Massive spin-2 particles and the swampland

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DFG
German-Israeli Project
Cooperation (DIP)
Holography and the
Swampland

Work in progress with S. Kundu and E. Palti

DIP meeting, 22nd March 2023

Motivation

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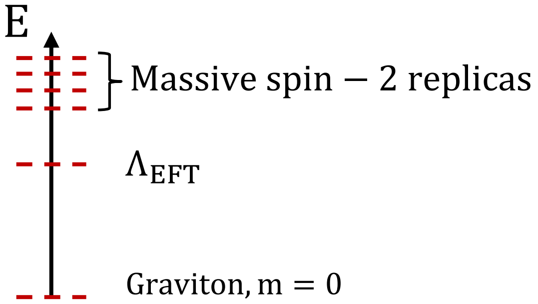
- Massive spin-2 particles appear in (string) compactifications
    - ➢ KK copies of the graviton
    - ➢ Ignore their effects at low energies
    - ➢ Top-down approach



E

$\text{Massive spin} - 2 \text{ replicas}$

Λ_{EFT}

$\text{Graviton}, \text{m} = 0$

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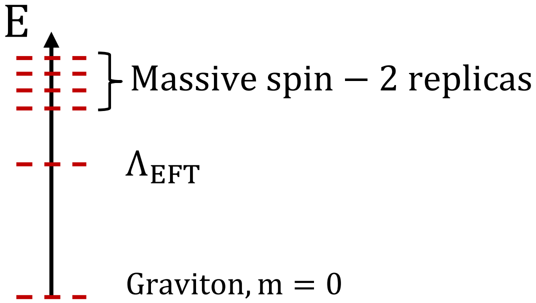
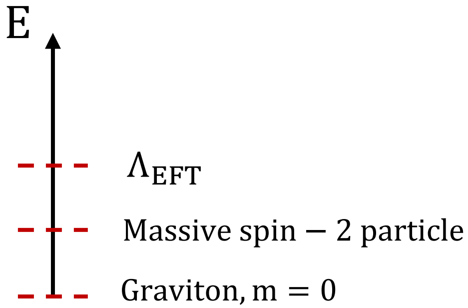
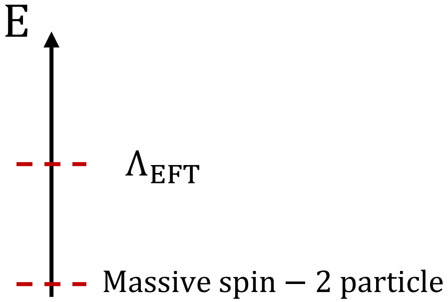


- But… from a bottom-up perspective
  - Gravity coupled to a massive spin-2 particle



  - Single massive spin-2 particle



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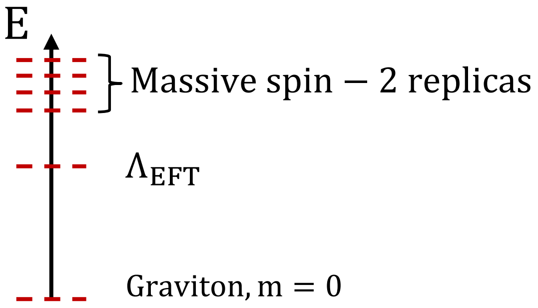

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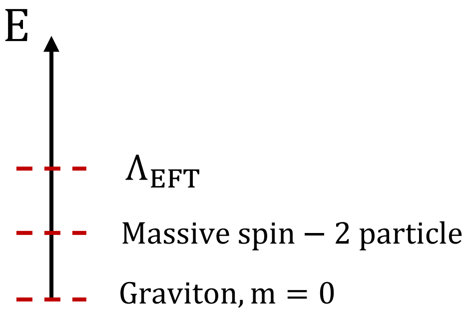

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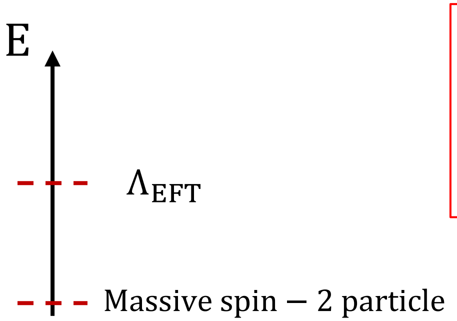

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(Swampland) constraints for massive spin-2 particles?

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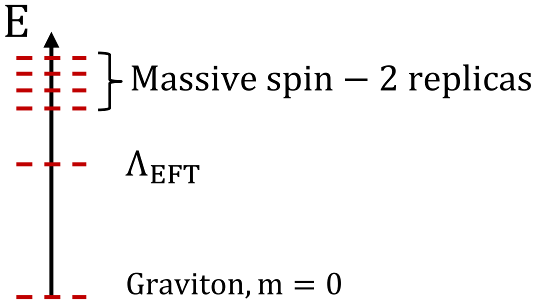

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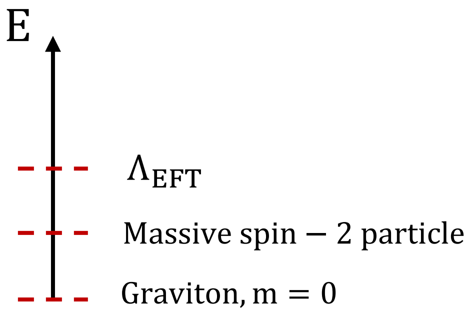

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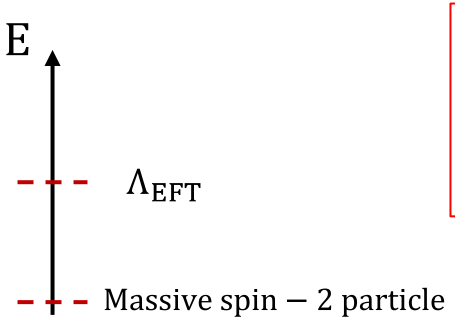

