Black Hole Entropy, Species Scale and Topological Strings

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Joint work with Niccolo Cribiori and Georgina Staudt (arXiv:2212.10286)

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Generic Features of Quantum Gravity :

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Mixing

$$\Lambda_{UV} \longleftrightarrow \Lambda_{IR}$$

- UV cut-off: Species scale:

$$\Lambda_{QG} = \frac{M_p}{N^{\frac{1}{d-2}}} \leq M_p \simeq 10^{19}\mathrm{GeV}$$

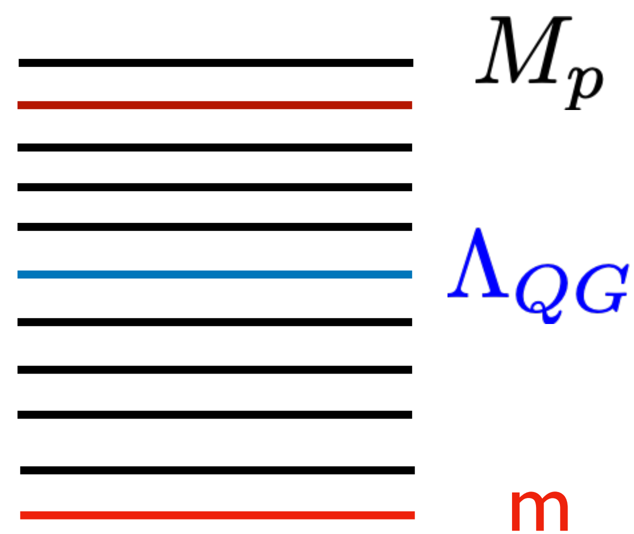
[G. Dvali (2007)]

N: Number of particles below Λ_{QG} .

.... it depends on scalar fields: $N = N(\phi)$

Swampland Distance Conjectures

At large distance Δ directions in the parameter space of string vacua there must be an infinite tower of states with mass scale m.



$$m = M_p e^{-\alpha\Delta}$$

[H. Ooguri, C.Vafa (2006)]

$m << M_p$ when $\Delta \to \infty$

Emergent String Conjecture :

The light tower of states at large distances is given by either light string excitations or light KK modes.

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String Species :

Regge tower: $$m_n \simeq \sqrt{n}\ M_s$$

Effective number of species: $$N = \frac{1}{g_s^2}$$

[G. Dvali, D.L. (2009); G. Dvali, C. Gomez (2010)]

Species scale: $$\Lambda_{QG} = M_p\ g_s^{\frac{2}{d-2}}$$

Agrees with the string scale M_s in d dimensions.

Geometric Species:
KK compactification with n large extra dimensions of radius R

Tower mass scale: $m_{KK} = 1/R \qquad \Delta = \log R$

KK tower: $$m_n \simeq \frac{n}{R}$$

Number of states: $$N = \left(\frac{\Lambda_{QG}}{m_{KK}}\right)^n = (\Lambda_{QG}\, R\ )^n = \mathcal{V}^{(n)}$$

Species scale: $$\Lambda_{QG} = (M_p)^{\frac{2}{n+2}} \ (m_{KK})^{\frac{n}{n+2}}$$
$$= (M_p)^{\frac{2}{n+2}} \ (1/R)^{\frac{n}{n+2}}$$

Agrees with the Planck mass in (4+n) dimensions.

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Agrees with the Planck mass in (4+n) dimensions.

Combined number of Species : $N = \mathcal{V}^{(n)} g_s^{-2}$

(Anti) de Sitter Conjecture - Dark Universe

[E. Palti, C.Vafa, D.L. (2019); M. Montero, C.Vafa, I.Valenzuela (2022)]

Tower mass scale: $m = \lambda^{-1}(\Lambda_{cc})^{\alpha}(M_p)^{1-4\alpha}$

Species scale: $\Lambda_{QG} = (M_p)^{\frac{2+n-4\alpha n}{n+2}}[\lambda^{-1}(\Lambda_{cc})^{\alpha}]^{\frac{n}{n+2}}$

