Strings, Species and the Weak Gravity Conjecture

- with Cesar Cota, Alessandro Mininno, and Max Wiesner: 2208.00009 and 2212.09758 (see A. Mininno's talk)

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Weak Gravity Conjecture

In every gauge theory coupled to gravity, there exists a particle with

$$\frac{g_{\rm YM}^2 q^2}{m^2} \geq \frac{g_{\rm YM}^2 Q^2}{M^2}\Big|_{\rm B.H.}$$

[Arkani-Hamed,Motl,Nicolis,Vafa'06]

Bottom-up motivation: Extremal black holes should decay
- especially (only?) at weak coupling

Tower WGC (tWGC): [Heidenreich,Reece,Rudelius] [Montero,Shiu,Soler]'16
[Andriolo,Junghans,Noumi,Shiu'18]

The super-extremal states must form an infinite tower.

Bottom-up motivation: [Heidenreich,Reece,Rudelius]'16-18

- Sufficient for consistency of the WGC under Kaluza-Klein reduction.
- But: The tower version is not strictly necesssary in presence of very super-extremal states, especially massless charged states.

Evidence for tWGC

1. Extremality condition = BPS condition

   $\Longrightarrow$ WGC tower must be a BPS tower

   [Ooguri,Vafa'16] [Grimm,Palti,Valenzuela'18] [Grimm,Palti,Li'18] [Gendler,Valenzuela'20]

   [Bastian,Grimm,Heisteeg'20] . . . [Alim,Heidenreich,Rudelius'21]

   [Gendler,Heidenreich,McAllister,Moritz,Rudelius'22]

2. In absence of BPS states:

   Focus on *asymptotic tWGC* in weak coupling limit $g_{\mathrm{YM}} \to 0$

   [Lee,Lerche,TW'18-20] [Kläwer,Lee,TW,Wiesner'20] . . .

Asymptotic Tower WGC:

Whenever a gauge theory admits a weak coupling limit in a suitable sense, then one can identify a super-extremal tower in every direction of the charge lattice associated with its gauge group.

Species scale

Asymptotic Tower WGC:

Whenever a gauge theory admits a weak coupling limit *in a suitable sense, then one can identify a super-extremal tower in every direction of the charge lattice associated with its gauge group.*

Suitable weak coupling limit (in 4d): [Cota,Mininno,TW,Wiesner'22 (1)+(2)]

$$\Lambda^2_{\text{WGC}}\,(U(1)_C) = g^2_{\text{YM,C}} M^2_{\text{Pl}}$$

$$\frac{\Lambda^2_{\text{WGC}}\,(U(1)_C)}{\Lambda^2_{\text{QG}}} \to 0 \qquad \Lambda_{\text{QG}} = \Lambda_{\text{sp.}} = \frac{M_{\text{Pl}}}{N_{\text{sp}}^{1/2}}$$: species scale for limit

We will see:

- Sufficient condition for a marginally super-extremal tower.
- Only in this case is a marginally super-extremal tower part of the EFT.

Species scale

Species scale depends on type of limit as characterised by Emergent String Conjecture [Lee,Lerche,TW'19] see talk by D. Lüst

1. Leading tower is (dually) a Kaluza-Klein tower

   Decompactification of n dimensions:

   N: number of states with $m^2 \leq k^2 M^2_{\mathsf{KK}}$: $N \sim k^n$ $\Lambda^2_{\mathsf{sp,KK}} = k^2_{\max} M^2_{\mathsf{KK}} \stackrel{!}{=} \frac{M^2_{\mathsf{Pl}}}{k^n_{\max}}$

   $$\frac{\Lambda^2_{\mathsf{sp,KK}}}{M^2_{\mathsf{Pl}}} = \left(\frac{M^2_{\mathsf{KK}}}{M^2_{\mathsf{Pl}}}\right)^{\frac{n}{2+n}}$$

   [Montero,Vafa,Valenzuela'22] [Cota,Mininno,TW,Wiesner'22 (1)]

2. Leading tower is an emergent critical string excitation tower

   cf. [Dvali,Lüst'09] [Marchesano,Melotti'22] [Castellano,Herraez,Ibanez'22]

   $$\Lambda^2_{\mathsf{sp}} \sim M^2_{\mathsf{string}} \log\left(\frac{M_{\mathsf{Pl}}}{M_{\mathsf{string}}}\right)$$

