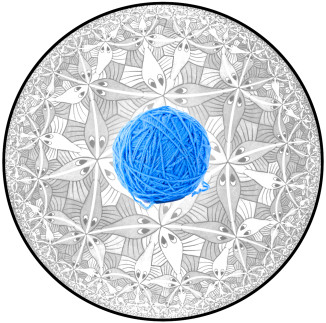
WEIZMANN INSTITUTE OF SCIENCE

THE BLACK HOLE/STRING TRANSITION IN ANTI DE SITTER SPACE

Erez Y. Urbach

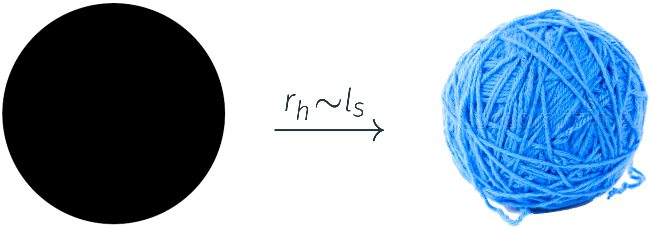
Based on 2202.06966 and 2303.09567.

DIP Swampland and Holography, BGU 2023



What is the black hole/string transition?

Consider weakly coupled $g_s \ll 1$ string theory in flat space R^{d+1}.



Conjecture: the microcanonical black hole/string transition

Black hole microstates in the limit $r_h \to l_s \equiv$ High energy string theory excitations.
[Veneziano '86] , [Susskind '93] , [Horowitz & Polchinski '96] ....

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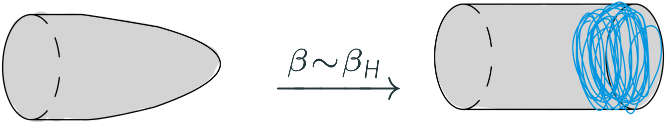


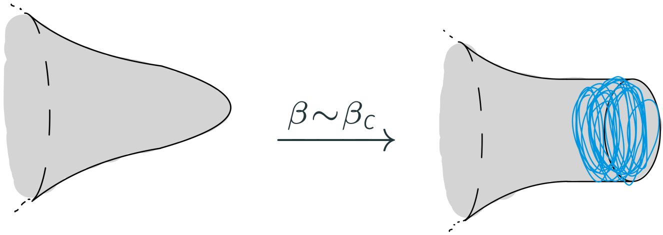
Euclidean Schwarzschild

String star
$R^d \times S^1_\beta$ + winding-strings

Conjecture: the canonical black hole/string transition

The Euclidean Schwarzschild saddle is continuously connected (by a line of worldsheet CFTs) to the thermal saddle $R^d \times S^1_\beta$ at $\beta = \beta_H \sim l_s$ (Hagedorn), via a normalizable condensate of strings winding on S^1_β.
[Horowitz & Polchinski '97] , [Chen, Maldacena, Witten '21] , [Brustein & Zigdon '21] ...

What is the black hole/string transition in AdS?



(small) AdS Black Hole

AdS string star

Thermal AdS + winding-strings

Conjecture: the AdS$_{d+1}$ black hole/string transition

The Euclidean AdS$_{d+1}$ black hole saddle is continuously connected (by a line of worldsheet CFTs) to the thermal AdS$_{d+1}$ at Hagedorn $\beta = \beta_c \sim l_s$ via a normalizable condensate of strings winding on the thermal circle.
[Alvarez-Gaume et al. '05-06] , [Jafferis et al. '22-23] , [EYU '22]

The plan

1. The **AdS_{d+1} string star**, for $d > 2$. [EYU '22]

2. The **AdS_3 string star**, with mixed RR+NS-NS or pure NS-NS backgrounds. [EYU '23]

THE ADS$_{d+1}$ STRING STAR, FOR $d > 2$

String theory on thermal AdS_{d+1}

Consider the string theory partition function on asymptotically thermal AdS_{d+1}:

$$Z_{AdS}(\beta) = \sum_{\text{saddles}} e^{-I(\beta)}(1 + O(G^0)),$$

with $I = \frac{1}{16\pi G} \int d^{d+1}x\sqrt{g}\,(2\Lambda - \mathcal{R} + ...)$, and $\Lambda = -d(d-1)/(2l_{ads}^2)$.
Holographically dual to the CFT$_d$ partition function on $S^{d-1} \times S^1_{\beta/l_{ads}}$.