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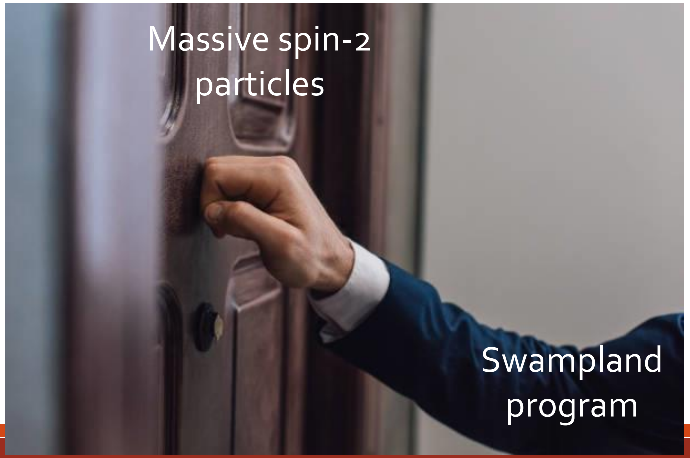
😈 Are these scenarios in the swampland?

(Swampland) constraints for massive spin-2 particles?



Contents

⚠ We will only consider $d = 4$ in this talk

QuickStart guide

- Massive spin-2 particles $w_{\mu\nu}$
  - 5 different polarisations (2 tensorial, 2 vectorial, 1 scalar)
  - Free (linearised) theory described by the Fierz-Pauli action Fierz, Pauli 1939

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    - de Rham, Gabadadze, and Tolley (dRGT) theory $L \sim \sqrt{-g}\left(R - \frac{m^2}{4} V(w, \alpha_1, \alpha_2)\right), \{\alpha_1, \alpha_2\} \in R$ Rham, Gabadadze, Tolley '10,'11
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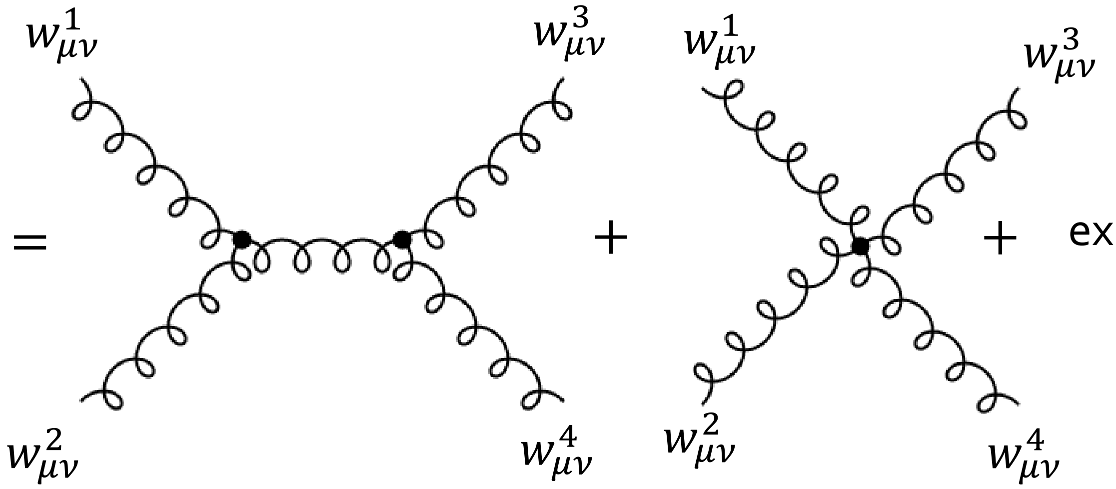

$\mathcal{A} =$ + exchange of other particles (if there are)

- ➢ Momentum conservation: $p_1 + p_2 = p_3 + p_4$

- ➢ Mandelstam variables: $s = -(p_1 + p_2)^2,\ t = -(p_1 + p_3)^2,\ u = -(p_1 - p_4)^2,\ s + t + u = 4m^2$

- ➢ $\mathcal{A} = \mathcal{A}(\mathrm{s}, \mathrm{t}) \rightarrow$ Large s limit (Regge limit), expansion around $s \rightarrow \infty$. Large $\{s, t\}$ limit (High energy limit), expansion around $\{s, t\} \rightarrow \infty$

Recap of previous constraints

- Massive spin-2 particles have been (and are) extensively studied

  Fierz, Pauli, Dam, Veltman, Zakharov, Vainsthein, Boulware, Deser, Dvali, Gabadadze, Gregory, Arkani-Hamed, Georgi, Schwartz, de Rham, Gabadadze, Tolley, Folkerts, Pritzel, Wintergerst , Hinterbichler, Bonifacio, Klaewer, Lüst, Palti, Heisenberg …

- Not a general review (very biased) . Highlight some works relevant for us:
  - Klaewer, Lust, Palti, 18: swampland conjecture involving massive spin-2 particles
  - Bonifacio, Hinterbichler, Hinterbichler, Joyce, Rosen ’17, ’18: constraints from superluminality
  - Bonifacio, Hinterbichler ’20 : constraints from dimensional reduction
  - Bonifacio, Hinterbichler ’18 & Rosen ’19: constraints on Λ_{EFT} ☜

