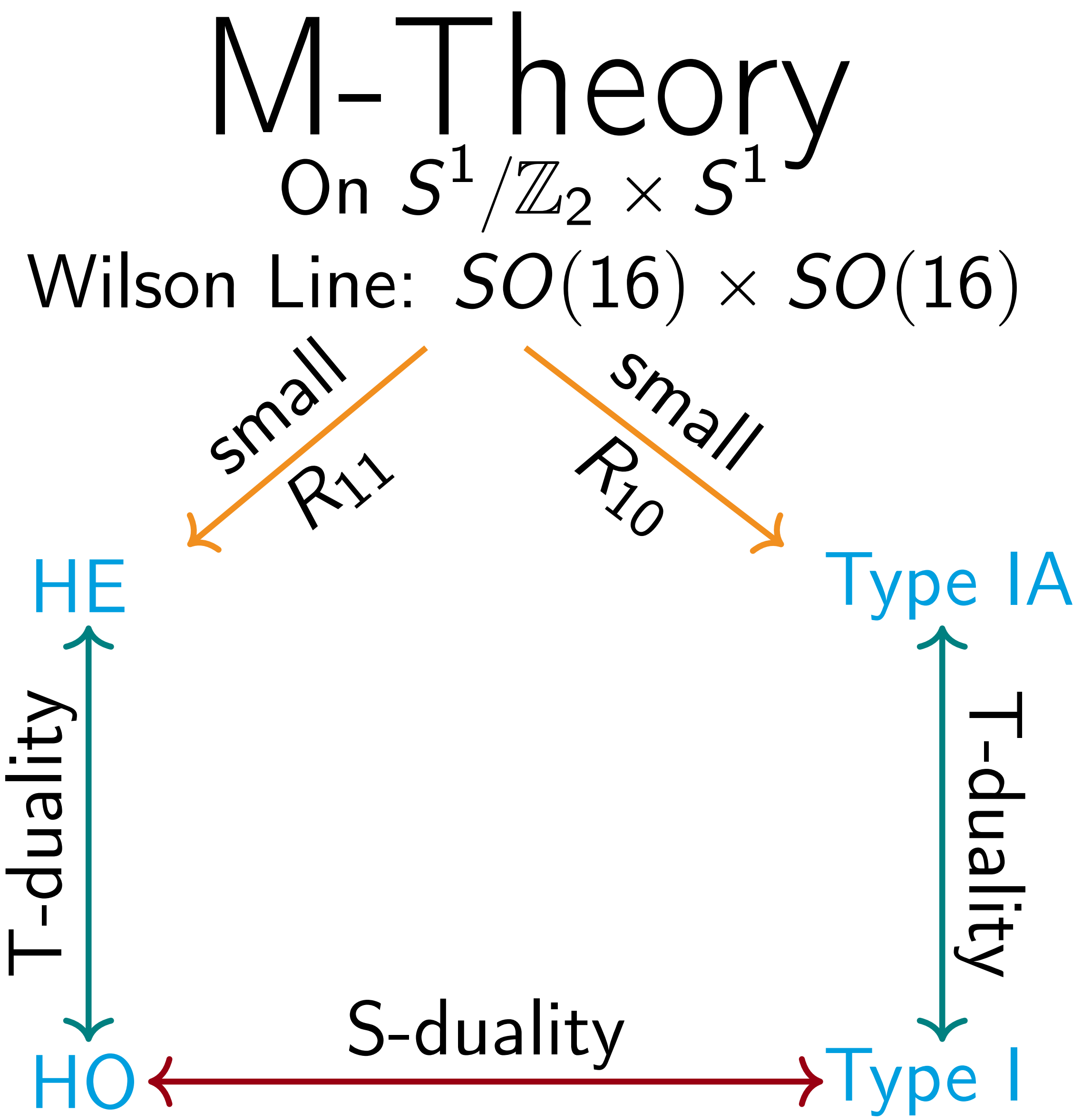
$$S_{HW} = S_{11D} + S_{YM} + S_B$$



evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

$S^1 : \ell_{11} R_{10}$



evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

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$$g_{he} = R_{11}^{3/2}$$
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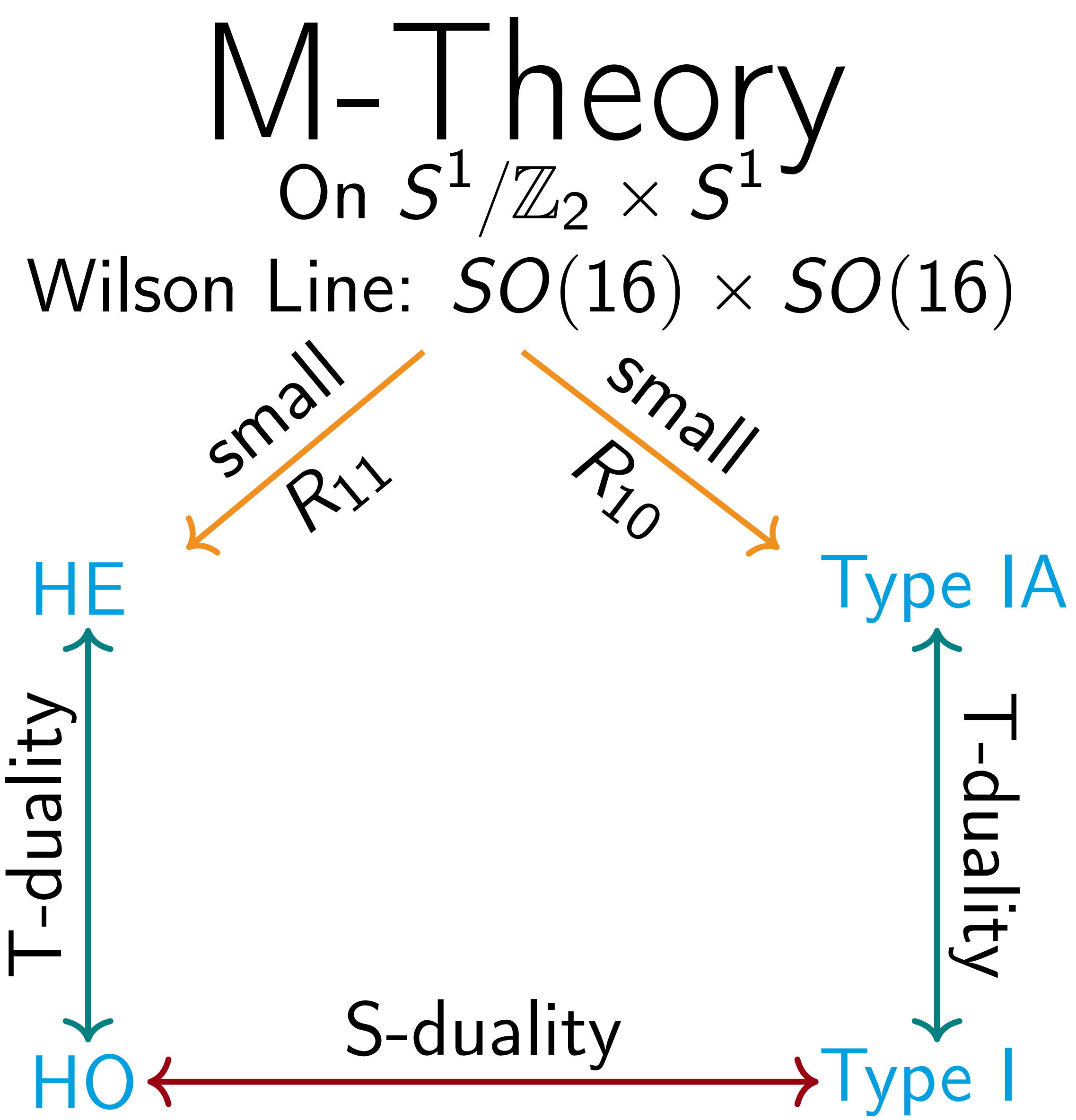

