Energy transport for thick holographic branes

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Defects have several applications

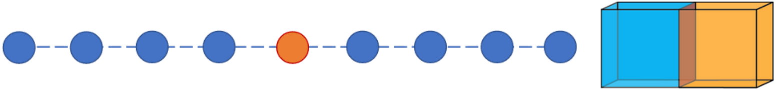
- Extended probes in QFT
- Condensed matter physics: junction of quantum wires [Wong, Affleck, 1993], quantum Hall liquids [Fal'ko, Iordanskii, 1999], impurities in spin chains [Rommer, Eggert, 2000]
- Holography: dynamical branes in AdS [Karch, Randall, 2000] [DeWolfe, Freedman, Ooguri, 2001]
- Supergravity solutions (Janus) [Bak, Gutperle, Hirano, 2007]
- Playgrounds for computations in quantum information [Almheiri, Engelhardt, Marolf, Maxfield, 2019][Penington, 2019][Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2019][Azeyanagi, Karch, Takayanagi, Thompson, 2007][Gutperle, Trivella, 2016][Chapman, Ge, Policastro, 2018][Braccia, Cotrone, Tonni, 2019][Auzzi, SB, Bonansea, Nardelli, Toccacelo, 2021]
- Relation to boundaries [Bachas, de Boer, Dijkgraaf, Ooguri, 2002]

Outline

1. 2d conformal interfaces
2. Thin brane model
3. Thick brane model (Janus)
4. Conclusions and perspectives

2d conformal interfaces preserve many symmetries

- **Interfaces**: codimension-one extended objects splitting a system in two parts



- **Conformal interface**: separate two critical systems and preserve $\mathrm{SO}(d,1)$ conformal subgroup
- **2d**: preserve one copy of Virasoro
- Energy conservation $\Rightarrow$ gluing condition

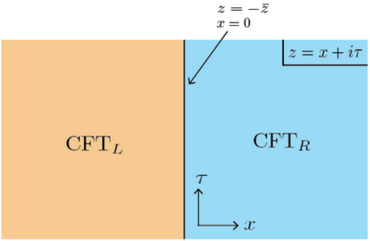
$$(T_L - \bar{T}_L)_{x=0^-} = (T_R - \bar{T}_R)_{x=0^+} \tag{1}$$

- 2pt-functions fixed by conformal symmetry

$$\langle T_L(z)T_L(w)\rangle_I = \frac{c_L}{2(z-w)^4}\,, \quad \langle T_R(z)T_R(w)\rangle_I = \frac{c_R}{2(z-w)^4}\,, \tag{2}$$

- New coefficient in left/right correlators

$$\langle T_L(z)T_R(w)\rangle_I = \frac{c_{LR}}{2(z-w)^4}\,, \tag{3}$$



2d conformal interfaces are universal

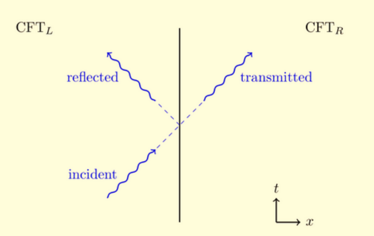
- Scattering experiment

$$\mathcal{T} = \frac{\text{transmitted energy}}{\text{incident energy}}\,, \quad \mathcal{R} = \frac{\text{reflected energy}}{\text{incident energy}} \tag{4}$$

- Different transmission from left/right

$$\mathcal{T}_L = \frac{c_{LR}}{c_L}\,, \quad \mathcal{T}_R = \frac{c_{LR}}{c_R}\,, \quad \mathcal{R}_{L(R)} = 1 - \mathcal{T}_{L(R)} \tag{5}$$

- **Universality**: energy transmitted independent of details of the excitation [Quella, Runkel, Watts, 2007] [Meineri, Penedones, Rousset, 2019]



Goal of the talk

Holographically compute the transmission coefficient

- **Thin brane model**: AdS_2 brane in AdS_3 [Bachas, Chapman, Ge, Policastro, 2020] [Baig, Karch, 2022]
- **Thick brane model**: continuous geometry with dilaton (Janus AdS_3) [Bachas, SB, Chapman, Policastro, Schwartzman, 2022]