This talk

Classify limits $g_{\rm YM} \to 0$ for 7-brane sector in 4d N=1 F-theory

1. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{QG}}} \to 0$

   - Emergent heterotic string limits
   - Non-BPS tower of heterotic excitations satisfies the asymptotic tower WGC - as in [Lee,Lerche,TW'19]

2. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{QG}}} \sim 1$

   - Decompactification limit to 6d or 8d gauge + gravity theory
   - No *obvious* super-extremal tower - in particular no weakly coupled such tower

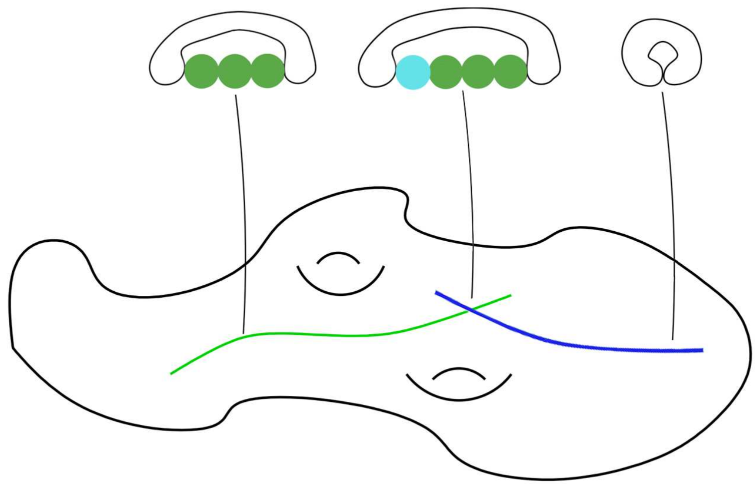
3. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{QG}}} \to \infty$

   - Decompactification to higher dimensional gravity theory with decoupled gauge sector (here: defect gauge theory)
   - No obvious super-extremal tower - tWGC makes no sense due to decoupling

F-theory in 4d

F-theory in 4d N=1 $\Longleftrightarrow$ Type IIB on $\mathbb{R}^{1,3} \times B_3$ with 7-branes

- B_3 = compact Kähler 3-fold
  $\Longrightarrow$ dynamical gravity
- 7-branes on complex surface $\mathbf{S} \subset B_3$
  $\Longrightarrow$ gauge symmetry



Couplings: (IIB Einstein frame)

$$\frac{\mathbf{M}_{\mathrm{Pl}}^{\mathbf{2}}}{\mathbf{M}_{\mathrm{IIB}}^{\mathbf{2}}} = \mathbf{4}\pi\mathcal{V}_{\mathbf{B_3}} \qquad \frac{\mathbf{1}}{\mathbf{g}_{\mathrm{YM}}^{\mathbf{2}}} = \frac{\mathbf{1}}{\mathbf{2}\pi}\mathcal{V}_{\mathbf{S}}$$

Aim: Understand N=1 Kähler moduli limits

$$g_{\mathrm{YM}}^2 \to 0 \Longleftrightarrow \mathcal{V}_{\mathbf{S}} \to \infty$$

F-theory in 4d

Couplings: (IIB Einstein frame)

$$\frac{\mathbf{M_{Pl}^2}}{\mathbf{M_{IIB}^2}} = 4\pi\mathcal{V}_{\mathbf{B_3}} \qquad \frac{\mathbf{1}}{\mathbf{g_{YM}^2}} = \frac{\mathbf{1}}{\mathbf{2}\pi}\mathcal{V}_{\mathbf{S}}$$

Aim: Understand limits with $\mathcal{V}_{\mathbf{S}} \to \infty$

Useful framework: **EFT strings** [Lanza,Marchesano,Martucci,Valenzuela'20-21]

1. Systematic (though not necessary) starting point for classification of limits
2. Tension naturally sits at WGC scale:

$$T_{\text{EFT}} \sim \Lambda_{\text{WGC}}^2 = g_{\text{YM}}^2 M_{\text{Pl}}^2$$

Is EFT string responsible for super-extremal tower?

cf. [Heidenreich,Reece,Rudelius'21] [Kaya,Rudelius'22]

EFT strings

$N = 1$ supergravity with chiral multiplets $T_i = s_i + ia_i$

Consider string charged magnetically under axion a_i

$$S = \int_{\text{string}} e_i \, B_{2i}^i + \ldots \qquad\qquad a_i \Longleftrightarrow B_2^i$$