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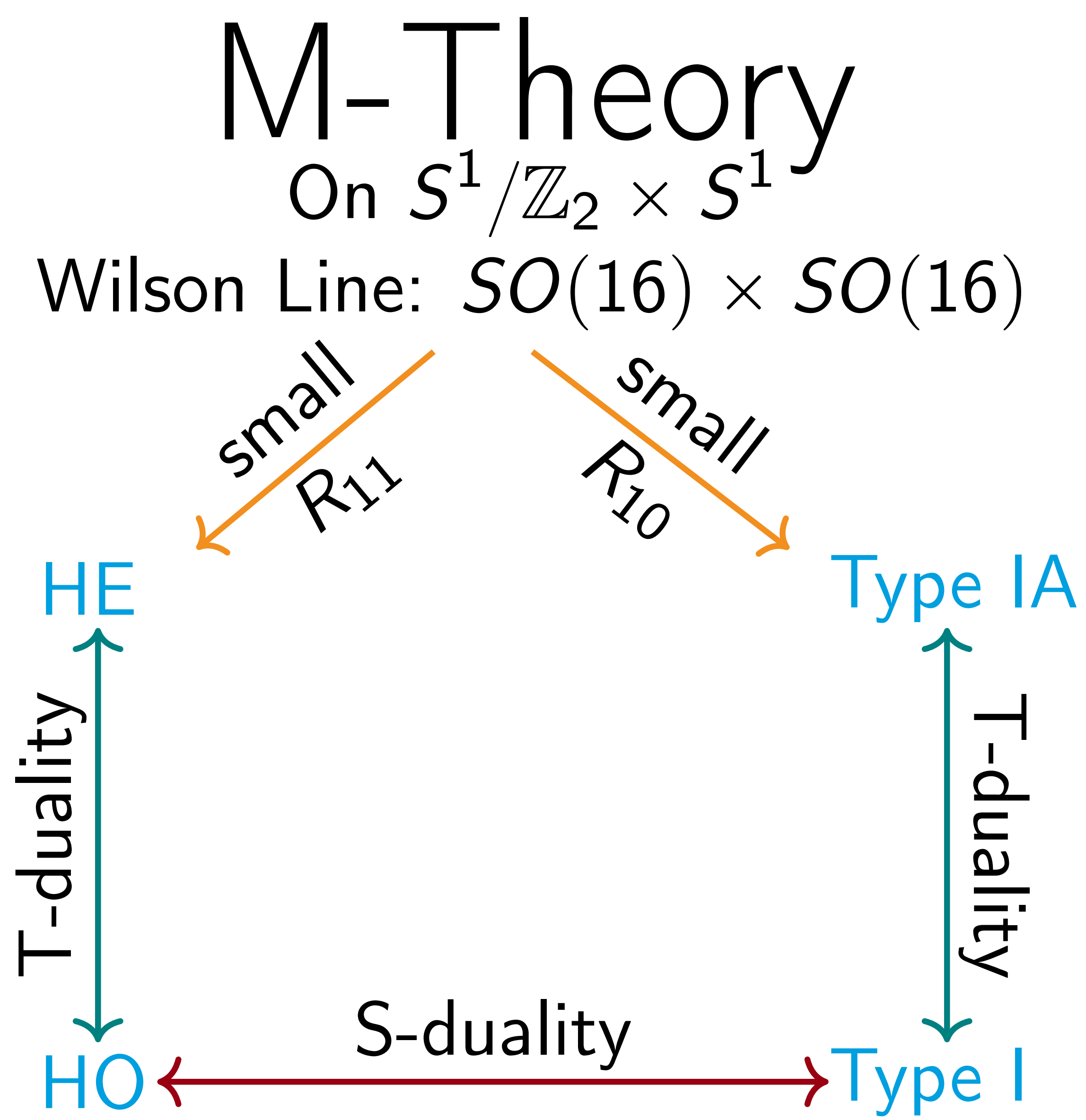

$$g_{ho} = R_{11}/R_{10}$$
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evidence for heterotic Shenker-like effects