Two saddles [Hawking & Page '83] :

1. **Thermal AdS** ($t \sim t+1$) with $I(\beta) = 0$ and

$$ds^2 = \beta^2 \cosh^2(\rho/l_{ads})dt^2 + d\rho^2 + l_{ads}^2 \sinh^2(\rho/l_{ads})d\Omega_{d-1}^2.$$

   ρ is the radial coordinate, $\rho \to \infty$ is the boundary.

2. **Euclidean AdS black holes**
   Reliable whenever curvature is small, $\beta \gg l_s$, otherwise needs higher derivative corrections.

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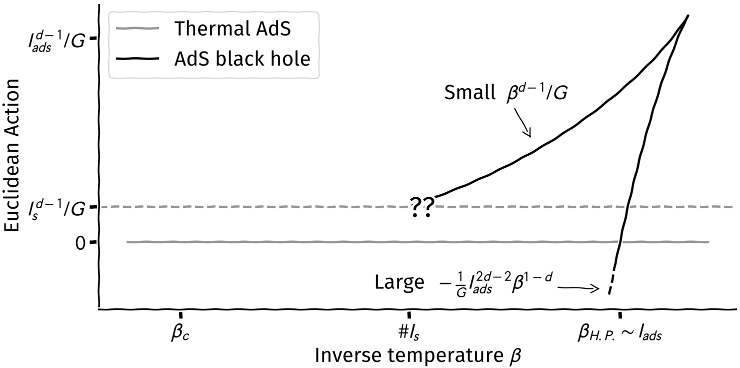
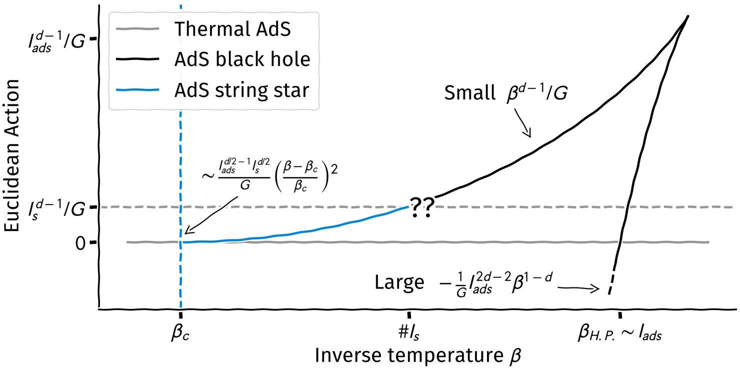


What happens to the small AdS black hole phase around $\beta \sim l_s$?



Conjecture: the AdS$_{d+1}$ black hole/string transition

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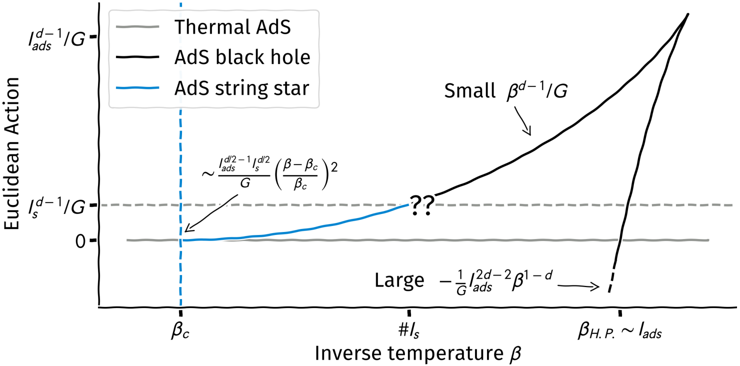
Holographic $\mathrm{CFT}_{(d)}$ on $S^{d-1} \times S^1$



Conjecture: a CFT_d black hole/string transition

On $S^{d-1} \times S^1$, the large N deconfined phase saddle is continously connected (via a line of large N saddles) to the confined phase (gas) at Hagedorn.
Support from weak coupling [Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk '03] etc.