Backreaction of such strings (codimension-two in $\mathbb{R}^{1,3}$)

$$T_i(z) = T_i^{(0)} - \frac{e_i}{2\pi} \log\left(\frac{z}{z_0}\right) \qquad\qquad z : \text{transverse} \subset \mathbb{R}^{1,3}$$

For $e_i > 0$: $T_i(z) \to \infty$ close to string $=$ infinite distance limit

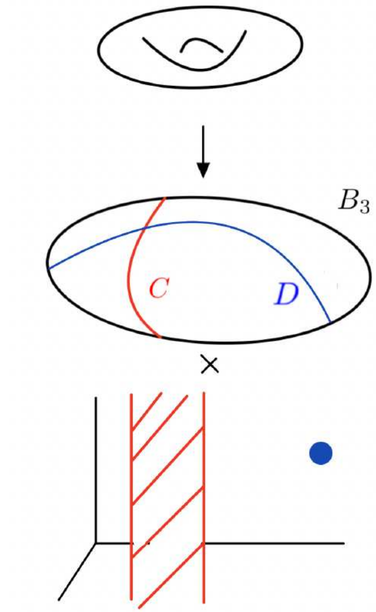
$\Longleftrightarrow$ **EFT strings** [Lanza,Marchesano,Martucci,Valenzuela'20-21]

Technical definition:

Cone of EFT string charges e_i $\overset{\text{dual}}{\Longleftrightarrow}$ Cone of BPS instantons with $S_{\text{inst}} = e^{-T_i}$

EFT strings from $\mathrm{Mov}_1(B_3)$

N=1 Kähler moduli space in F-theory: [Lanza,Marchesano,Martucci,Valenzuela'20-21]



- **Instantons:**
  Euclidean D3 on effective divisors
  $D \in \mathrm{Eff}^1(B_3)$
- **EFT Strings:**
  D3 on curves C in dual cone of **movable curves $\mathrm{Mov}_1(B_3)$**
- **Movable curves** can probe entire base
  (live in a family that covers dense open subset of B_3)
- EFT strings sensitive to gravity

Characterisation of movable curves on B_3 and associated EFT string limits in [Cota,Mininno,TW,Wiesner'22]

EFT string limits

EFT string limit:

For a subset $\mathcal{I} \subset \mathrm{Eff}^1(B_3)$ of generators of the effective cone:

$$\mathcal{V}_D \sim \lambda \to \infty\,, \quad \forall D \in \mathcal{I}\,, \qquad\qquad \mathcal{V}_{\hat{D}} < \infty \text{ for } \hat{D} \notin \mathcal{I}$$

***primitive* EFT string**: $|\mathcal{I}| = 1$ (single expanding divisor D_0)

***quasi-primitive* EFT string**: minimal number of expanding divisors (≡ building block of EFT string limits)

Note: Not all limits are EFT string limits (inhomogeneous scaling)
cf [Grimm,Lanza,Li'22]

Strategy: First quasi-primitive EFT limits - generalisation thereafter

[Cota,Mininno,TW,Wiesner'22 (1)]

(Quasi-)Primitive EFT Strings

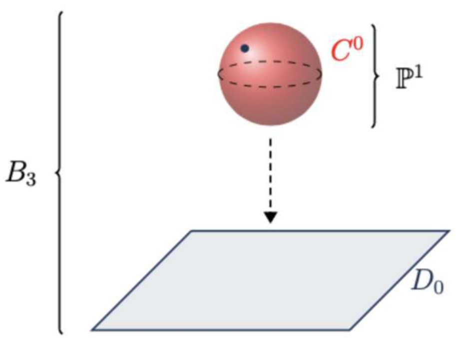
Classification of EFT string limits [Cota,Mininno,TW,Wiesner'22 (1)]

- EFT string on curve C^0 $\Longleftrightarrow$ expansion of divisor D_0 with $D_0 \cdot C^0 \neq 0$
- **3 types of quasi-primitive EFT strings** — classified by $q = 0, 1, 2$:

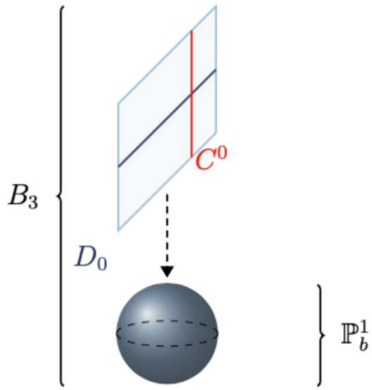
For curve $C^0 = D_1 \cdot D_2$ define $Q_1 = D_1^2 \cdot D_2$, $\qquad Q_2 = D_1 \cdot D_2^2$

$$q(C^0) := \Theta(Q_1) + \Theta(Q_2)$$

$q = 0$:
C^0 is a $\mathbb{P}^1$ fiber



$q = 1$:
$C^0 \subset$ surface fiber

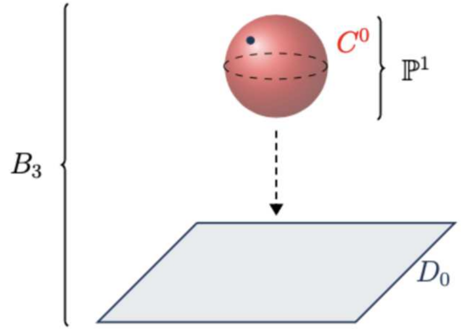


$q = 2$:
No fiber structure

Quasi-Primitive EFT Limits

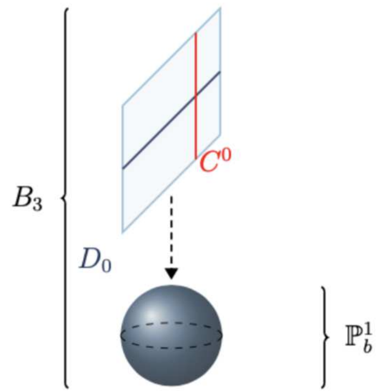
$q = 0$:

- $\mathcal{V}_{C^0} \sim \frac{1}{\lambda}, \quad \mathcal{V}_{D_0} \sim \lambda^2 \qquad \lambda \to \infty$
- always primitive:
  Emergent heterotic string limit of [Lerche,Lee,TW'19] [Klawer,Lee,TW,Wiesner'20]:
  EFT string = critical heterotic string



$q = 1$:

- $\mathcal{V}_{C^0} \sim 1, \quad \mathcal{V}_{D_0} \sim \lambda$
- EFT string not critical string



$q = 2$:

- $\mathcal{V}_{C^0} \sim \lambda, \quad \mathcal{V}_{D_0} \sim \lambda^2$
- EFT string not critical string

Primitive Weak Coupling Limits

First: **primitive** EFT limits - generalisations later

Consider now gauge theory on 7-branes along divisor $\mathbf{S} = \kappa D_0 + \ldots$:

$$\frac{2\pi}{g^2_{\rm YM}} = \mathcal{V}_{\mathbf{S}} = \kappa \mathrm{Re}(T_0) + \ldots \to \infty$$

in a primitive limit $\mathrm{Re}(T_0) \to \infty$: D_0 dual to curve C^0 of type q

$$S_{4d} = \frac{M^2_{\rm Pl}}{2} \int \left[R \star 1 - (1+q) \frac{\mathrm{d}T_0 \wedge \star \mathrm{d}\bar{T}_0}{(T_0 + \bar{T}_0)^2} \right] - \frac{\kappa M^2_{\rm Pl}}{8} \int \left[\mathrm{Re}\, T_0 \ \mathrm{tr}|F|^2 - i \mathrm{Im}\, T_0 \ \mathrm{tr} F \wedge F \right] + \ldots .$$

✓ $g_{\rm YM} \to 0$ for gauge theory   ✓ $T_{\rm EFT} = g^2_{\rm YM} M^2_{\rm Pl} = \Lambda^2_{\rm WGC}$

$\Longrightarrow$ seems similar to weak coupling limit of a heterotic string $(q = 0)$
where het string excitation give WGC tower [Lee,Lerche,TW'19]

Is the tower WGC satisfied by the 'excitations' of the primitive EFT string?

cf. [Heidenreich,Reece,Rudelius'21] [Kaya,Rudelius'22]

Weak Gravity Conjecture

Worldsheet theory of string: $N = (0, 2)$ with fermions charged under G

[Lawrie,Schäfer-Nameki,TW'16]; cf [Heidenreich,Reece,Rudelius'21]

Provided that the EFT string can be treated as a perturbative string:

- $M_k^2 = 8\pi T_{\text{EFT}}(n_k - E_0)$
- From elliptic genus/modularity: [Lee,Lerche,TW'18/19]

$$\text{max. charge-to-mass ratio :} \quad q_k^2 \geq 4mn_k \qquad m = \frac{1}{2}\mathbf{S}\cdot C^0$$

Exact WGC relation (Repulsive Force Conjecture [Palti'17]):

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\rm Pl}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\right]$$

Coulomb $\overset{!}{\geq}$ **Gravity** $+$ **Yukawa**

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$$\frac{1}{1+q}\frac{1}{M_{\rm Pl}^2} \overset{!}{\geq} \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\rm Pl}^2}$$

Satisfied only for $q=0$ EFT strings $\leftrightarrow$ critical heterotic string!