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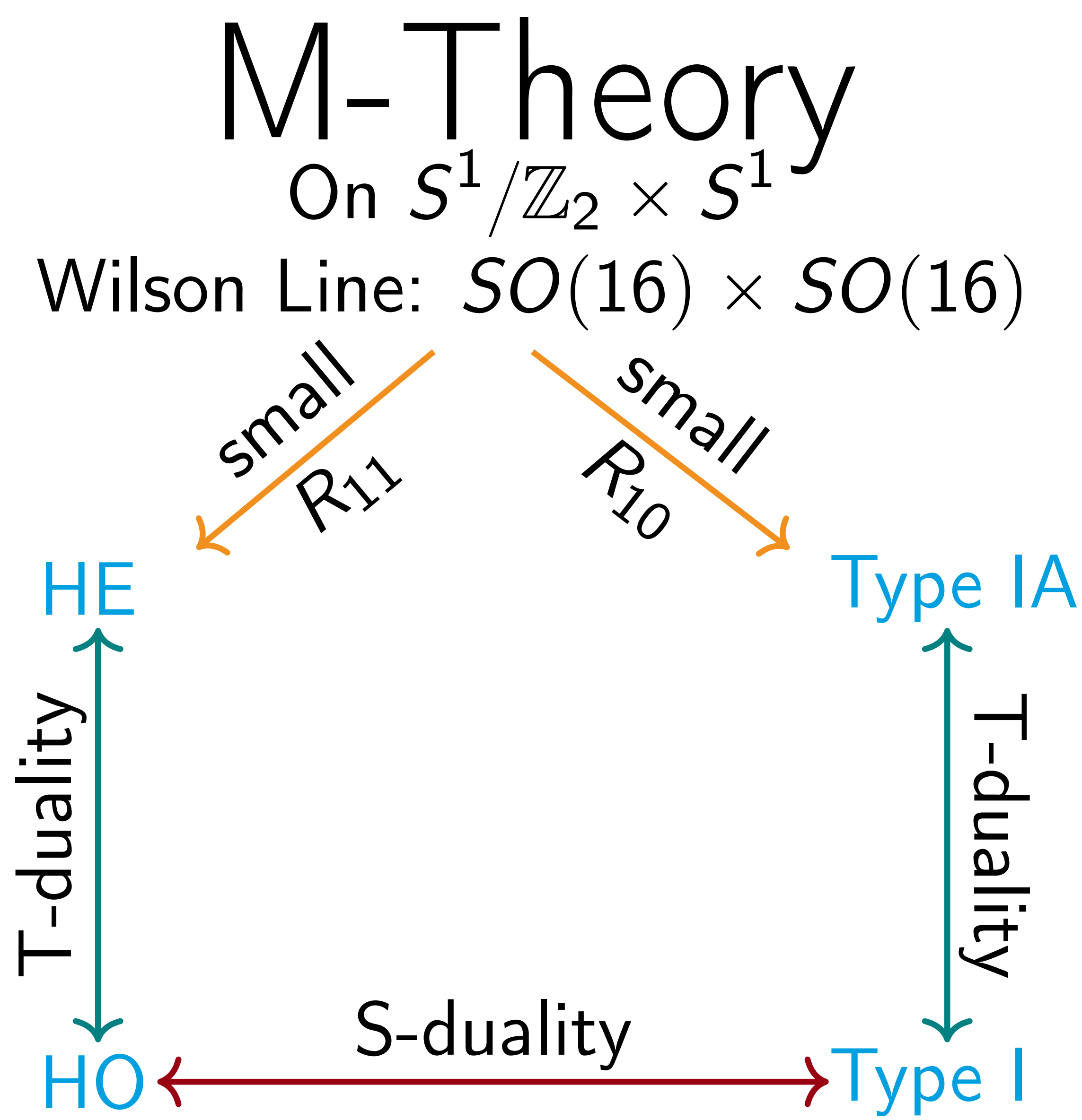

$$g_{IA} = R_{10}^{3/2}$$
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evidence for heterotic Shenker-like effects

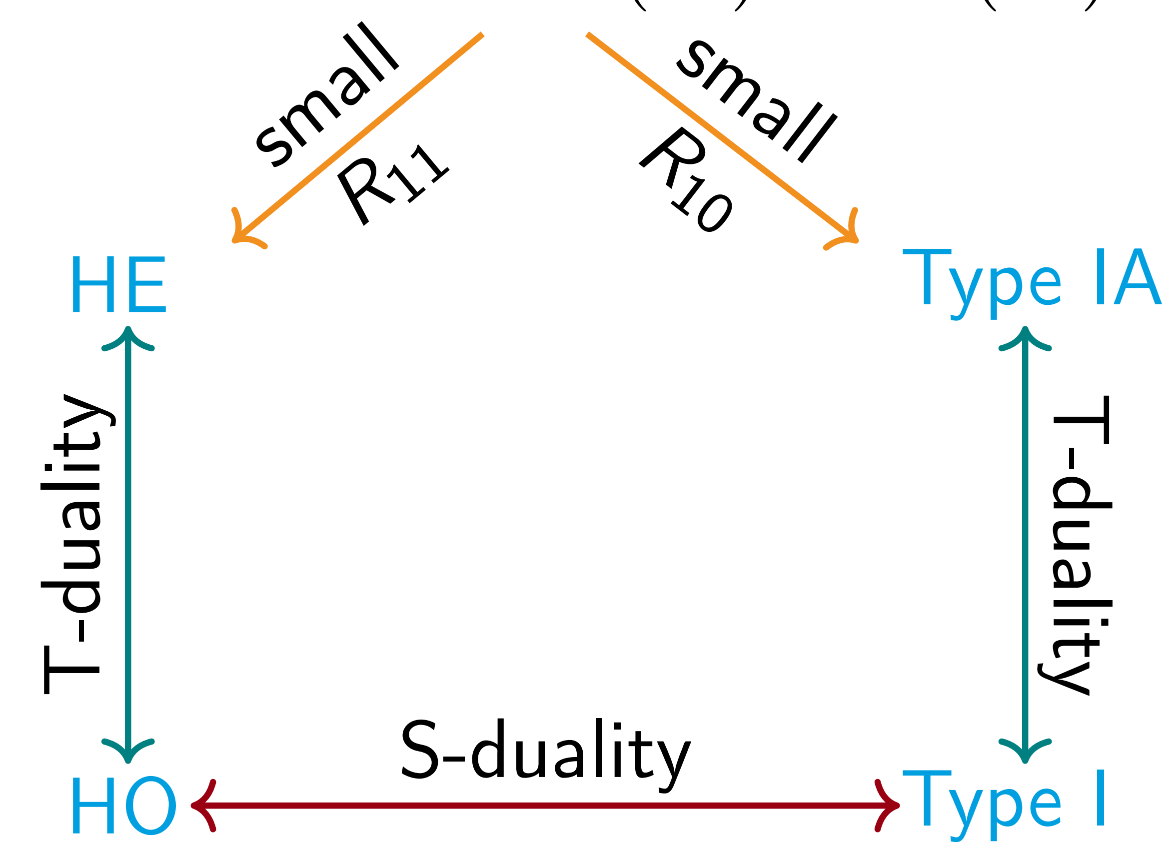
$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