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The spin-2 conjecture

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- An EFT with gravity $g_{\mu\nu}$ + massive spin-2 particle $w_{\mu\nu}$ with mass m and interaction scale of the helicity-1 mode M_w, has a cut-off scale Λ_m

$$\boxed{\Lambda_m \sim \frac{m\, M_p}{M_w}}$$

WGC

$\Lambda_{WGC} \sim g_{U(1)} M_p,$

$g_{U(1)} = \frac{\mathrm{m}}{\sqrt{2} M_w}$

➢ Follows from an application of the WGC conjecture to the helicity-1 mode of the massive spin-2 field:

$$w_{\mu\nu} = \underset{\substack{\uparrow \\ \text{Helicity-2} \\ \text{mode}}}{h_{\mu\nu}} + 2\underset{\substack{\uparrow \\ \text{Helicity-1} \\ \text{mode}}}{\partial_{(\mu}\chi_{\nu)}} + \underset{\substack{\uparrow \\ \text{Helicity-0} \\ \text{mode}}}{\Pi^L_{\mu\nu}\pi} \xrightarrow[\text{Fierz-Pauli action}]{\text{Plugging into the}} L_{FP} \ni -\frac{1}{8} m^2 F_{\mu\nu}\, F^{\mu\nu} \xrightarrow[\text{current through}]{\text{Couples to matter}} L_{int} = \frac{m^2}{M_w}\chi_\mu J^\mu$$

$$F_{\mu\nu} = \partial_\mu \chi_\nu - \partial_\nu \chi_\mu$$

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Helicity-2 mode

Helicity-1 mode

Helicity-0 mode

$$F_{\mu\nu} = \partial_\mu \chi_\nu - \partial_\nu \chi_\mu$$

- Strong form: if the graviton itself is massive

$$\boxed{\Lambda_m \sim m}$$

Experimental bounds: $m_{grav} < 10^{-22} eV$; Ligo, Virgo ’16

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- Some objections in de Rham, Heisenberg, Tolley '18

Constraints on Λ_{EFT}

By Λ_{EFT} we mean the scale at which perturbative unitarity breaks down (strong coupling scale): news degrees of freedom or strong coupling effects must be taken into account at this scale.

Constraints on Λ_{EFT}

Klaewer, Lust, Palti, 18

- The spin-2 conjecture: bound on Λ_{EFT} using WGC. Vast literature on the topic.
  - Arkani-Hamed, Georgi, Schwartz '02: linearised theory of massive spin-2: $\Lambda \equiv \Lambda_5 = \left(M_p m^4\right)^{1/5}$. Particular Non-linear terms, raised up to $\Lambda \equiv \Lambda_3 = \left(M_p m^2\right)^{1/3}$
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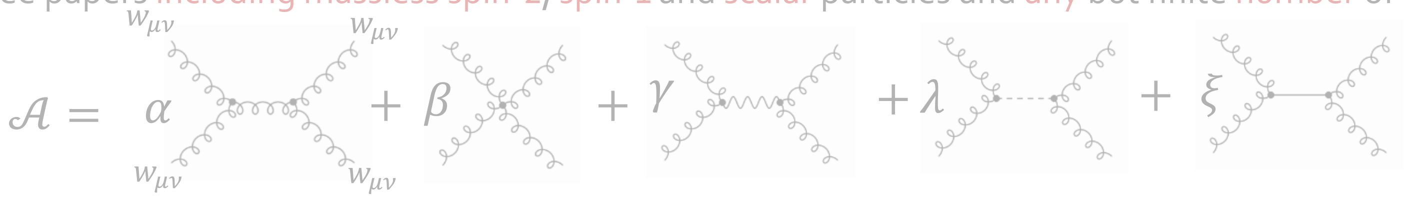
  - ➢ How? Studying when perturbative unitarity breaks down: $\mathcal{A} \sim \frac{E^n}{\Lambda^n} \to E \sim \Lambda \to \mathcal{A} \sim 1$
  - ➢ Study tree-level four-particle scattering amplitudes of massive spin-2 particles: can be done in general, without reference to any Lagrangian
  - ➢ Series of three papers including massless spin-2, spin-1 and scalar particles and any but finite number of derivatives.



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  - ➢ Maximize $\Lambda_k \sim \left(M_p m^{k+1}\right)^{1/(k+2)}$ Construct the theory with the lowest possible k: cancellations among the different contributions

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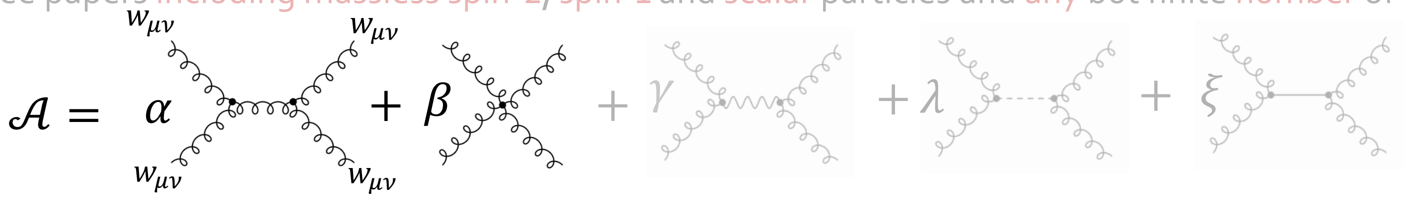

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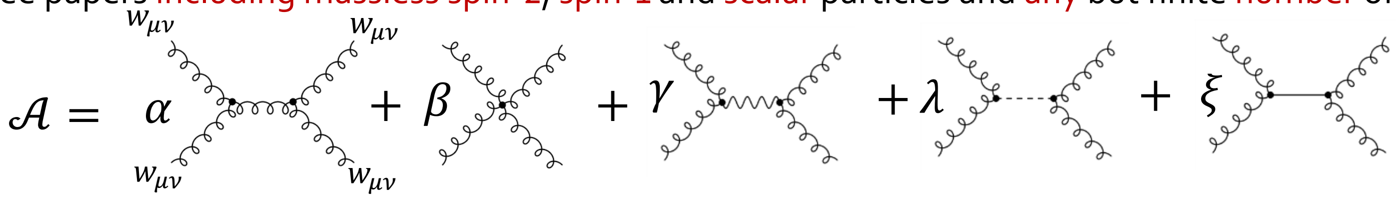