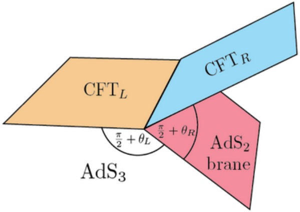
$$\mathcal{T}_{L(R)} \longleftrightarrow c_{LR}$$

Thin brane model

Holographic interfaces: bottom-up approach

- Thin AdS_2 brane in AdS_3 [Bachas, 2002][Azeyanagi, Karch, Takayanagi, Thompson, 2007]

$$S = \frac{1}{16\pi G_N}\int d^3x_L\sqrt{-g}\left(R+\frac{2}{\ell_L^2}\right) + \frac{1}{16\pi G_N}\int d^3x_R\sqrt{-g}\left(R+\frac{2}{\ell_R^2}\right) - \sigma\int d^2x\sqrt{-\gamma}$$



- Solve Einstein's equations in the left/right

$$ds^2_{L(R)} = \frac{\ell^2_{L(R)}}{\xi^2_{L(R)}}\left(-dt^2_{L(R)} + d\xi^2_{L(R)} + du^2_{L(R)}\right) \tag{6}$$

- Israel matching conditions determine the location of the brane [Israel, 1966]

$$\begin{cases}\gamma^L_{ij} = \gamma^R_{ij} \\ [K_{ij}] - [\mathrm{tr}\, K]\gamma_{ij} = 8\pi G_N \sigma \gamma_{ij}\end{cases} \quad \Rightarrow \quad \frac{\ell_L}{\cos\theta_L} = \frac{\ell_R}{\cos\theta_R} = \frac{\tan\theta_L + \tan\theta_R}{8\pi G_N \sigma} \tag{7}$$

Scattering experiments can be built at the boundary

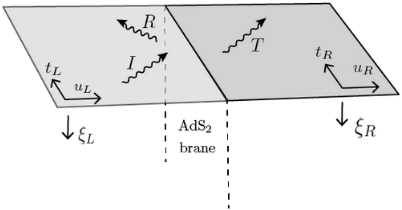
- Prepare a stress-tensor with left and right-moving waves

$$\langle T^L_{\alpha\beta}\rangle dx^\alpha_L dx^\beta_L = \varepsilon \left[1 e^{i\omega(t_L-u_L)} d(t_L-u_L)^2 + \mathcal{R}_L e^{i\omega(t_L+u_L)} d(t_L+u_L)^2\right] + \text{c.c.}$$
$$\langle T^R_{\alpha\beta}\rangle dx^\alpha_L dx^\beta_L = \varepsilon\, \mathcal{T}_L e^{i\omega(t_R-u_R)} d(t_R-u_R)^2 + \text{c.c.} \tag{8}$$

- 3d bulk solution is completely fixed

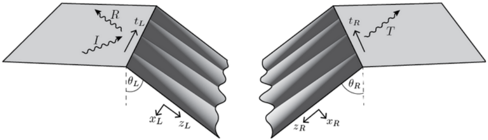
$$ds^2 = \frac{\ell^2}{\xi^2}\left[d\xi^2 + \left(g^{(0)}_{\alpha\beta} + \xi^2 g^{(2)}_{\alpha\beta} + \frac{\xi^4}{4\ell^2} g^{(4)}_{\alpha\beta}\right) dw^\alpha dw^\beta\right], \quad w^\pm = u \pm t \tag{9}$$

$$g^{(2)}_{\alpha\beta} = 4G_N \ell \langle T_{\alpha\beta}\rangle, \qquad g^{(4)} = g^{(2)}(g^{(0)})^{-1} g^{(2)} \tag{10}$$



The brane fluctuates

The perturbation changes the shape of the brane



Method:

- Impose Israel matching conditions
- Impose no-outgoing wave condition at the horizon (in the IR)

$$\hat{K}_{\pm\pm} = \frac{a_{\pm}\varepsilon\omega^2}{2\pi\sigma\ell}\, e^{i(t\pm z)} + \mathcal{O}(\varepsilon^2) \quad \Rightarrow \quad a_+ = 0$$

$\hat{K}$ traceless part of the extrinsic curvature

Transmission in single brane model

[Bachas, Chapman, Ge, Policastro, 2020]

$$\boxed{\mathcal{T}_{L(R)} = \frac{2}{\ell_{L(R)}} \left(\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N \sigma \right)^{-1}} \tag{11}$$