$S^1 : \ell_{11} R_{10}$

M-Theory

On $S^1/\mathbb{Z}_2 \times S^1$

Wilson Line: $SO(16) \times SO(16)$



$$g_{he} = R_{11}^{3/2}$$
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$$g_{I} = R_{10}/R_{11}$$
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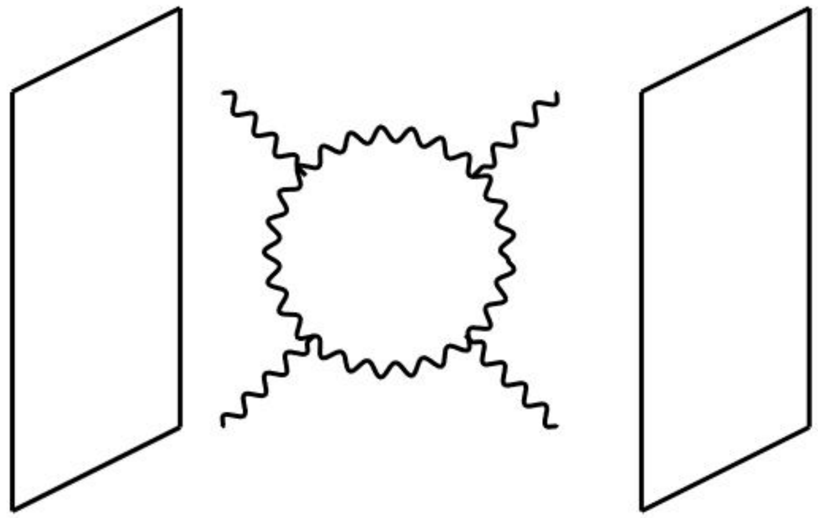
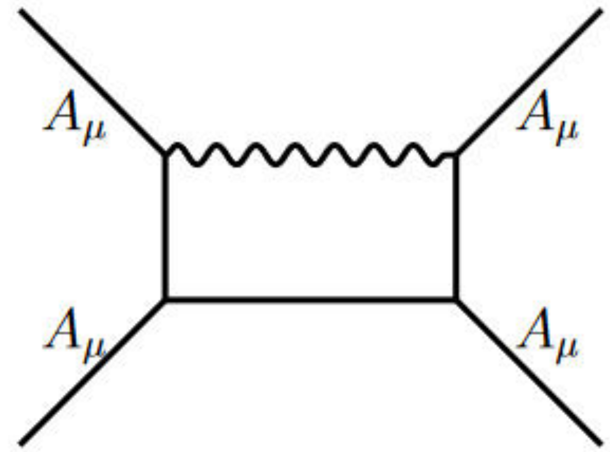
evidence for heterotic Shenker-like effects

Calculations from [Green, Rudra, '16]





In 10D HO:

$$S_{10D}^{HO} \supset \frac{g_{ho}^{-1/2}}{2^9(2\pi)^7 4! \ell_H^2} \int_{\mathcal{M}_{10}} d^{10}x\sqrt{-G}\, t_8 t_8 R^4 E_{3/2}(i g_{ho}^{-1})$$

evidence for heterotic Shenker-like effects

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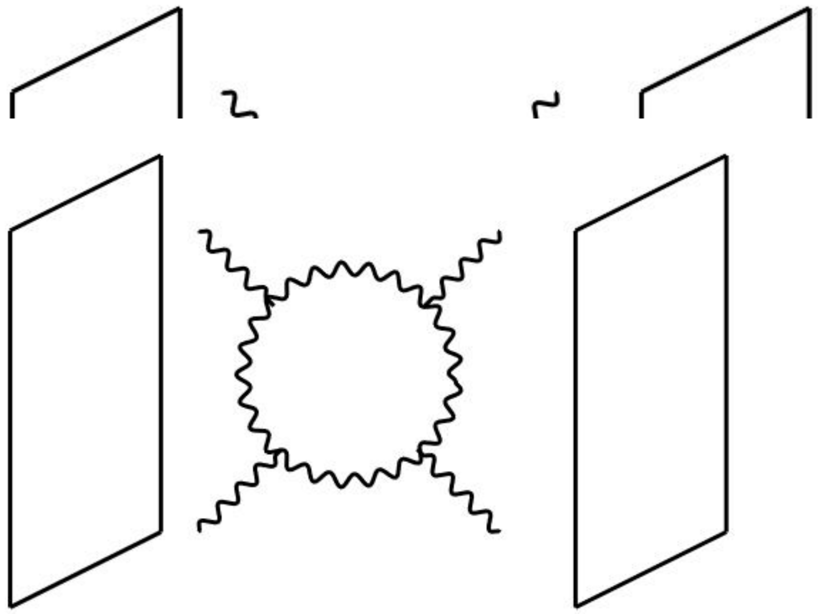

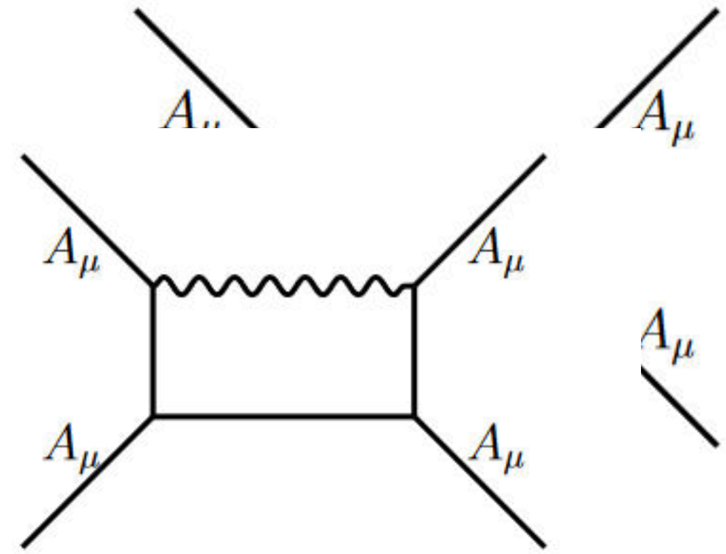

In 1(

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$$E_s(\tau) = \sum_{(c,d)}{}' \frac{y^s}{|c\tau + d|^{2s}}$$