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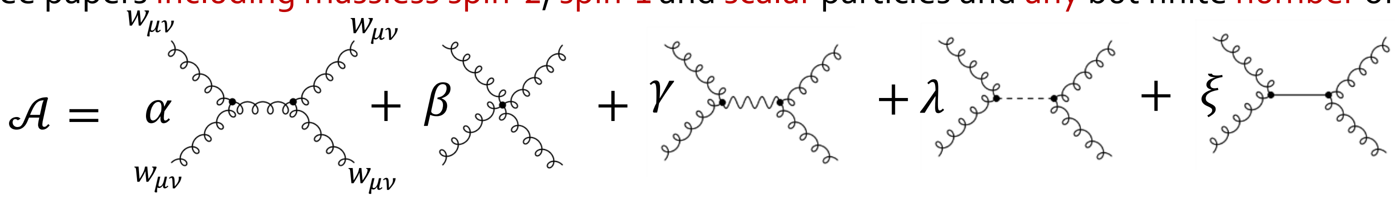
Klaewer, Lust, Palti, 18

- The spin-2 conjecture: bound on Λ_{EFT} using WGC. Vast literature on the topic.
  - ➢ Arkani-Hamed, Georgi, Schwartz '02: linearised theory of massive spin−2: $\Lambda \equiv \Lambda_5 = \left(M_p m^4\right)^{1/5}$. Particular Non-linear terms, raised up to $\Lambda \equiv \Lambda_3 = \left(M_p m^2\right)^{1/3}$
  - ➢ This happens for dRGT (de Rham, Gabadadze Tolley, '11) and the pseudolinear theory (Hinterbichler '13)

- Λ_3 not showed to be the highest possible scale . Model independent approach to this problem Bonifacio, Hinterbichler '18 & Rosen '19
  - ➢ How? Studying when perturbative unitarity breaks down: $\mathcal{A} \sim \frac{E^n}{\Lambda^n} \to E \sim \Lambda \to \mathcal{A} \sim 1$
  - ➢ Study tree-level four-particle scattering amplitudes of massive spin-2 particles: can be done in general, without reference to any Lagrangian
  - ➢ Series of three papers including massless spin-2, spin-1 and scalar particles and any but finite number of derivatives.



  - ➢ Scattering of 2 → 2 massive spin-2 particles, amplitudes go like: $\mathcal{A} \sim \frac{E^{2(k+2)}}{\left(M_p m^{k+1}\right)^2} \to \mathcal{A} \sim 1$ when $E \equiv \Lambda_k \sim \left(M_p m^{k+1}\right)^{1/(k+2)}$
  - ➢ Maximize $\Lambda_k \sim \left(M_p m^{k+1}\right)^{1/(k+2)}$ Construct the theory with the lowest possible k: cancellations among the different contributions

Constraints on Λ_{EFT}

- $\Lambda \equiv \Lambda_3 = \left(M_p m^2\right)^{1/3}$ highest possible scale for a theory containing one massive spin-2 particle (and no higher spin particles)
  - $\mathcal{A} \sim \frac{E^6}{\left(M_p m^2\right)^2}$. Contributions with $\mathcal{A} \sim E^m$, with $m > 6$ chosen to cancel each other
  - Can only be improved including additional massive particles with spin 2 or higher spin
  - If one insists on cancelling the terms producing E^6 → only trivial solution, all cubic couplings (the theory) must vanish

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- Our work will be along the same lines. Instead of the behaviour $E^m, m \geq 6$, we will only be interested in the amplitudes going like $s^m, \mathrm{m} \geq 3$.

Classical Regge Growth (CRG) Conjecture

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Chandorkar, Chowdhury, Kundu, Minwalla ’21

- ➢ It can be argued that in the ‘impact parameter (δ) space’: $S(\delta, s) \sim s^m, \mathrm{m} \leq 2$. Subtleties changing to the usual $S(t, s)$.

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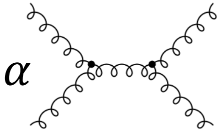
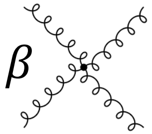
$\mathcal{A} = \alpha$  $+ \beta$  + exchange of other particles $\sim s^n, n \leq 2$

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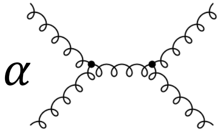
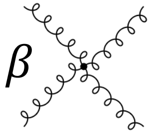
$\mathcal{A} = \alpha$  $+ \beta$  + exchange of other particles $\sim s^n, n \leq 2$

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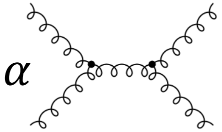
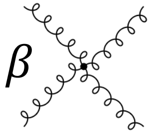
$\mathcal{A} = \alpha$  $+ \beta$  $+$ exchange of other particles $\sim s^n, n \leq 2$

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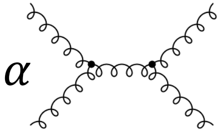
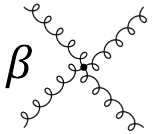
$\mathcal{A} = \alpha$  $+ \beta$  $+$ exchange of other particles $\sim s^n, n \leq 2$