Range of tension [Coleman, De Luccia, 1980][Karch, Randall, 2000]:

$$\left| \frac{1}{\ell_R} - \frac{1}{\ell_L} \right| \leq 8\pi G_N \sigma \leq \frac{1}{\ell_L} + \frac{1}{\ell_R} \tag{12}$$

- Monotonically decreases with the tension
- **Universality**: the result does not depend on the frequency
- Transmission in empty AdS_3

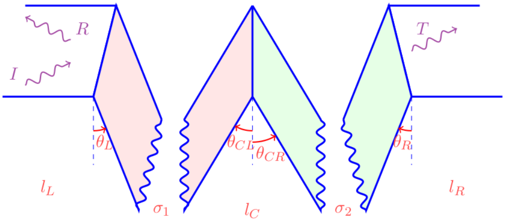
$$\sigma = 0\,, \quad \ell_L = \ell_R \quad \Rightarrow \quad \mathcal{T}_{L(R)} = 1 \tag{13}$$

- Holographic model has only one parameter
  $\Rightarrow$ Transmission and boundary entropy $\log g$ fixed in terms of the tension!

Transmission in double brane model

Fuse two branes and perform the same computation [Baig, Karch, 2022]:

$$\boxed{\mathcal{T}_{L(R)} = \frac{2}{\ell_{L(R)}} \left(\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N (\sigma_1 + \sigma_2) \right)^{-1}} \quad (14)$$



- Depends on the sum of tensions
- Transmission and boundary entropy $\log g$ can vary independently

Thick brane model (Janus)

Holographic interfaces: top-down models

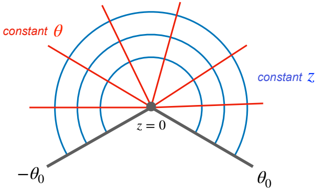
- Einstein gravity coupled to a dilaton

$$S = \frac{1}{16\pi G_N} \int d^3x \sqrt{-g}\, \left(R - \partial^\mu \phi \partial_\mu \phi - 2V(\phi)\right) \tag{15}$$

- Continuous geometries dual to vacuum states of the ICFT

$$ds^2 = a^2(\theta) \left[d\theta^2 + \frac{1}{z^2}\left(-dt^2 + dz^2\right)\right], \qquad \phi = \phi(\theta) \tag{16}$$

- Special case: non-supersymmetric 3d Janus AdS solution from reduction of 10d solution in type IIB SUGRA [Freedman, Nunez, Schnabl, Skenderis, 2003][Bak, Gutperle, Hirano, 2007]



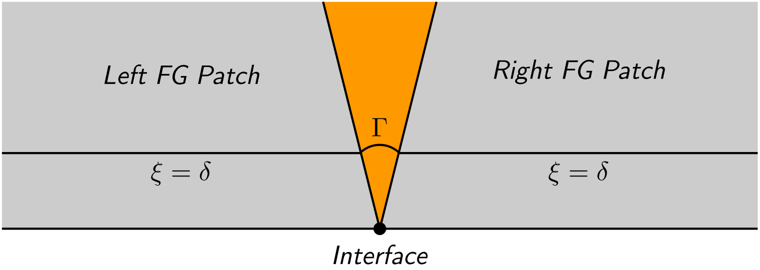
Perturbation with plane waves is difficult

Method:

- Add a perturbation for the stress-tensor at the boundary
- Solve the Einstein's equations with perturbation

Problems:

- Fefferman-Graham coordinates are not defined everywhere [Papadimitriou, SKenderis, 2004]
- Hard to study Einstein's equations

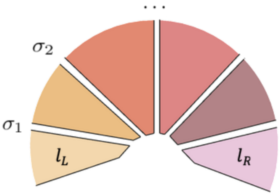


Discrete geometries are simpler!