$$\tau = x + iy$$
$$E_s(\gamma \cdot \tau) = E_s(\tau)$$

evidence for heterotic Shenker-like effects

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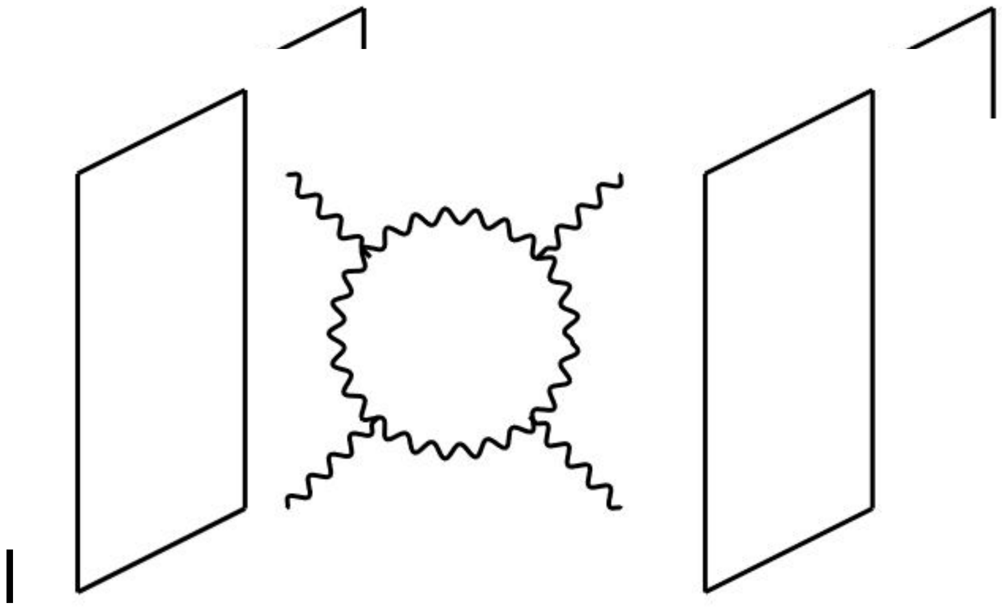

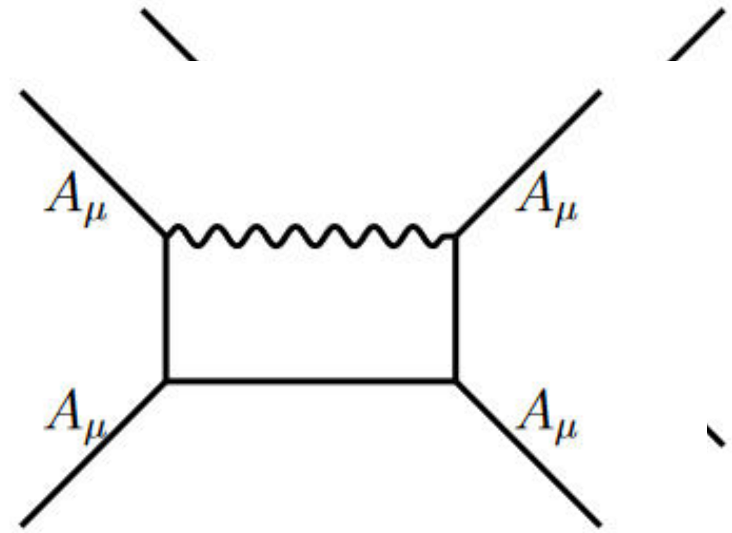

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evidence for heterotic Shenker-like effects

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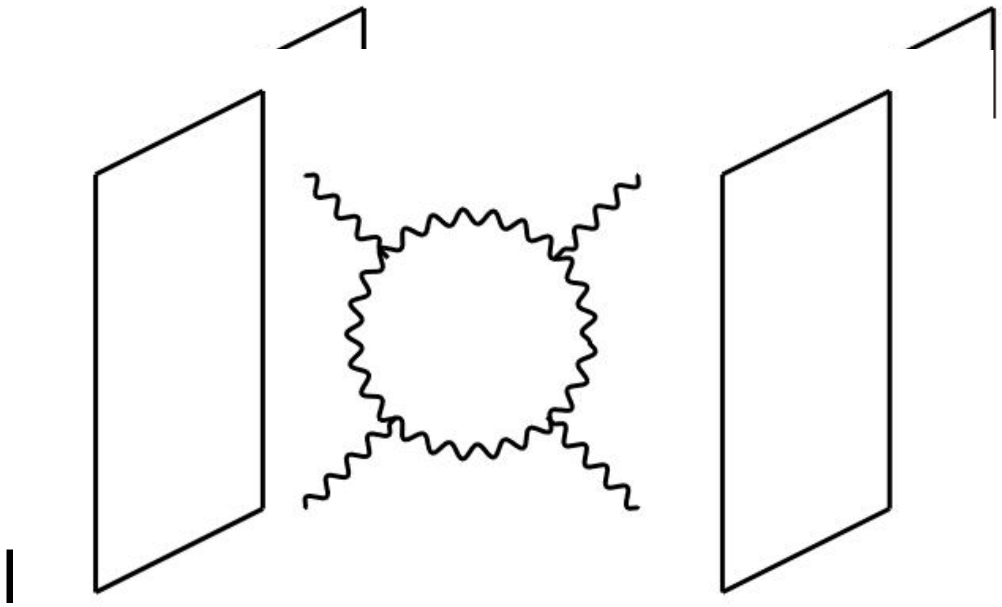