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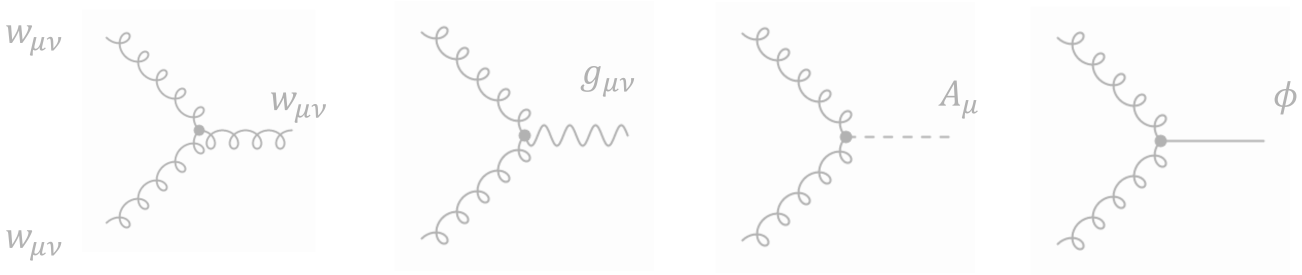
- Previous results related to our question but not exactly the same ☝🏼
  - Terms like $\mathcal{A} \sim s^4 \sim E^8$ cancel in both cases
  - Terms like $\mathcal{A} \sim t^4$ cancel in Bonifacio, Hinterbichler '18 & Rosen '19 both noth in our case
  - Terms like $\mathcal{A} \sim \frac{s^3}{t} \sim E^4$ cancel in our case but ok in Bonifacio, Hinterbichler '18 & Rosen '19

Results

- Model independent approach: construct directly the tree-level amplitudes. How?

1. Find all possible Lorentz-invariant cubic vertices Costa, Penedones, Poland, Rychkov `11. Impose symmetry under $w^1 \leftrightarrow w^2 \leftrightarrow w^3$



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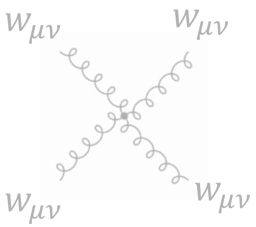

3. Compute the Amplitude $A_{\text{tree}}(s,t) = A_{\text{exchange}} + A_{\text{contact}}$ for any polarisation: $5^4 = 625$ choices (not all different)

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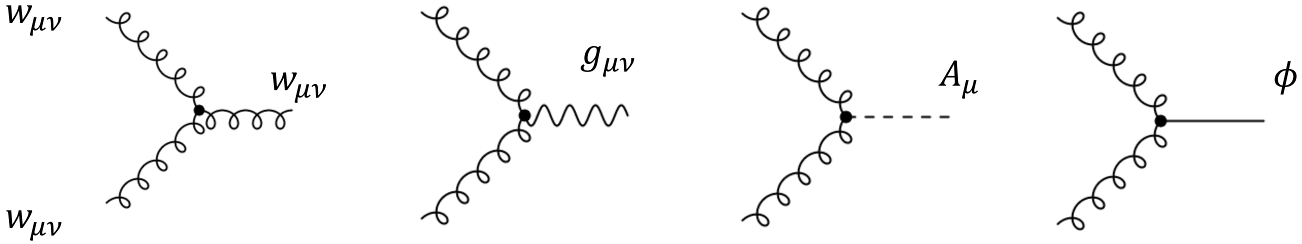

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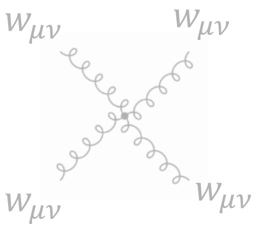

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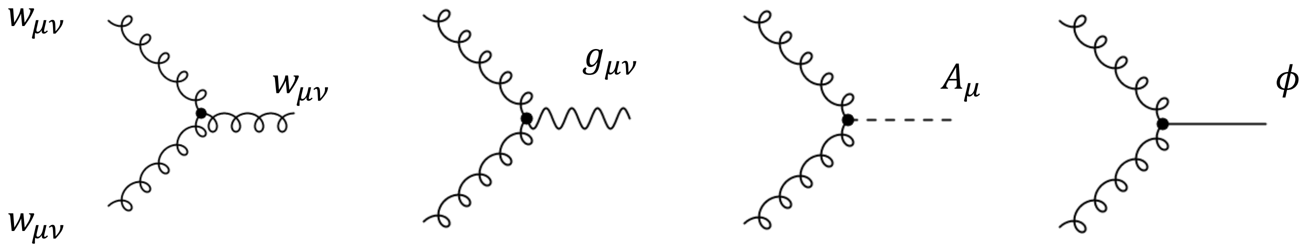

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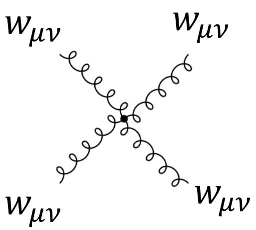

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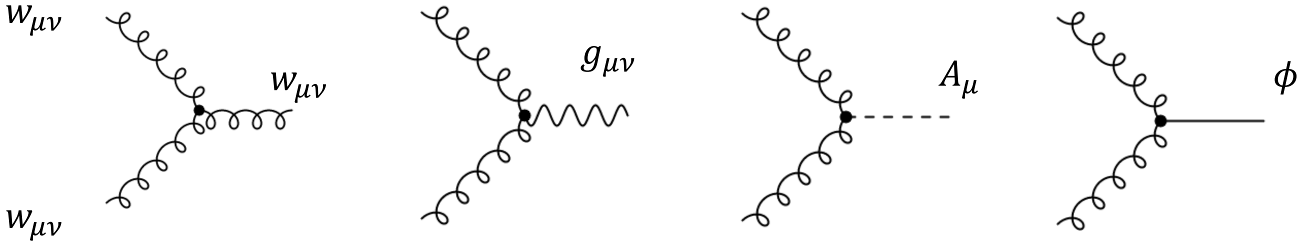