Weak Gravity Conjecture

$$\frac{g_{\mathrm{YM}}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\mathrm{Pl}}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\mathrm{Pl}}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\mathrm{Pl}}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\mathrm{Pl}}^2}\right)\right]$$

LHS:

$$\frac{2\pi}{g_{\mathrm{YM}}^2} = \mathcal{V}_{\mathbf{S}} = \kappa \mathrm{Re}(T_0) + \dots\,, \qquad M_k^2 = 8\pi T_{\mathsf{EFT}}(n_k - E_0)$$

$$q_k^2 \geq 4mn_k\,, \quad m = \frac{1}{2}\mathbf{S}\cdot C^0 = \kappa e_0\,, \quad \frac{T_{\mathsf{EFT}}}{M_{\mathrm{Pl}}^2} = e_0 L^0 = -\frac{e_0}{2}\frac{\partial K}{\partial \mathsf{Re}\, T_0} = \frac{e_0\,(1+q)}{2\,\mathsf{Re}\, T_0}$$

$$\Longrightarrow \frac{q_k^2 g_{\mathrm{YM}}^2}{M_k^2} = \frac{1}{1+q}\frac{1}{M_{\mathrm{Pl}}^2}$$

RHS:

$$g^{00} = -2\frac{\partial^2 K}{\partial(\mathsf{Re}\, T_0)^2} = \frac{2(1+q)}{(\mathsf{Re}\, T_0)^2} \Longrightarrow \frac{1}{4}\frac{M_{\mathrm{Pl}}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\mathrm{Pl}}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\mathrm{Pl}}^2}\right) = \frac{1}{2(1+q)}$$

$$\Longrightarrow \mathrm{RHS} = \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\mathrm{Pl}}^2}$$

Weak Gravity Conjecture

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\rm Pl}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\right]$$

$$\frac{1}{1+q}\frac{1}{M_{\rm Pl}^2} \overset{!}{\geq} \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\rm Pl}^2}$$

One *could* save it by *postulating* that

$$M_k^2 = 8\pi T_{\rm EFT}\,\mathfrak{n}(q)(n_k - E_0) \qquad \text{for} \quad \mathfrak{n}(q) = \frac{2}{2+q}$$

Instead we claim:

This is not the solution.

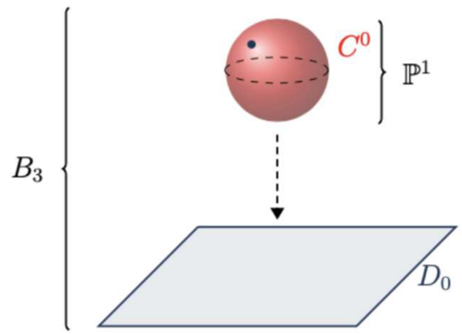
The tWGC is not asymptotically satisfied by perturbative EFT string excitations unless $q = 0$ (critical heterotic string) – for good reason.

Interpretation of limits

Why does the tWGC relation for excitations of EFT string with $q \neq 0$ fail?

$q = 0$: Emergent heterotic string limit

$$\mathcal{V}_{C^0} \sim \frac{1}{\lambda} \qquad \mathcal{V}_{S=D_0} \sim \lambda^2 \qquad \mathcal{V}_{B_3} \sim \lambda$$



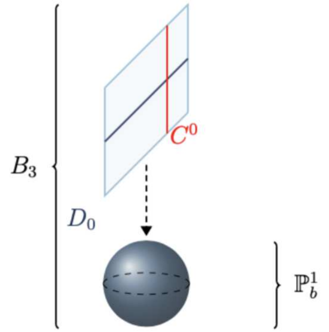
- $\Lambda^2_{\text{WGC}} = g^2_{\text{YM}} M^2_{\text{Pl}} \sim T_{\text{EFT}} \sim M^2_{\text{het}}$
- Species scale set by tension of emergent heterotic string cf. [Dvali,Lüst'09] [Marchesano,Melotti'22] [Castellano,Herraez,Ibanez'22]

$$\Lambda^2_{\text{sp}} \sim M^2_{\text{het}} \log\left(\frac{M_{\text{Pl}}}{M_{\text{het}}}\right)$$

$$\Longrightarrow \frac{\Lambda^2_{\text{WGC}}(U(1))}{\Lambda^2_{\text{sp}}} \to 0$$ logarithmic suppression