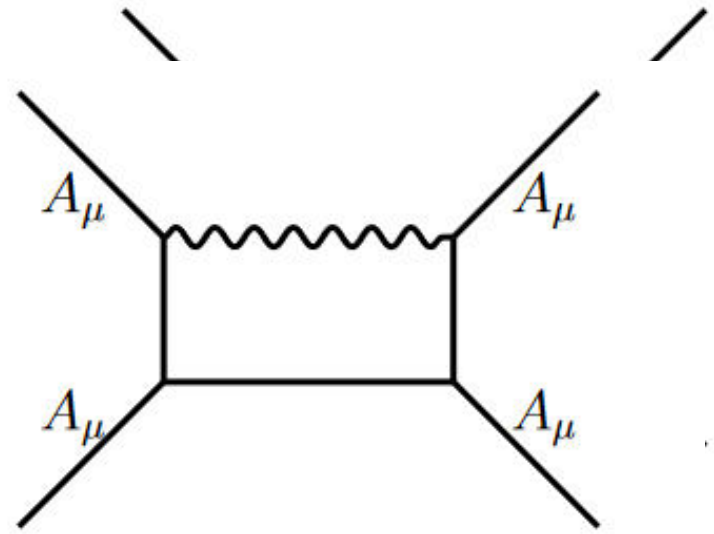

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Shenker-like Terms

ote: Similar terms vanish in 10D HE

evidence for heterotic Shenker-like effects

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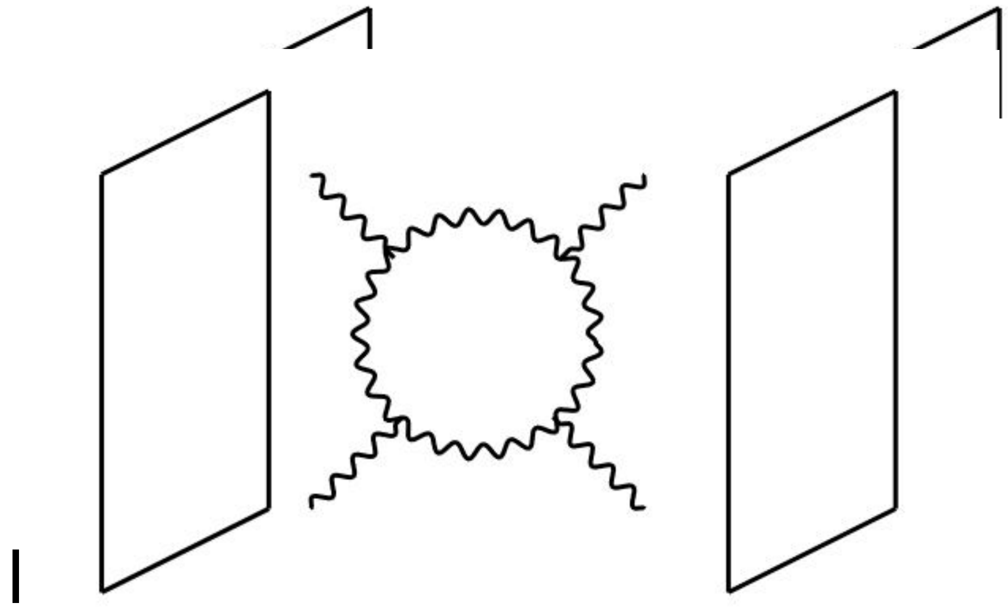

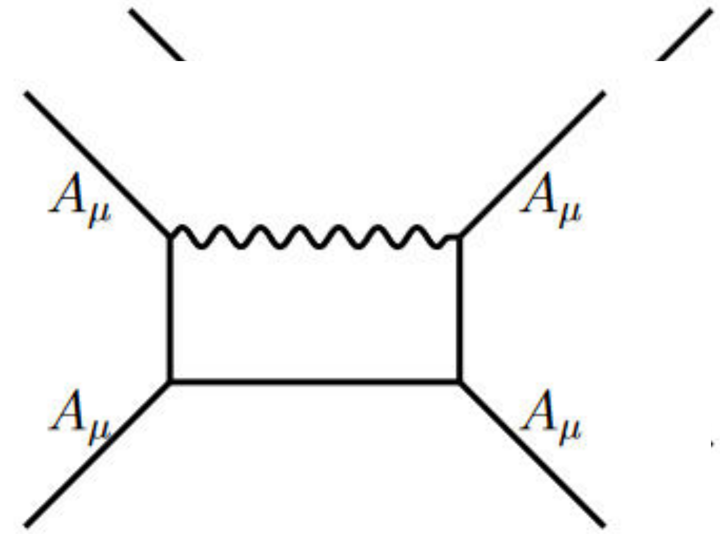

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Shenker-like Terms

Note: Similar terms vanish in 10D HE

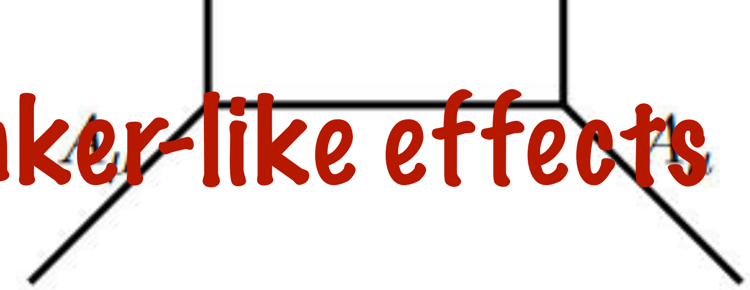
evidence for heterotic Shenker-like effects

- But Why? Dualities

  Back to 9D: SO & Type I are S-Dual via $g_{ho} \leftrightarrow g_I^{-1}$

$$\Big(\qquad \Big)$$

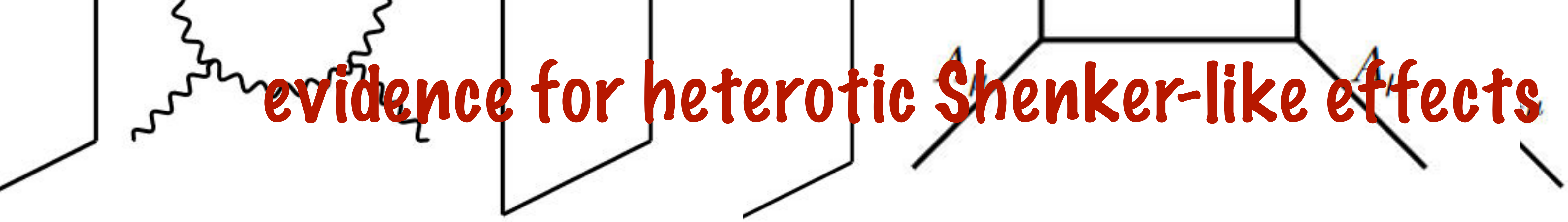
evidence for heterotic Shenker-like effects