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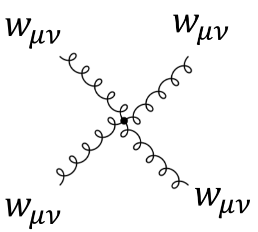

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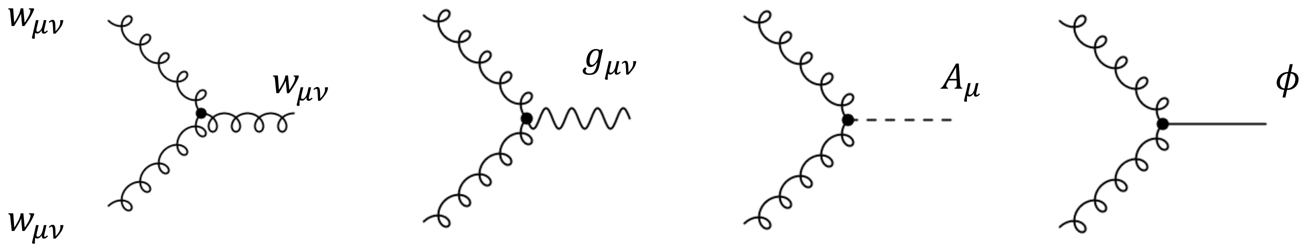

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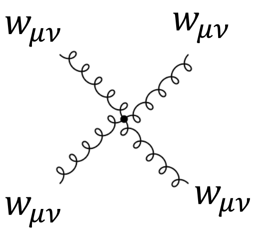

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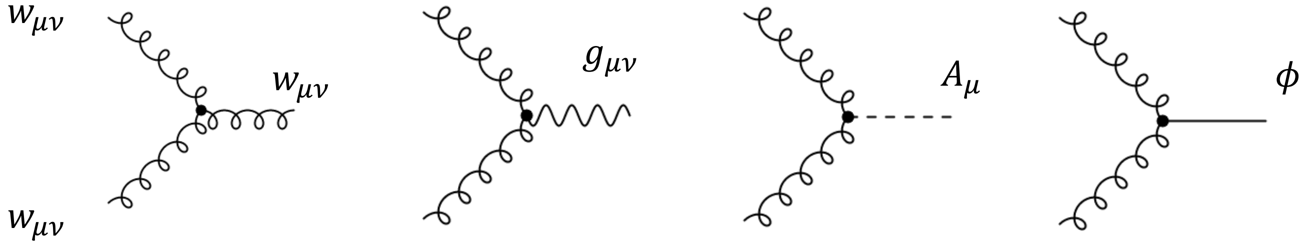

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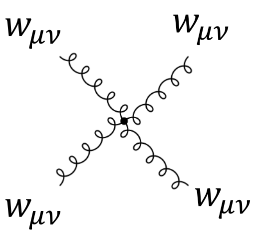

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Results

- Considering only parity-even terms:

  - 4 independent cubic vertices involving three massive spin-2 particles (E.g. $w^{1\nu}_{\mu} w^{2\alpha}_{\nu} w^{3\mu}_{\alpha}, R^{1\mu\nu}{}_{\alpha\beta}, R^{2\alpha\beta}{}_{\gamma\delta} R^{3\gamma\delta}{}_{\mu\nu}, \dots$)
  - ∞ number of contact terms. E.g: $w^1 w^2 w^3 w^4 \rightarrow (\partial^\mu \partial_\mu)^n w^1 w^2 w^3 w^4, n = 1, \dots, \infty$
    - Different from the cubic case, where $p_1 + p_2 + p_3 = 0 \rightarrow (\mathrm{p_i} \cdot p_j)^n = f(m^n)$
    - More derivatives → higher s^n → most of the terms will vanish
    - In Bonifacio, Hinterbichler '18: algorithm to deal with an arbitrary but finite number of derivatives
    - Can be expanded in a basis of tensor structures: $\sum_{n,m} a_{m,n} s^n t^m (\sum_i f_i(s,t))$. In the $f_i(s,t)$ the derivatives are always contracted with the indices of the massive spin-2 particles, e.g $\partial^\mu \partial^\nu w^1_{\mu\nu} w^{2\gamma}{}_{\delta}\, w^{3\delta}{}_{\xi}\, w^{4\xi}{}_{\gamma}$
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    - Can be expanded in a basis of tensor structures: $\sum_{n,m} a_{m,n} s^n t^m (\sum_i f_i(s,t))$. In the $f_i(s,t)$ the derivatives are always contracted with the indices of the massive spin-2 particles, e.g. $\partial^\mu \partial^\nu w^1_{\mu\nu} {w^2}^\gamma_\delta \, {w^3}^\delta_\xi \, {w^4}^\xi_\gamma$
    - Finite contribution at fixed $s^a t^b$
  - Results only considering the $f_i(s,t)$.
  - Exchange of a massless spin-2, massive spin-1 and scalar particle

Results

- Considering only parity-even terms:
  - 4 independent cubic vertices involving three massive spin-2 particles (E.g. $w_{\mu}^{1\nu} w_{\nu}^{2\alpha} w_{\alpha}^{3\mu}, R{^1}^{\mu\nu}_{\alpha\beta}, R{^2}^{\alpha\beta}_{\gamma\delta} R{^3}^{\gamma\delta}_{\mu\nu}, \ldots$)
  - ∞ number of contact terms. E.g: $w^1 w^2 w^3 w^4 \rightarrow (\partial^{\mu}\partial_{\mu})^n w^1 w^2 w^3 w^4, n = 1, \ldots, \infty$
    - Different from the cubic case, where $p_1 + p_2 + p_3 = 0 \rightarrow \left(\mathrm{p_i} \cdot p_j\right)^n = f(m^n)$
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- To satisfy the CRG: all cubic vertices must vanish→ the theory is trivial (there is no theory)

If the CRG conjecture is true, it does not seem possible to construct a theory containing a single (interacting) massive spin-2 particle (with no higher spin particles).