Interpretation of limits

q=1: decompactification to 6d gauge + gravity
which generically is not weakly coupled



$$\mathcal{V}_{\mathbb{P}^1_b} \sim \lambda \qquad \mathcal{V}_{\text{fiber}} \sim 1 \qquad \mathcal{V}_{B_3} \sim \lambda$$

- $\Lambda^2_{\mathsf{WGC}} = g^2_{\mathsf{YM}} M^2_{\mathsf{Pl}} \sim T_{\mathsf{EFT}} \sim M^2_{\mathsf{IIB}}$
- $\frac{M^2_{\mathsf{KK}}}{M^2_{\mathsf{Pl}}} \sim \frac{1}{\lambda^2} \implies \frac{\Lambda^2_{\mathsf{sp,KK}}}{M^2_{\mathsf{Pl}}} \sim \left(\frac{M^2_{\mathsf{KK}}}{M^2_{\mathsf{Pl}}}\right)^{\frac{n}{n+2}} \Big|_{n=2} \sim \left(\frac{1}{\lambda^2}\right)^{1/2} \sim \frac{M^2_{\mathsf{IIB}}}{M^2_{\mathsf{Pl}}}$

$$\implies \frac{\Lambda^2_{\mathsf{WGC}}\,(U(1))}{\Lambda^2_{\mathsf{sp}}} \sim 1$$

- Gauge theory and EFT string are strongly coupled in asymptotic 6d frame
- Tower for WGC not strictly speaking necessary

Interpretation of Limits

- q=2: decompactification to 10d, with 8d defect gauge theory

$$\mathcal{V}_{B_3} \sim \lambda^3 \qquad \text{homogeneously}$$

- $\Lambda^2_{\text{WGC}} = g^2_{\text{YM}} M^2_{\text{Pl}} \sim T_{\text{EFT}} \sim \lambda$
- $\frac{M^2_{\text{KK}}}{M^2_{\text{Pl}}} \sim \frac{1}{\lambda^4} \implies \frac{\Lambda^2_{\text{sp,KK}}}{M^2_{\text{Pl}}} \sim \left(\frac{M^2_{\text{KK}}}{M^2_{\text{Pl}}}\right)^{\frac{n}{n+2}} \Big|_{n=6} \sim \frac{1}{\lambda^3}$

$$\implies \frac{\Lambda^2_{\text{WGC}}\left(U(1)\right)}{\Lambda^2_{\text{sp}}} \to \infty$$

- complete decoupling between gravity and gauge sector
- Here: 8d gauge sector as defect in 10d gravity
- WGC does not apply

Generalisations

Result: [Cota,Mininno,TW,Wiesner'22]

- Conclusions carry over to general, non-EFT string limits
- Example: Combine q=1 quasi-primitive limit with weak coupling limit in 6d $\Longrightarrow$ 6d emergent string limits with tWGC

Asymptotic tWGC from perturbative string excitations requires

1. limit towards weak coupling of gauge theory and of the EFT string
2. species scale to be set by EFT string

$$\Lambda_{\rm sp} \overset{!}{=} \Lambda_{\rm sp,string} \sim T_{\rm string}$$

Both requirements together are only satisfied for $q = 0$ primitive EFT strings limits (or their generalisations):

These are always the heterotic critical strings (i.e. emergent string limits)

Generalisations

General pattern (also confirmed in M-theory [see talk by Mininno])

1. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{sp}}} \ll 1$: Decompactification limit with gauge group the KK U(1)

   ✓ WGC tower: KK tower

2. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{sp}}} \leq 1$: Emergent heterotic string limit ✓ WGC tower: het

3. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{sp}}} \sim 1$: Decompactification to higher dim. gauge-gravity theory

   in which gauge sector not weakly coupled

   ??? WGC Tower ???

4. $\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{sp}}} \gg 1$: Decoupling of gauge and gravity limit:

   Either decompactification to defect theory or $g_s \to 0$ limit

   ✓ No WGC tower needed

Case 3 remains mysterious:

Do black holes furnish WGC tower? If non-BPS, then hard to tell!