- But

Back to 9D: SO & Type I are S-Dual via $g_{ho} \leftrightarrow g_I^{-1}$

$$S_{9D}^{HO} \supset \frac{r_{ho}}{2^9(2\pi)^6 4!\ell_H} \int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_{ho}^2}\right) t_8 t_8 R^4 + \cdots \qquad S_{9D}^{I} \supset \frac{r_I}{2^9(2\pi)^6 4!\ell_I} \int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_I^2}\right) t_8 t_8 R^4 + \cdots$$



evidence for heterotic Shenker-like effects

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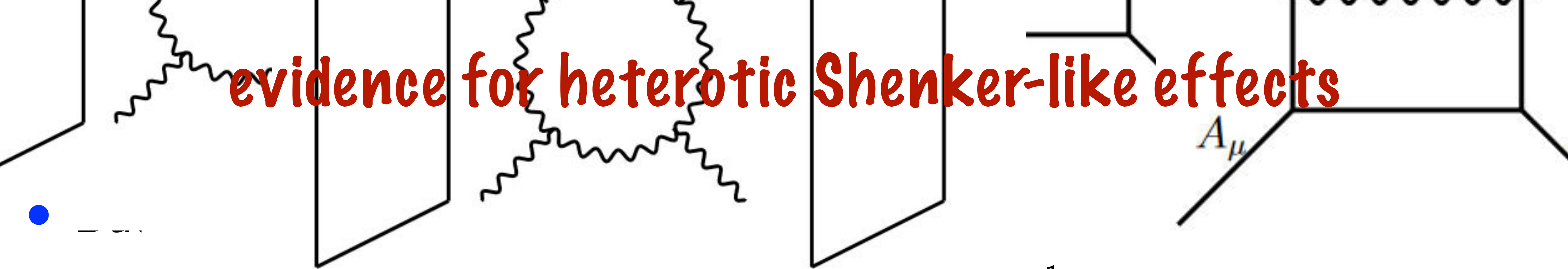
$$\Longrightarrow \frac{r_{ho}}{\ell_H} 2\zeta(3) g_{ho} t_8 t_8 R^4$$



evidence for heterotic Shenker-like effects

- Back t

$\leftrightarrow$ $^{-1}$

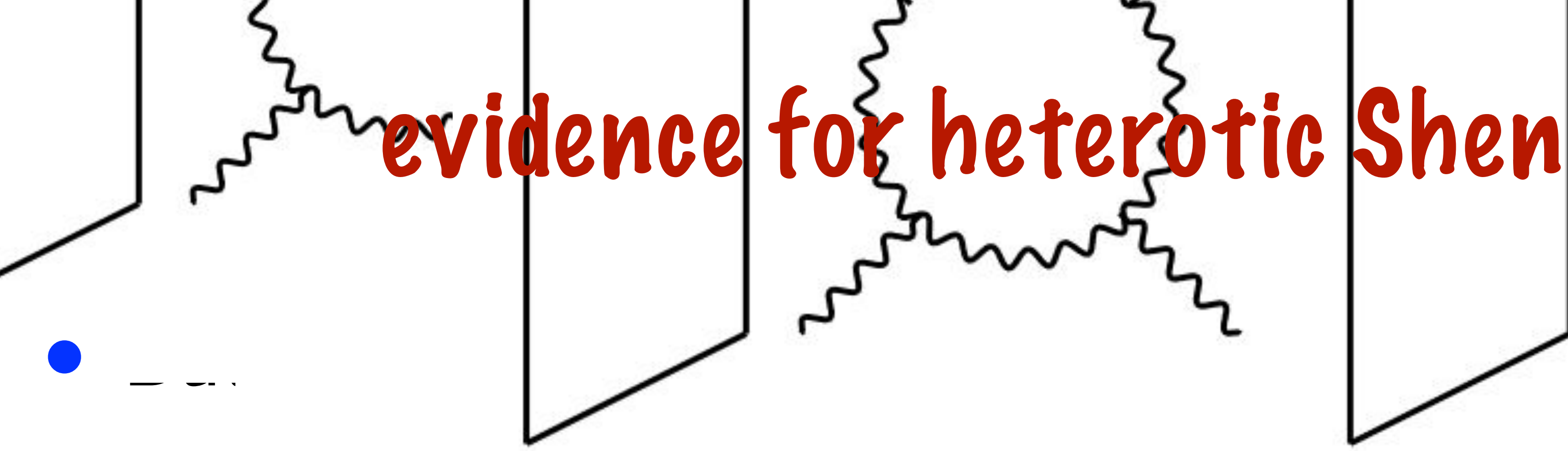
$$S^{HO}_{9D} \supset \frac{r_{ho}}{2^9(2\pi)^6 4!\ell_H}\int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_{ho}^2}\right)t_8t_8R^4+\cdots \qquad S^{I}_{9D} \supset \frac{r_{I}}{2^9(2\pi)^6 4!\ell_I}\int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_{I}^2}\right)t_8t_8R^4+\cdots$$

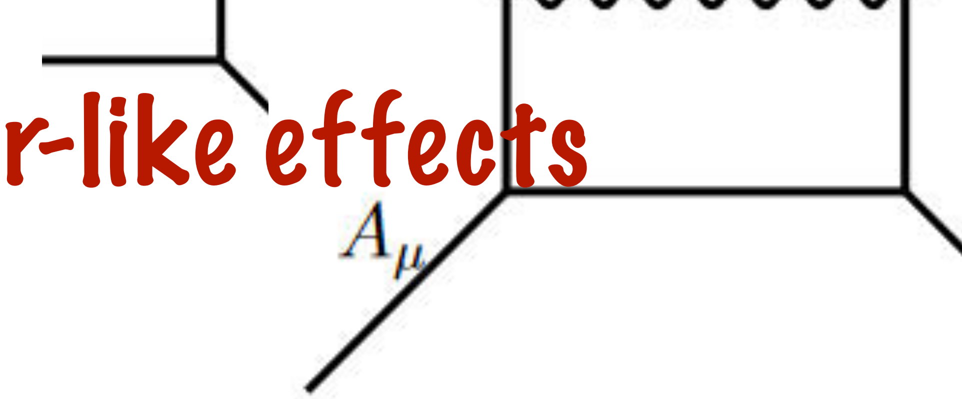
$$\Longrightarrow \frac{r_{ho}}{\ell_H}2\zeta(3)g_{ho}t_8t_8R^4$$