Method: Atick-Witten EFT around thermal AdS

For temperatures $0 < \beta - \beta_H \ll l_s$ we can reduce on the thermal circle, and get the effective field theory [Atick & Witten '88]

$$I^{(d)} = \frac{1}{16\pi G} \int d^d x \sqrt{g^{(d+1)}} \Big\{ (\nabla\varphi)^2 + |\nabla\chi|^2 + m^2(x)|\chi|^2 + \frac{\kappa}{\alpha'} \varphi |\chi|^2 \Big\}.$$

1. $\varphi(x)$ is the metric back-reaction: $g_{tt} = \beta^2 \cosh^2(\rho/l_{ads}) \exp(2\varphi)$.

2. $\chi(x)$ is the field associated to the first winding string around S^1_β. The mass

$$\alpha' m^2(x) = \kappa \frac{\sqrt{g_{tt}} - \beta_H}{\beta_H} = \kappa \frac{\beta \cosh(\rho/l_{ads}) - \beta_H}{\beta_H},$$

with κ an $O(1)$ number, and $\beta_H \sim l_s$ the flat-space Hagedorn temperature.

3. The EFT is reliable only near the Hagedorn temperature $0 < \beta - \beta_H \ll l_s$, $|\chi|, |\varphi| \ll 1$.

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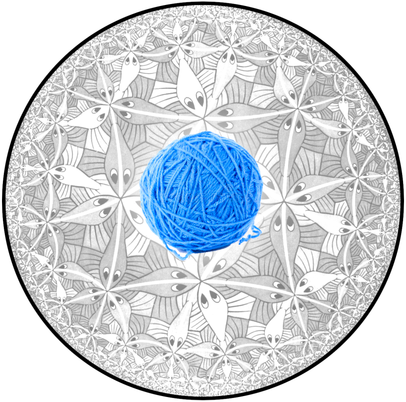
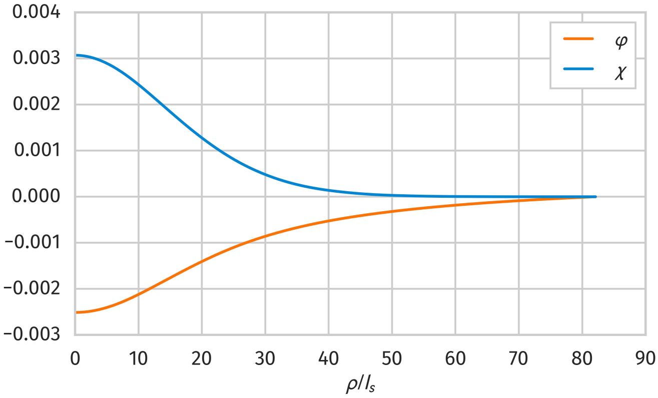
The AdS string star

The "**AdS string star**" is the spherically-symmetric bound state solution $\chi(\rho), \varphi(\rho)$ of the EFT, centered around the origin $\rho = 0$:



Possible interpretation: a gravitational bound state of hot strings.

The AdS string star - example



Small solutions $l_s/l_{ads} \ll \frac{\beta-\beta_H}{\beta_H} \ll 1$

- The AdS string star coincides with the flat-space solution of [Horowitz & Polchinski '97] , [Chen, Maldacena, Witten '21] .
  The saddle's amplitude, size L and Euclidean action

$$|\chi|, |\varphi| \sim \frac{\beta-\beta_H}{\beta_H}, \quad L/l_s \sim \left(\frac{\beta-\beta_H}{\beta_H}\right)^{-\frac{1}{2}}, \quad I(\beta) \sim \frac{l_s^{d-1}}{G} \cdot \left(\frac{\beta-\beta_H}{\beta_H}\right)^{\frac{4-d}{2}}.$$

- The small AdS black hole $l_s \ll \beta \ll l_{ads}$ coincide with Schwarzschild:

$$r_h \sim \beta, \quad I(\beta) \sim \frac{\beta^{d-1}}{G}. \tag{1}$$

- Different regimes of validity. Agree when extrapolated to $\frac{\beta-\beta_H}{\beta_H} \sim O(1)$:

$$L, r_H \sim l_s, \quad I(\beta) \sim \frac{l_s^{d-1}}{G}. \tag{2}$$

Gives a possible black hole/string transition in AdS, based on flat space.