Dark Universe: $n = 1 \quad \alpha = 1/4 \quad \lambda \sim 10^{-3}$

Radius of dark dimension: $R \sim \lambda\Lambda_{cc}^{-1/4} \sim 1\mu m$

Related species scale: $\Lambda_{QG} \simeq 10^{10}\ GeV$

Gravitino Conjecture - Supersymmetry Breaking

There is a tower of states related to the gravitino:

$$m \simeq (M_{3/2})^{\beta}(M_p)^{1-\beta}$$

[N. Cribiori, M. Scalisi, D.L. ; A. Castellano, A. Font. A. Herraez, L. Ibanez (2021)]

[See also: I. Antoniadis, C. Bachas, D. Lewellen, T. Tomaras (1988)]

$$\beta = 1: \quad M_{SUSY} = \mathcal{O}(\Lambda_{cc}^{1/8}) = \mathcal{O}(1 - 10 \text{ TeV})$$

[L. Anchordoqui, I. Antoniadis, N. Cribiori, D.L., M. Scalisi (2023)]

„Topological“ number of species:

So far we considered the number of species at large radii or at weak string coupling.

For a general CY compactification, there is topological contribution to the number of species in terms of the genus-one free energy:

[D. van de Heisteeg, C.Vafa, M.Wiesner, D.Wu (2022)]

$$N = F_1(\phi, \bar{\phi})$$

F_1 is part of the topological string partition function:

$$Z_{\rm top} = e^F \quad \text{with} \quad F = \sum_{g=0}^{\infty} F_g$$

$$F_1 = \frac{1}{12}\int_{CY} J \wedge c_2 + \cdots = \frac{1}{12} c_{2i}\, {\rm Im} z^i$$

[M. Bershadsky, S. Cecotti, H. Ooguri, C.Vafa (1993);
I. Antoniadis, E. Gava, K. Narain, T.R. Taylor (1993)]

Arguments for the topological number species hypothesis:

A) Spectrum of laplacian in topological B-model

B) a-function of a 4D CFT counts number of degrees of freedom.

Here: black hole argument for this hypothesis

[N. Cribiori, D.L., G. Staudt (2022]

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Black hole entropy contains information about number of species.

BH entropy: $$\mathcal{S} = (R_{BH})^{d-2}\, M_p^{d-2}$$

Black hole mass scale: $$\Lambda_{BH} = \frac{1}{R_{BH}} = \frac{M_p}{\mathcal{S}^{\frac{1}{d-2}}}$$

$$\Lambda_{QG} \geq \Lambda_{BH} \qquad \Longrightarrow \qquad N \leq \mathcal{S}$$

Smallest possible BH with minimal entropy ⟶ number of species

Black Hole Entropy Distance Conjecture:

Black holes are also related to a tower of states:

BH entropy conjecture $\mathcal{S} \to \infty$

$$m = \left(\frac{1}{\mathcal{S}}\right)^{\gamma}, \qquad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

- Test geometry by connecting BH-entropy related to internal moduli space.

[M. Delgado, M. Montero, C. Vafa (2022)]

- Tool: Attractor equations for black holes in $\mathcal{N} = 2$ supergravity.

$$\Rightarrow \qquad \mathcal{S} = \mathcal{S}(\phi_{\text{hor}})$$

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- Our strategy for deriving the number of species: look for smallest possible black holes probes with particular charges.

$$N = S_{\rm min}$$

Black holes that probe string states:

Heterotic dyonic black holes with charges p and q:

$$\mathcal{S} = 8\pi^2 p^2 (g_s)^{-2}$$

[M. Cvetic, D. Youm(1995);
M. Cvetic, A. Tseytlin (1995)]

String coupling at BH Horizon:

$$(g_s)^{-2} = \frac{q}{p}$$

Smallest possible black hole:

$$p = 1 \qquad \Longrightarrow \qquad \mathcal{S}_{\rm min} = 8\pi^2 (g_s)^{-2} = N$$

q arbitrary

N=2 supergravity:

Bosonic part of the action:

$$S = \int dx_4^2 \left(M_p^2 R + K_{ij}(z^i)\partial z^i \partial z^j + \mathcal{N}_{\Lambda\Lambda'}(z^i) F_\Lambda F_{\Lambda'} + \frac{1}{96\pi} \underbrace{c_{2i} \, \mathrm{Im}\, z^i}_{F_1} \mathrm{Tr}\, R \wedge *R + \ldots \right)$$

Vector multiplets: X^Λ $\qquad \Lambda = 0, \ldots, n_V$

Scalar fields: $z^i = X^i / X^0$ $\qquad i = 1, \ldots, n_V$

Second Chern numbers:

$$c_{2i} \equiv \int_M c_2(TM) \wedge \omega_i \qquad \omega_i \in H^2(M; \mathbb{Z})$$

Vector multiplet couplings: degree two function:

Prepotential:

$$F(X,A) = \sum_{g=0}^{\infty} F_g(X)\, A^g$$

Classical prepotential: F_0 - two derivative couplings:

First order correction: $F_1 = \frac{1}{12} c_{2i}\ \mathrm{Im} z^i$ - four derivative couplings.

Graviphoton field: A

Kähler potential:

$$K = -\log\ i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)$$

$$F_\Lambda = \frac{\partial F(X,A)}{\partial X^\Lambda}$$

Extremal black hole solutions in N=2 supergravity:

Electric and magnetic charges: (q_Λ, p^Λ)

$\mathcal{N}=2$ central charge: $$Z = e^{K/2}(X^\Lambda q_\Lambda - F_\Lambda p^\Lambda)$$

Attractor equations: scalar fields on horizon: $$p^\Lambda = i(X^\Lambda - \bar{X}^\Lambda)$$

$$q_\Lambda = i\left(F_\Lambda(X,A) - \bar{F}_\Lambda(\bar{X},\bar{A})\right)$$

[S. Ferrara, R. Kallosh (1996)]

Background for graviphoton: $$A = \bar{A} = -64$$

Area of horizon: $$Z\bar{Z} = e^{-K(X,A)} = X^0 \bar{X}^0 e^{-K(z,A)}$$

However the Bekenstein - Hawking area does not provide the entire entropy !

Wald formula: $$\mathcal{S} = 2\pi \oint_{S^2} \epsilon_{ab}\epsilon_{cd} \frac{\delta \mathcal{L}}{\delta R_{abcd}}$$ [R. Wald (1993)]

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 = \pi \left[Z\bar{Z} - 256 \, \mathrm{Im} \, F_A(X, A) \right]$$ with $F_A = \partial_A F$

$$= \pi \left[Z\bar{Z} + 4\mathrm{Im} \left(A F_A \right) \right]$$

$$= \pi \left[(X^\Lambda + \bar{X}^\Lambda) q_\Lambda + 4\mathrm{Im} \, F \right]$$

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Introduce $\frac{1}{2\pi}\phi^\Lambda = X^\Lambda + \bar{X}^\Lambda \,, \quad \mathcal{F}(\phi, p) = 4\pi \,\mathrm{Im}\, F$ such that $\frac{\partial \mathcal{F}}{\partial \phi^\Lambda} = -q_\Lambda ,$

Entropy is the Legendre transform of the free black hole energy:

[H. Ooguri, A. Strominger, C. Vafa (2004);
G. Cardoso, B. de Wit, J. Käppeli, T. Mohaupt (2006)]

$$\mathcal{S} = -\phi^\Lambda \frac{\partial \mathcal{F}}{\partial \phi^\Lambda} + \mathcal{F}(\phi, p)$$

BH partition function:

$$\mathcal{Z}_{BH} = \exp \mathcal{F}$$

IIA superstring on a CY manifold:

$$F(X,A) = F_0 + F_1 A$$

$$F_0 = -\frac{1}{6}\frac{C_{ijk}X^i X^j X^k}{X^0}\,, \qquad F_1 = -\frac{1}{24}\frac{1}{64}c_{2i}\frac{X^i}{X^0}$$

Electric charge q , magnetic charges p^i

Attractor equation → 2-cycle volume at horizon:

$$z^i = ip^i\sqrt{\frac{q}{\frac{1}{6}C_{ijk}p^i p^j p^k + c_{2i}p^i}}$$

Entropy: $$\mathcal{S} = 2\pi\sqrt{\frac{1}{6}q\left(C_{ijk}p^i p^j p^k + c_{2i}p^i\right)}$$

[K. Behrndt, G. Lopes Cardoso, B. De Wit, R. Kallosh, D.L. , T. Mohaupt (1996); C. Lopes Cardoso, B. De Wit, T. Mohaupt (1999)]

Agrees with microscopic entropy counting of wrapped M5-branes.