The $\frac{r_i}{\ell_i\sqrt{g_i}}f(g_i)t_8t_8R^4$ term requires a coefficient such that

$$f(ig_{ho}^{-1}) = f(ig_I^{-1})$$

evidence for heterotic Shenker-like effects





- Back t

-1

$\leftrightarrow$

$$S_{9D}^{HO} \supset \frac{r_{ho}}{2^9(2\pi)^6 4!\ell_H} \int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_{ho}^2}\right) t_8 t_8 R^4 + \cdots \qquad S_{9D}^{I} \supset \frac{r_{I}}{2^9(2\pi)^6 4!\ell_I} \int d^9x\sqrt{-G}\left(\frac{2\zeta(3)}{g_{I}^2}\right) t_8 t_8 R^4 + \cdots$$

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$$f(ig_{ho}^{-1}) = f(ig_I^{-1})$$

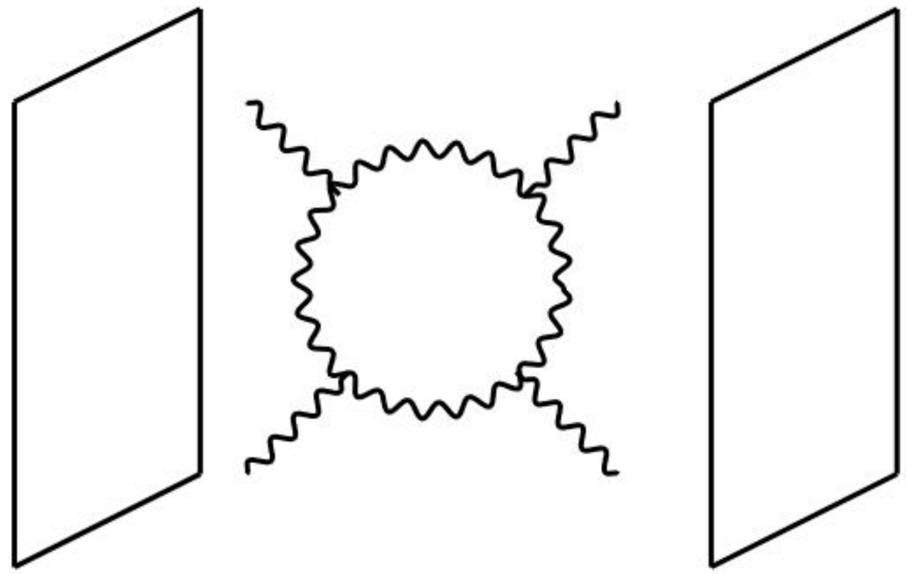
satisfied by the real-analytic Eisenstein series $E_s(\tau)$

s is determined by matching the perturbative part

evidence for heterotic Shenker-like effects

- And Self-Duality

A 9D



on T^2

$g_{IIB}^{-\frac{1}{2}}$ E

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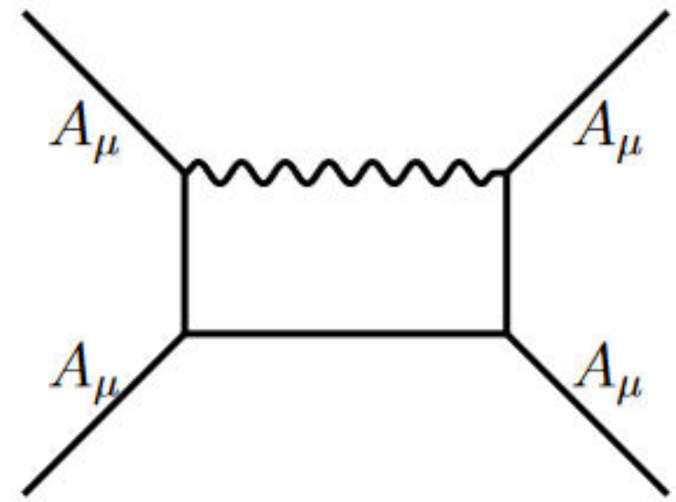

t C_0,

eories

- From What?

In Type I:

Non-BPS type I D-instantons. Responsible for O(32) $\Rightarrow$ SO(32) [Witten, '98]

In T-dual IIA frame, these are D-particles winding around the orbifolded x^{11} direction [Dasgupta, Gaberdiel, Green, '00]

In Heterotic: Unclear

sketch of proof of theorem 1 (& similarly, 2)

Proof: The proof by contradiction – assume ①-④ are true at (T_0, S_0)

$$\partial_S V(T,S) = \frac{F_S}{W} V(T,S) + \left\{ e^{k(S,\bar{S})} |\Omega(S)|^2 |H(T)|^2 Z(T,\bar{T}) \right\} \partial_S A(S,\bar{S})$$ $\Rightarrow$ vanishes by ③

$$\Rightarrow \partial_T^k \partial_{\bar{T}}^l \partial_S V(T_0, S_0) = 0$$ $\Rightarrow$ Hessian is block diagonal

d: To satisfy ①, introduce $\Lambda > 0$ such that

$$V(T_0, S_0) = e^{k_0} |\Omega_0|^2 Z_0 \Lambda^4$$

which yields an expression for $H_T(T_0)$:

$$H_T(T_0) = \frac{3i}{2\pi} H_0 \widehat{G}_2(T_0, \bar{T}_0) \pm \frac{\sqrt{3}i}{T_0 - \bar{T}_0} \left(\Lambda^2 \pm i \sqrt{|H_0|^2 \left(3 - A(S_0, \bar{S}_0)\right)} \right)$$

$A(S_0, \bar{S}_0) = 0$ by (iii)

The 2nd condition in ② gives a (long) expression for $H_{TT}(T_0)$

Plug these into the T-modulus sector of the Hessian:

$$(\partial_T \partial_{\bar{T}} V)_0 \propto -2\Lambda^4 < 0 \Rightarrow \quad \begin{aligned} \partial_t^2 V &= 2\partial_T \partial_{\bar{T}} V - 2\mathrm{Re}(\partial_T^2 V) \\ \partial_a^2 V &= 2\partial_T \partial_{\bar{T}} V + 2\mathrm{Re}(\partial_T^2 V) \\ \partial_t \partial_a V &= -2\mathrm{Im}(\partial_T^2 V) \end{aligned}$$

Cannot both be positive

$\Updownarrow$

dS minima not possible