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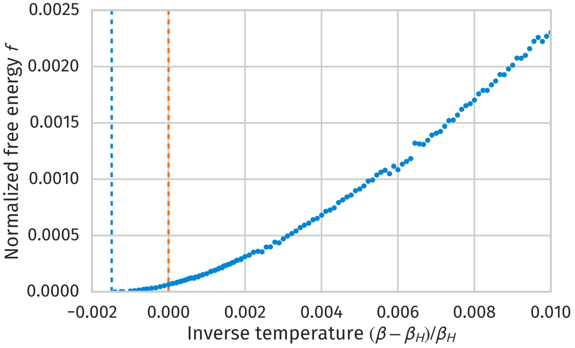
- The amplitude and size L of the solution is

$$|\chi|, |\varphi| \sim \frac{l_s}{l_{ads}}, \quad L \sim \sqrt{l_s l_{ads}}.$$

- Close to the AdS Hagedorn temperature β_c, the Euclidean action is

$$I(\beta) \sim \frac{l_{ads}^{d/2-1} l_s^{d/2}}{G} \left(\frac{\beta-\beta_c}{\beta_c}\right)^2.$$



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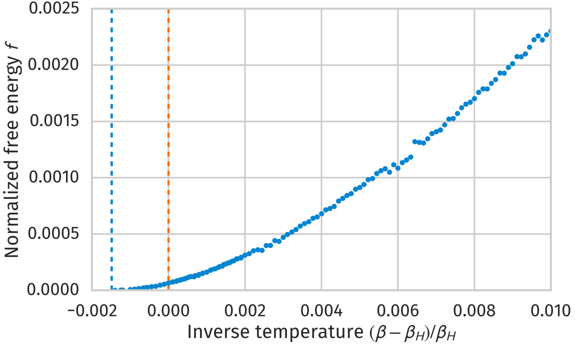

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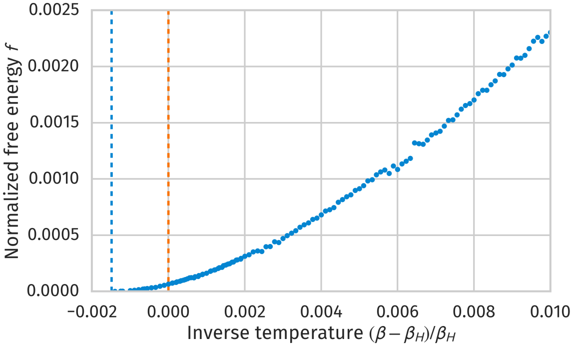

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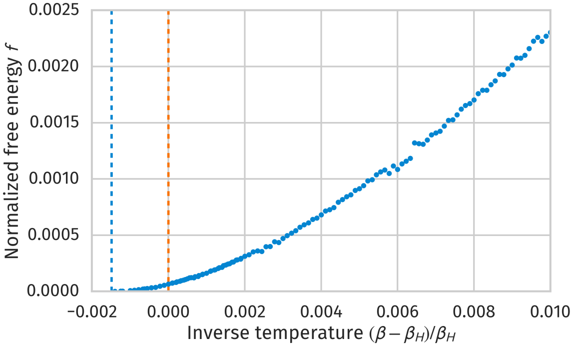

Thermodynamic instabilities

- The AdS string star is non-perturbatively meta-stable, just like small AdS black holes.

- Close to β_c the AdS string star has negative specific heat, and possess a normalizable negative eigenmode (similar to black holes [Gross, Perry, Yaffe '82]).

- In 10d, there exist a Gregory-Laflamme-like instability whenever extra dimensions are present at the AdS curvature scale, just like small black holes.

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- For IIB on $AdS_5 \times S^5$ ($d = 4$), it gives the $\mathcal{N} = 4$ SYM Hagedorn temperature (also [Maldacena, unpublished])

$$T_c = \frac{1}{\sqrt{8\pi^2}}\lambda^{1/4} + \frac{1}{2\pi} + O(\lambda^{-1/4}),$$

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- In three-dimensional flat space:
  1. No Schwarzschild solutions.
  2. No string star solutions.

  No need for a black hole/string transition...

- But, in AdS_3 there are BTZ black holes!
  What about AdS_3 string stars?

