The double scaled SYK model

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Based on work in with N. Brukner, M. Isachenkov, P. Narayan, V. Narovlansky, A. Raz, J. Simon, J. Torrents, M. Watanabe... ;

Consider the SYK model with interaction length p.

(*Sachdev, Ye; Georges, Pacollet, Sachdev; Kitaev; Polchinksi, Rosenhaus; Maldacena, Stanford, Jensen; Almheiri, Verbaarschot, Jia, Altland, Bagrets, Kamenev, Yang, Saad, Verlinde, Turiaci, ...*).

The "double scaled" limit (*Erdos and Schroder; also Cotler et al; Garcia-Garcia, Jia, Verbaarschot...; Math side: Speicher, Bozejko,..)*

$$N \to \infty,\ \lambda = 2\frac{p^2}{N}\ fixed \qquad (alternate\ parameter\ q = e^{-2\lambda})$$

Standard low energy JT limit: take then $\lambda \to 0$ ($q \to 1$) and $E - E_0 \to 0$ at the same time. We recover the Schwarzian theory ("triple scaled SYK", *Cotler et al*).

Scrambling time: OTOC $\sim \lambda^{\alpha} e^{\lambda_L t}$, so λ replaces the $1/N$. So solving for any λ means solving exactly in $1/M_p$.

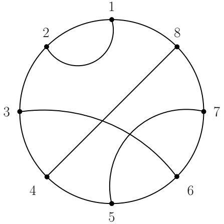
Main points

- The double scaled theory can be solved exactly at large N using chord diagrams (at all energy scales, finite times).
- Crank the handle and out comes the bulk.
- Action and auxiliary Hilbert space (generzliation of the Schwarzian to all energy scales).
- Gravitational dual = q-version of free probability theory (beyind the 't Hooft's double line).
- Judicious choice of observables which are universal throughout many models.
- Observables and Correlation functions are governed by the quantum groups $SL_{q^{1/2}}(2)$.
- Quantized geometry - AdS_2 is replaced by a q-deformed $AdS_{2,\sqrt{q}}$

Basic technique: Evaluate the averaged partition function by expanding and computing moments of H

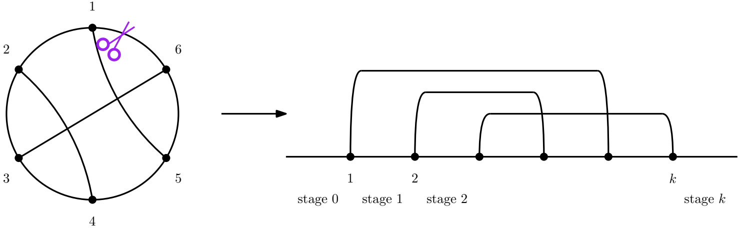
$$\langle \operatorname{tr} e^{-\beta H} \rangle_J = \Sigma \frac{(-\beta)^k}{k!} \langle \operatorname{tr} H^k \rangle_J = lim_{L\to\infty} \langle (1-H)^{\beta/\epsilon} \rangle_J$$

The trace is normalized $\operatorname{tr} 1 = 1$, $\langle \rangle_J$ stands for ensemble average.

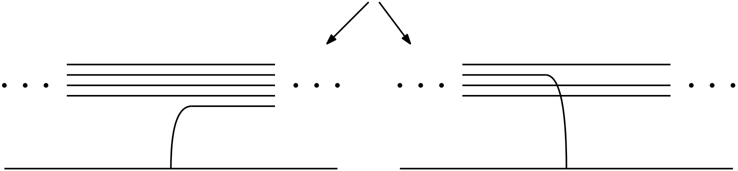


$$\langle \operatorname{tr} H^k \rangle_J = \sum_{\text{Chord diagrams}} q^{\#\text{ intersections}}$$

To evaluate the sum over chord diagrams efficiently we cut open the chord diagrams



and write a "transfer matrix" that implements this as process going from left to right. At each stage we can either "open" a new chord (at the bottom of the stack) or "close" one of the chords from the stack.