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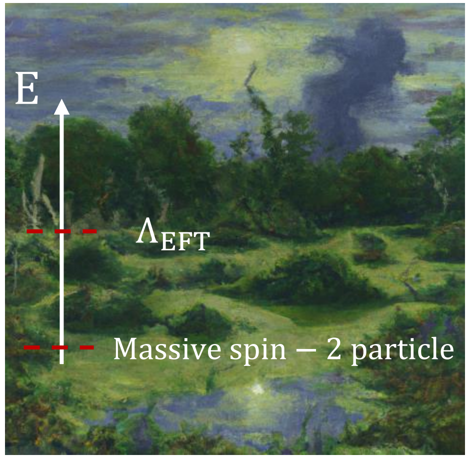

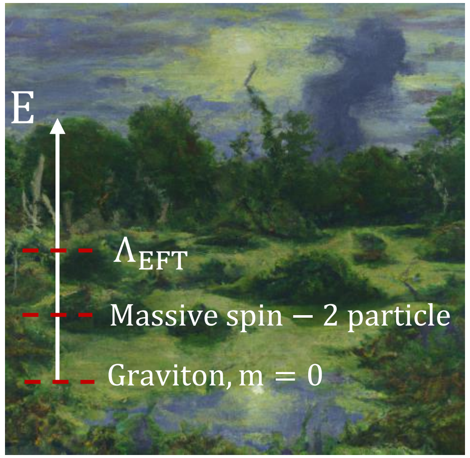

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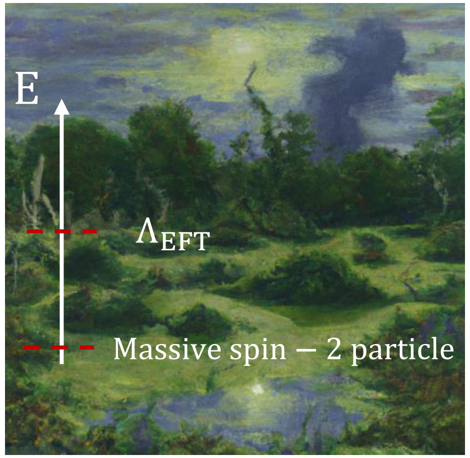

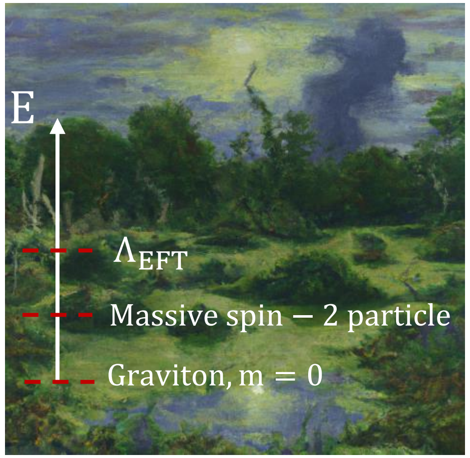

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- Cautious: need to include all contact terms and parity-odd terms. Previous results in the literature: expect our results to hold.

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Thank you for your attention! 🙂

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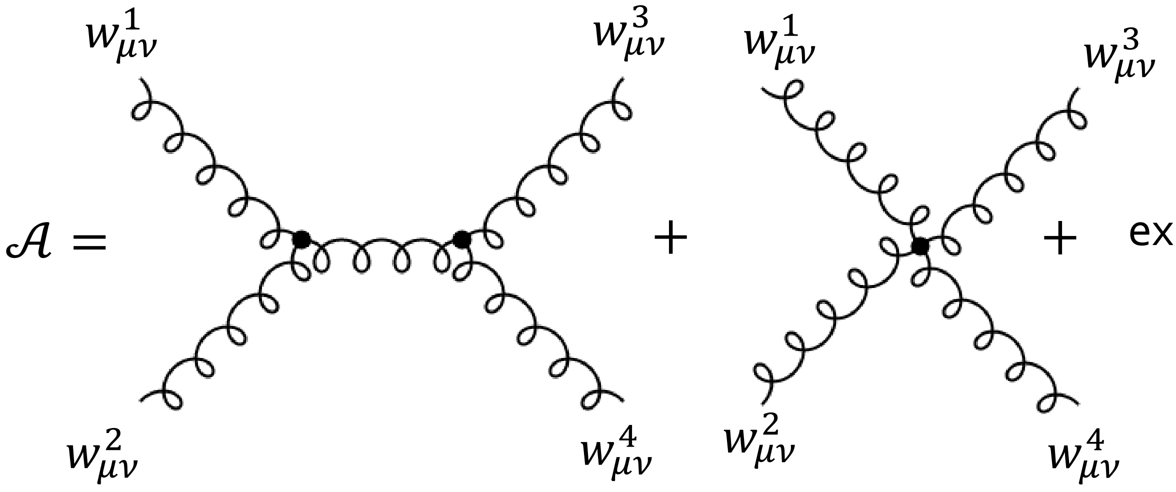
QuickStart guide

- $2 \rightarrow 2$ tree level scattering of massive spin-2 particles



exchange of other particles (if there are)

- ➢ Momentum conservation: $p_1 + p_2 = p_3 + p_4$
- ➢ Mandelstam variables: $s = -(p_1 + p_2)^2,\ t = -(p_1 + p_3)^2,\ u = -(p_1 - p_4)^2,\ s + t + u = 4m^2$
- ➢ $\mathcal{A} = \mathcal{A}(\mathrm{s},\mathrm{t}) \rightarrow$ Large s limit (Regge limit), expansion around $s \rightarrow \infty$. Large $\{s, t\}$ limit (High energy limit), expansion around $\{s, t\} \rightarrow \infty$