[J. Maldacena, A. Strominger, E. Witten (1997)]

Classical case: F1=0

$$\mathcal{S}_0 = 2\pi \mathcal{V}_0^{1/3} \left(\frac{1}{6} C_{ijk} p^i p^j p^k \right)^{2/3}$$

[N. Cribiori, M. Dierigl, A. Gnecchi, D.L. , M. Scalisi (2022)]

Classical volume (at horizon): $$\mathcal{V}_0 = \sqrt{\frac{6q^3}{C_{ijk} p^i p^j p^k}}$$

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Classical volume (at horizon): $$\mathcal{V}_0 = \sqrt{\frac{6q^3}{C_{ijk} p^i p^j p^k}}$$

Minimal entropy:

$$\frac{1}{6} C_{ijk} p^i p^j p^k = 1 \qquad \Longrightarrow \qquad \mathcal{S}_{0,\min} = 2\pi \mathcal{V}_0^{1/3}$$

q arbitrary

Number of species: $$N = 2\pi \mathcal{V}_0^{1/3}$$

⟶ BH probes KK tower of two-cycle, minimal cycle on a CY.

Special case: K3 fibred CYs:

$$F_0 = (X^0)^2 z^1 C_{ab} z^a z^b$$

z^1 : Volume of P^1 base of fibration.

$$\mathcal{S} = \sqrt{q p^1 C_{ab} p^a p^b} = z^1 C_{ab} p^a p^b$$

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Minimal entropy:

$$C_{ab} p^a p^b = 1 \qquad \Rightarrow \qquad \mathcal{S}_{\min} = z^1$$

q, p^1 arbitrary

Number of species: $N = z^1$ ⟶ BH probes KK tower of P^1 base

$\simeq$ Tower of dual heterotic strings $\mathcal{S}_{\min} = N \sim \dfrac{1}{g_s^2}$

[S. Lee, W. Lerche, T. Weigand (2019)]

Higher order case: F1 non-zero

$$\mathcal{S} = 2\pi\sqrt{\frac{1}{6}q\left(C_{ijk}p^ip^jp^k + c_{2i}p^i\right)}$$

Minimal entropy: $C_{ijk}p^ip^jp^k = 0 \quad (p^j = 0\,,\ j \neq i)$

q, p^i arbitrary This choice is possible for K3 and elliptic fibrations.

⟶ Shrunk K3 or elliptic fibre.

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Number of species:

$$\mathcal{S}_{\min} = 2\pi\sqrt{\frac{1}{6}qc_{2i}p^i} = 2\pi F_1 \Longrightarrow \boxed{N = 2\pi F_1}$$

[N. Cribiori, D.L., G. Staudt (2022)]

Note that for this choice of charges: $\mathcal{V}_0 = \infty\,, \qquad \mathcal{S}_0 = 0$

But finite „quantum" volume: $\mathcal{V} = \frac{1}{2}\frac{q^{3/2}}{\sqrt{\frac{1}{6}c_{2i}p^i}} = \frac{1}{2}\frac{q^2}{F_1}$

OSV conjecture

Here F1 is the higher order correction to the prepotential, but it is strictly speaking not yet connected to genus-one partition function of the topological string.

So to make the connection to the topological string we have to make the assertion:

$$\mathcal{Z}_{BH} = \exp \mathcal{F} = |\mathcal{Z}_{\text{top}}|^2$$

This is just the OSV conjecture !

[H. Ooguri, A. Strominger, C. Vafa (2004)]

Summary

Black Holes and their entropies provide information about species and tower of states in quantum gravity:

Minimal black hole entropy:

- KK species of internal geometry
- String species
- Species of topological string

Thank you !