At each stage have a number of chords $l = 0, 1, 2, \cdots$. Introduce

$$\mathcal{H}_{aux} = Span\{|l\rangle, \quad l = 0, 1, 2..\} \tag{1}$$

This will become he gravitational Hilbert space.

The recursion relation then becomes

$$T = \begin{pmatrix} 0 & \frac{1-q}{1-q} & 0 & 0 & \cdots \\ 1 & 0 & \frac{1-q^2}{1-q} & 0 & \cdots \\ 0 & 1 & 0 & \frac{1-q^3}{1-q} & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \langle \operatorname{tr} H^k \rangle_J = \langle 0 | T^k | 0 \rangle$$

In any correlation function

$$QM \to GR$$

$$\langle Tr(...e^{itH}...)\rangle_J \to \langle ...e^{it\hat{T}}...\rangle_{\mathcal{H}_{aux}}$$

$$"H" \to \hat{T} = T \ conjugated \ to \ be \ Hermitian$$

The ordinary geometric limit (+ JT gravity=Schwarzian) is obtained

- $\beta \to \infty$ - many chords
- $q \to 0$ - can go long "distances"

A picture very similar to Euclidean AdS_2 (Euclidean).

I kid you not.

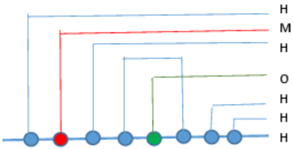
Actually this limit contains more since we can stay away from the strict Schwarzian limit - new kinds of geometries (non maximal chaos).

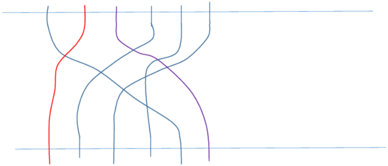
Particles: $p' \sim \sqrt{N}$

$$M = i^{p_A/2} \sum_{1 \leq i_1 < \cdots < i_{p_A} \leq N} {J'}_{i_1 \cdots i_{p_A}} \psi_{i_1} \cdots \psi_{i_{p_A}}$$

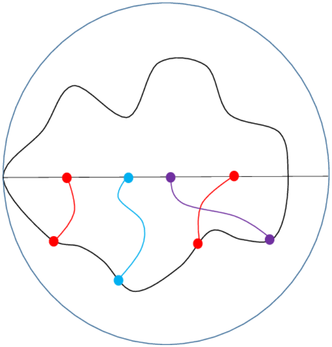
New J''s are again random, independent, with zero mean. The IR conformal dimension will turn out to be p'/p_0. Main argument: all operators should be a similar statistical class as H, maybe with a different conformal dimension.

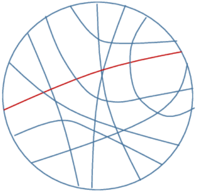
We obtain the Hilbert space in the bulk on the nose (+ natural definition of norm).





We can also be more specific about how the Hilbert space shows up in the bulk (see also *Lin*)





Renormalized 2-pt function

$$GR: \quad \langle OO \rangle \sim e^{-Ml} \sim e^{-\Delta l/R_{AdS}}$$

$$DS-SYK: \; = e^{-\frac{p_0 p'}{N^2}K} = e^{-\Delta \frac{p_0^2}{N}K} = e^{-\Delta |ln(q)| K}$$

K - no of intersections. Any Δ

$$l = |ln(q)| K R_{AdS}$$

Lines are "geodesics". Length is quantized by $|ln(q)|R_{AdS}|$ and determined by no of intersecting chords (not a metric).