- Also, string theory on AdS_3 is interesting by itself, as we can turn on NS-NS flux H_3.

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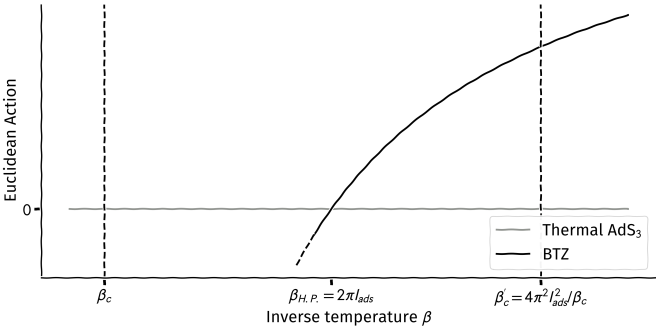
A BTZ/string transition for AdS_3?



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- The small BTZ solution has very low temperature $\beta = 2\pi/r_h$, not high! No simple black hole/string transition!
- Thermal AdS_3 is tachyoinic above the Hagedorn temperature $\beta_c \sim l_s$. The BTZ is tachyonic below temperature $\beta_c' = 4\pi^2 l_{ads}^2/\beta_c$.

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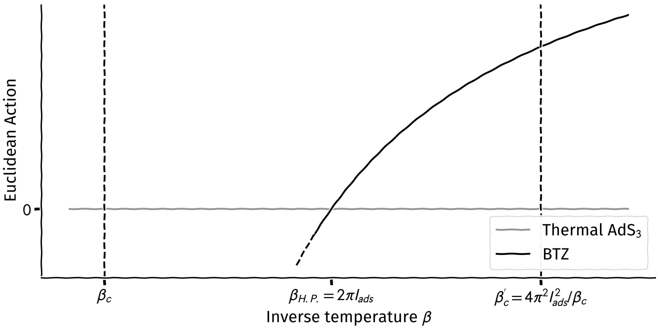

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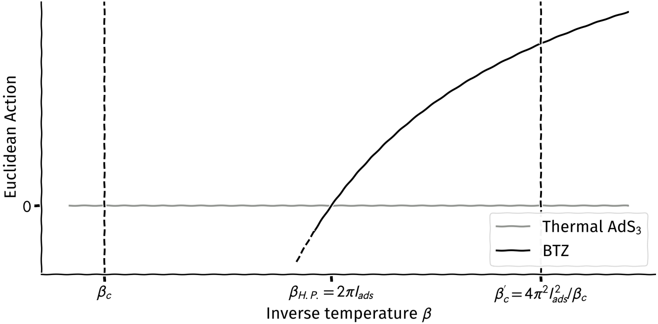

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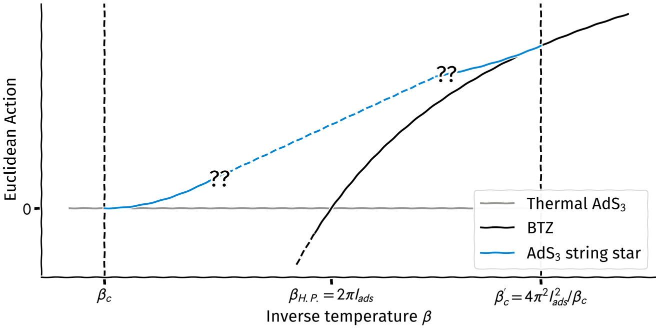

Conjecture: the BTZ/string transition

There exist a line of worldsheet CFTs connecting thermal AdS_3 at Hagedorn β_c (via a normalizable winding condensate on S^1_β) to the Euclidean BTZ saddle at $\beta_c' = 4\pi^2/\beta_c$ (via a normalizable winding condensate on S^1_ϕ).
[Berkooz, Komargodski, Reichmann ’07] [Halder, Jafferis, Kolchmeyer ’22] [EYU ’23]

The mixed RR and NS-NS $AdS_3 \times S^3 \times M_4$ background

Consider the type IIB background of $\mathrm{AdS}_3 \times S^3 \times M_4$ (M_4 is T^4 or K3).