[Bachas, SB, Chapman, Policastro, Schwartzman, 2022]

- Consider a pizza geometry with multiple branes and extrapolate Karch's result

$$\boxed{\mathcal{T}_{L(R)} = \frac{2}{\ell_{L(R)}} \left(\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N \sum_i \sigma_i \right)^{-1}} \qquad (17)$$



- Take the continuum limit

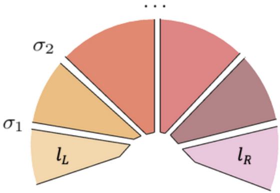
$$\sum_i \sigma_i \to \int_{-\infty}^{\infty} \frac{d\sigma}{dy} dy \qquad (dy = a(\theta)d\theta) \qquad (18)$$

Discretization method

- Take a collection of empty AdS_3 regions

$$ds_j^2 = \tilde{a}_j^2(\theta)\left[d\theta^2 + \frac{1}{z^2}\left(-dt^2 + dz^2\right)\right]$$

$$\tilde{a}(\theta) = \frac{\ell_j}{\cos(\theta - \delta_j)} \qquad \text{for } (j-1)\varepsilon < \theta < j\varepsilon$$



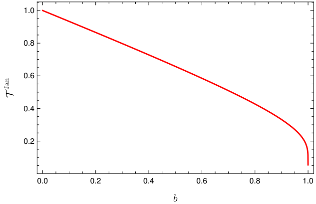
- Impose Israel matching conditions for $\tilde{a}_j(\theta)$ at each brane

$$\sigma_j a_j = \sqrt{\left(\frac{a_j}{\ell_j}\right)^2 - 1} - \sqrt{\left(\frac{a_j}{\ell_{j+1}}\right)^2 - 1} \tag{19}$$

- Impose that we recover $a(\theta)$ for Janus geometry in the continuum limit

Transmission of Janus interface

$$\mathcal{T}_{\mathrm{Jan}} = \frac{1}{2}\sqrt{b(2-b)}\left[\operatorname{arctanh}\left(\sqrt{\frac{b}{2-b}}\right)\right]^{-1}$$



- Monotonically decreasing function of the deformation parameter b
- Transmission in empty AdS_3

$$b = 0 \quad \Rightarrow \quad \mathcal{T}_{\mathrm{Jan}} = 1 \tag{20}$$

- Infinitely strongly coupled case (linear dilaton)

$$b \to 1 \quad \Rightarrow \quad \mathcal{T}_{\mathrm{Jan}} \to 0 \tag{21}$$

Discrete branes are equivalent to dilaton

- **Background solution**: discrete model with branes in the continuum limit and geometry with dilaton solve Einstein's equations with the same metric

$$T^{\rm mat}_{\mu\nu} = -\Lambda(y) g_{\mu\nu} - \frac{d\sigma}{dy}\Pi_{\mu\nu}\,, \quad T^{\phi}_{\mu\nu} = -(\partial^\rho\phi\partial_\rho\phi) + g_{\mu\nu}\left(\frac{1}{2}\partial^\rho\phi\partial_\rho\phi - V(\phi)\right) \tag{22}$$

$$\Rightarrow \quad \Lambda(y) = -\frac{1}{2}\left(\frac{d\phi}{dy}\right)^2 + V(\phi)\,, \qquad \frac{d\sigma}{dy} = \left(\frac{d\phi}{dy}\right)^2 \tag{23}$$

- **Perturbed solution**: scattering experiment can be prepared with 2d transverse traceless modes (2d universality)

$$ds^2 = a^2(\theta)\left[d\theta^2 + (\bar\gamma_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta\right] \tag{24}$$

$$\bar\gamma^{\alpha\beta} h_{\alpha\beta} = 0\,, \qquad \bar\nabla^\alpha h_{\alpha\beta} = 0 \tag{25}$$

- The computation only involves $a(\theta) \Rightarrow$ discretization does not change the result in the continuum limit!

Conclusions

- General technique to compute transmission coefficients for holographic ICFTs with Einstein-dilaton action

$$ds^2 = a^2(\theta)\left[d\theta^2 + \frac{1}{z^2}\left(-dt^2 + dz^2\right)\right], \qquad \phi = \phi(\theta) \tag{26}$$

- Discretization provides a simpler method than a direct computation

Future developments

- Top-down supersymmetric models in higher dimensions [Chiodaroli, Gutperle, Krym, 2009][Chen, Gutperle, 2020][Lozano, Nunez, Ramirez, 2021]
- Holographic transport of electric charge
- Application to cosmology
- Monotonicity theorems along RG flows [Cuomo, Komargodski, Raviv-Moshe, 2021]

Thank you!