Note that the transfer matrix can be reduced to the Liouville description of the Schwarzian.

$$Z(\beta) \sim \int_{\phi(\pm\beta/2)\to-\infty} D\phi e^{-\int_{-\beta/2}^{\beta/2}\left((\dot{\phi})^2 - e^{-\phi}\right)}$$

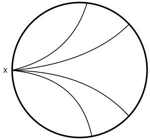


Figure: Equal Poincaré time slicing of the disk. X special point on the left

The transfer matrix goes over to the Liouville action at large n, low energies

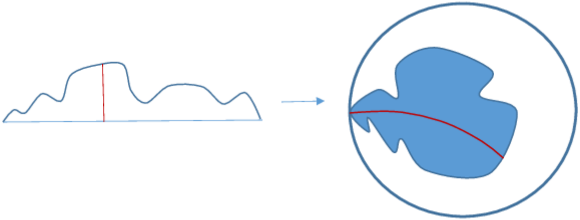
$$e^{-\phi} = q^n$$

Recall that:

$$T = \frac{1}{\sqrt{1-q}} \begin{pmatrix} 0 & 1-q & 0 & 0 & \cdots \\ 1 & 0 & 1-q^2 & 0 & \cdots \\ 0 & 1 & 0 & 1-q^3 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \langle \mathrm{tr}\, H^k \rangle_J = \langle 0 | T^k | 0 \rangle$$

The envelope

$$Env(CD) = \{n(t) |\ t = k\epsilon, k = 0, \ldots, \beta/\epsilon\},$$
$$n(0) = n(\beta/\epsilon) = 0,\ n(t) > 0,\ n(t+\epsilon) - n(t) = 0, \pm 1$$



Before $H_{boundary\ particle} = C_2$ of $SL(2,R)$. Here the symmetry is (from the 4-pt functions) $\hat{q} = q^{1/2}$:

$$Sl(2,R) \to Sl_{2,\sqrt{q}}$$

$$AD = DA = 1,\ AB = \hat{q}BA,\ AC = \hat{q}^{-1}CA,$$
$$[B,C] = \frac{A^2 - D^2}{\hat{q} - \hat{q}^{-1}}$$
$$\Omega = \left(\frac{\hat{q}^{-1}A^2 + \hat{q}D^2 - 2}{(\hat{q}^{-1} - \hat{q}^2} + BC \right)$$

We want to replac AdS_2 by some "homogeneous space" with respect to this algebra. The "boundary curve" will be propagated by Ω.

Approach 1:

$$T\ F(\tilde{H},\tilde{z}) = F(\hat{q}^{\frac{1}{2}}\tilde{H},\tilde{z})$$
$$R\ F(\tilde{H},\tilde{z}) = F(\tilde{H},\hat{q}\tilde{z})\,.$$
$$\tilde{H} \sim e^{-\phi/2} \sim q^{n/2}$$

$$A = TR^{-1}$$
$$B = \frac{R^{a+2} - R^{a}}{\tilde{z}(\hat{q}^{a+2} - \hat{q}^{a})}$$
$$C = \frac{\tilde{z}}{\hat{q}^{-1} - \hat{q}}\left(T^{-2}R^{-a} - T^{2}R^{-a-2}\right) + i\mu\tilde{H}^{-2}T^{\tilde{a}}$$
$$\Omega = \frac{\hat{q}^{-1}T^{-2} + \hat{q}T^{2} - 2}{(\hat{q}^{-1} - \hat{q})^{2}} + i\mu\tilde{H}^{-2}\ T^{\tilde{a}}\ B = \ transfer\ matrix$$

Approach 2 (Olshanetsky-Rogov) gives a subset of the above:

Start with q-deformed AdS_3 and carry out a reduction to an AdS_2

$$H = H^*, \; Hz = \hat{q}^2 zH, \; z^* H = \hat{q}^2 H z^*$$
$$zz^* = \hat{q}^2 z^* z + (\hat{q}^2 - 1)(1 - H^{-2})\,.$$

or

$$zH = \kappa^{-1} H z,$$
$$zz^* = a z^* z - b H^{-2}, \; a = \frac{\kappa^2}{\hat{q}^2}, \; b = \frac{\kappa}{\hat{q}^2}(1 - \hat{q}^2)\,.$$

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Thank you