The thermal AdS_3 component [Cho, Collier, Yin '20]

$$\begin{aligned} ds^2 &= d\rho^2 + \sinh^2(\rho) d\phi^2 + \beta^2 \cosh^2(\rho) dt^2, \\ B &= i\beta\lambda \sinh^2(\rho) d\phi \wedge dt, \\ C &= i\beta\sqrt{1-\lambda^2} \sinh^2(\rho) d\phi \wedge dt, \end{aligned}$$

with $t \sim t+1$ and $0 \leq \lambda \leq 1$.
$\lambda = 1$ is the pure NS-NS solution.

Atick-Witten 2d EFT close to Hagedorn

The NS-NS flux is important!
The F1 string couples electrically to B. As a result, the string winding mode $\chi(x)$ couples to B at leading order.

1. The background NS-NS flux reduces the mass by

$$\alpha' m_{\text{eff}}^2(x) = \frac{4\pi^2}{\alpha'} \left(\beta^2 \left(\cosh^2(\rho) - \lambda^2 \sinh^2(\rho)\right) - \beta_H^2\right); \quad \beta_H^2 = 8\pi^2\alpha'.$$

2. Define the back-reacted gauge-field $\zeta(x)$

$$B \equiv i\beta\lambda \sinh^2(\rho)(1 + \zeta(x)) d\phi \wedge dt.$$

The EFT include a leading cubic term $\zeta|\chi|^2$, just like the metric $\varphi(x)$.
Interpretation: the hot strings attract gravitationally and repel electrically.

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Finite RR flux: $1-\lambda^2 \gg \alpha'/l_{ads}^2$.

For near-Hagedorn temperatures $\frac{\beta-\beta_H}{\beta_H} \sim l_s/l_{ads}$:

1. ζ contributes at subleading $1/L^2 \ll 1$ order, and can be neglected.
2. The solution for χ and φ has amplitude and size

$$|\chi|, |\varphi| \sim \frac{l_s}{l_{ads}}\sqrt{1-\lambda^2}, \quad L \sim \sqrt{l_s l_{ads}}(1-\lambda^2)^{-1/4}.$$

3. The AdS$_3$ Hagedorn temperature

$$\beta_c = 2^{3/2}\pi l_s \left(1-\sqrt{\frac{1-\lambda^2}{2}} l_s/l_{ads} + O(l_s^2/l_{ads}^2)\right).$$

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Almost pure NS-NS: $1-\lambda^2 \sim \alpha'/l_{ads}^2$

1. ζ is no longer negligible.
2. The amplitude and size L of the solution are

$$|\chi|, |\varphi|, |\zeta| \sim \frac{\alpha'}{l_{ads}^2}, \quad L \sim l_{ads}.$$

   The amplitude is much smaller, and the solution is now AdS-sized!

A qualitatively different type of AdS string star!

At the pure NS-NS $\lambda = 1$:

- The effective mass is a constant $\alpha' m_{\text{eff}}^2 = \frac{1}{4\pi^2\alpha'}(\beta^2 - \beta_H^2)$, just like in flat-space.
- Close to Hagedorn, the string star (and the corresponding Atick-Witten Tachyon) is delta-function normalizable. [Berkooz, Komargodski, Reichmann '07] Lorentzian interpretation: due to the (continuous) long strings spectrum in AdS_3. [Maldacena, Michelson, Strominger '98] , [Seiberg & Witten '99]

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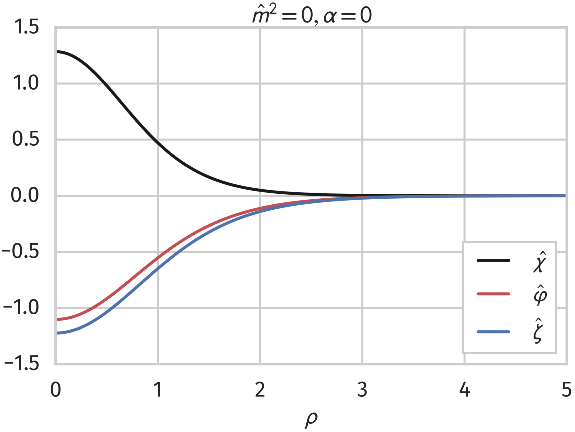

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    - Coincides with flat space at lower temperature, allows for a black hole/string transition in AdS.
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