Recap of previous constraints

- Massive spin-2 particles have been (and are) extensively studied

  Fierz, Pauli, Dam, Veltman, Zakharov, Vainsthein, Boulware, Deser, Dvali, Gabadadze, Gregory, Arkani-Hamed, Georgi, Schwartz, de Rham, Gabadadze, Tolley, Folkerts, Pritzel, Wintergerst , Hinterbichler, Bonifacio, Klaewer, Lüst, Palti, Heisenberg …

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  - Klaewer, Lust, Palti, 18: swampland conjecture involving massive spin-2 particles
  - Bonifacio, Hinterbichler, Hinterbichler, Joyce, Rosen '17, '18: constraints from superluminality
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- An EFT with gravity $g_{\mu\nu}$ + massive spin-2 particle $w_{\mu\nu}$ with mass m and interaction scale of the helicity-1 mode M_w, has a cut-off scale Λ_m

$$\boxed{\Lambda_m \sim \frac{m\, M_p}{M_w}}$$

WGC

$\Lambda_{WGC} \sim g_{U(1)} M_p,$

$g_{U(1)} = \frac{\mathrm{m}}{\sqrt{2} M_w}$

- ➢ Follows from an application of the WGC conjecture to the helicity-1 mode of the massive spin-2 field:

$$w_{\mu\nu} = \underbrace{h_{\mu\nu}}_{\text{Helicity-2 mode}} + 2\underbrace{\partial_{(\mu}\chi_{\nu)}}_{\text{Helicity-1 mode}} + \underbrace{\Pi^L_{\mu\nu}\pi}_{\text{Helicity-0 mode}} \xrightarrow[\text{Fierz-Pauli action}]{\text{Plugging into the}} L_{FP} \ni -\frac{1}{8} m^2 F_{\mu\nu}\, F^{\mu\nu} \xrightarrow[\text{current through}]{\text{Couples to matter}} L_{int} = \frac{m^2}{M_w} \chi_\mu J^\mu$$

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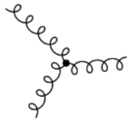
- Some objections in de Rham, Heisenberg, Tolley '18

Superluminality constraints

Bonifacio, Hinterbichler, Joyce, Rosen '17, '18

- Demanding absence of time advance in eikonal scattering of 2 → 2 massive spin-2 particles (similar to Arkani-Hamed, Georgi, Schwartz '02) :
  - Model independent: S-matrix approach
  - Constraints on the cubic vertices of the theory



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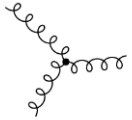

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- Single massive spin-2 particle

  - ➢ Cubic vertices must appear in a specific linear combination → specific linear combination for the terms in the Lagrangian producing them

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Space of parameters is reduce but theories are not ruled out

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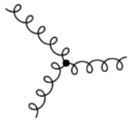

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Bonifacio, Hinterbichler ’20

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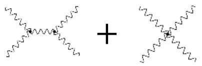

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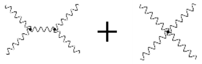

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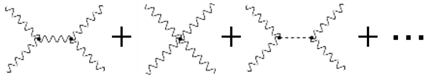

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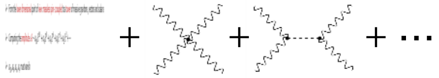

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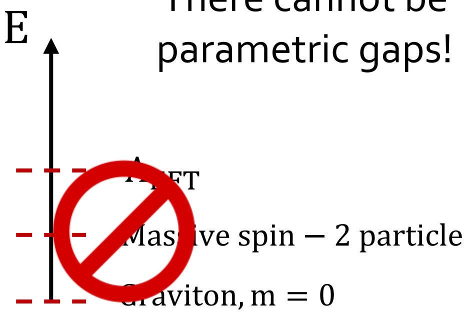
- D-dim GR in $\mathcal{M}_D = \mathbb{R}^{1,\mathrm{D-d}} \times \mathcal{N}^d$, with $\mathcal{N}$ closed, smooth, connected, orientable, Ricci flat Riemannian manifold. Terms up to two derivatives.
  - Tower of massive spin-2 particles $w^a_{\mu\nu}$ with mass $m_a^2 = \lambda_a$
  - $\mathcal{A} \sim E^2$ for the four-point graviton amplitude in $\mathbb{R}^{1,\mathrm{D-d}}$ imposes: $\boxed{\frac{\lambda_{a+1}}{\lambda_a} \leq 4 \rightarrow \frac{m_{a+1}}{m_a} \leq 2}$

There cannot be parametric gaps!



Constraints on Λ_{EFT}

Bonifacio, Hinterbichler '18 & Rosen '19

By Λ_{EFT} we mean the scale at which perturbative unitarity breaks down (strong coupling scale): news degrees of freedom or strong coupling effects must be taken into account at this scale.

Summary so far

- Massive spin-2 particles
  - Linearised level: unique theory, Fierz-Pauli theory.
  - Non-linear extension: dangerous! Ghosts may appear! Two known ghost-free non-linear theories: dRGT (massive gravity) and pseudolinear theory
  - Conjectured that $\Lambda_m \sim \frac{m\, M_p}{M_w}$ (coupled to gravity), strong form: $\Lambda_m \sim m$ (massive gravity); motivated by WGC.
  - Shown that at least $A \sim E^6$ for any theory containing one massive spin-2 particle and no higher spin particles.
  - No parametric gaps between massive spin-2 particles coming from a tower produced when dimensional reducing GR (at two derivative level, some assumptions)

A swampland menace…