Solid State Communications, Vol. 32, pp. 695–698.
Pergamon Press Ltd. 1979. Printed in Great Britain.

ELECTRON–PHONON INTERACTION AND CYCLOTRON RESONANCE IN TWO-DIMENSIONAL ELECTRON GAS*

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(*Received* 23 *March* 1979; *in revised form* 6 *July* 1979 *by R.H. Silsbee*)

Results for the influence of electron–phonon interaction on the cyclotron effective mass and the resonance linewidth in a two-dimensional electron gas are presented. The temperature and magnetic field dependence is studied and the existence of quantum oscillations is demonstrated. It is shown that the relevant phonon frequency in typical MOS inversion layers is very small so that magneto-transport properties are temperature dependent even at a few degrees Kelvin. Results are consistent with the observed temperature, magnetic field and frequency dependence in Si(100) inversion layers.

THE AIM OF THIS PAPER is to present some results and considerations of the influence of electron–phonon interaction on the magneto-optic properties of two-dimensional electron gases. Experimental realization and study [1] of two-dimensional charged gases has been found mostly in the inversion/accumulation layers created in the semiconductor at the semiconductor–oxide interface of a metal–oxide–semiconductor (MOS) sandwich. Most such studies are based on inversion layers in silicon, although a few isolated studies on compound semiconductors have also been reported. It has been noted that the ability to continuously vary the carrier density, along with usual parameters like temperature, applied magnetic field, etc. affords a rather unique opportunity for the study of basic interaction phenomena in two dimensions. However, the knowledge gained from such studies depends, amongst other things, upon the use of appropriate theoretical formulation for the interpretation of the particular experiment [2]. Of the two basic interactions intrinsic to such a system, namely electron–electron and electron–phonon, a considerable emphasis has been placed upon studies involving only the former, to the neglect of the latter. To a large extent this has been motivated by the expected and claimed dominance of the *e–e* interaction over the electron–phonon interaction in silicon inversion layers, although no definitive evidence for this has been presented. On the contrary, extensions of certain well known results of Fermi liquid theory from three to two dimensions, when applied [3] to inversion layers in the appropriate range of their applicability, prove sufficient reasons to suspect significant *e–ph* contributions. For the compound semiconductors it has been argued by one of us that the polar nature of the materials suggests, *a priori*, a stronger electron–phonon coupling in the inversion/accumulation region. In addition, very low values of r_s indicate insignificant *e–e* interaction effects. In either case, a realistic understanding of the *e–ph* effects, small or large, requires a meaningful and realistic investigation of its influence and distinguishing features.

With this aim, in this paper we consider the simplest model – a single parabolic 2-dimensional electron band interacting with bulk phonons in the presence of static magnetic field applied normal to the plane of the electron gas. Even for this simplest of all situations, the results presented here are the first and reveal significant dependences on temperature, magnetic field, etc. The particular results obtained are the position of the main cyclotron resonance peak, as well as its linewidth. The density, temperature and magnetic field dependence of both is investigated. In contrast to popular belief, but consistent with relevant experiments, it is shown that significant temperature dependence can be present at very low temperatures since the frequency of relevant acoustic phonons is of the order of a few degrees Kelvin. It is also found that in the regime where the intrinsic Landau level width is dominated by short ranged impurity scatters, the cyclotron resonance linewidth due to such phonons has a temperature dependent component which shows a $\sqrt{H}$ dependence on the magnetic field. This remarkably unexpected result further strengthens our suggestions regarding the relevance of

* Work supported by AFOSR (Contract No. AFSOR78-3530), and in part by ONR (Contract No. N00014-77-C-0397).

† Alfred P. Sloan Foundation Fellow.

electron–phonon interactions in inversion layers by providing both, temperature and magnetic field dependence of the linewidth in conformity with relevant data.

The effective mass appearing in the main cyclotron resonance is found to have a leading order enhancement given by a dimensionless electron–phonon coupling strength, confirming an earlier suggestion [3] made by one of us on the basis of the analogy with the result of Fermi liquid theory in three dimensions. However, of equal significance is the appearance of quantum oscillations caused by the Fermi energy passing through various Landau levels as the magnetic field is varied. Finally, the effective mass is found to increase with temperature in agreement with relevant experiments.

We consider a two-dimensional electron gas with a parabolic band and in a perpendicular magnetic field H interacting with phonons, the system being characterized by the Hamiltonian,

$$\mathscr{H} = \sum_{nk} \hbar\omega_c(n+\tfrac{1}{2})c^+_{n,k}c_{n,k} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} b^+_{\mathbf{q}} b_{\mathbf{q}} + \sum_{nn'} g_{\mathbf{q},n,n'} c^+_{n,k} c_{n',k-q_y}(b_{\mathbf{q}} + b^+_{-\mathbf{q}}) \,. \quad (1)$$

The first term describes free electrons in a magnetic field with a wavefunction [4] $e^{iky}u_n(x + kl^2)$, where $l = c\hbar/eH$ and u_n is the harmonic oscillator eigenfunction. The free electron energies are $\hbar\omega_c(n+\frac{1}{2})$ where, $\omega_c = (eH/m^*_b c)$, is the bare cyclotron frequency and m^*_b the band effective mass. The energies are independent of k, leading to a degeneracy of $(1/\pi l^2)$. The second and third terms correspond to bulk phonons and their interaction with the electrons. For typical inversion layers the electron wavefunction in the z direction is spread over many lattice constants; thus we expect the bulk phonons with small q_z momentum to couple strongly with the electrons. The interacting phonons are then effectively 2-D with the coupling constant

$$g_{\mathbf{q},n,n'} = g(\mathbf{q}) \int dx\, u_n(x)\, e^{iq_x(x-kl^2)} u_{n'}(x - q_y l^2) \quad (2)$$

where $g(\mathbf{q})$ is the electron–phonon coupling when electrons are represented by plane waves. For bulk-like phonons [5], $|g_q|^2 \propto q$ so that we can define a dimensionless coupling λ by

$$\lambda = \frac{|g(\mathbf{q})|^2}{\omega_q} N(E_F) = \frac{|g(\mathbf{q})|^2}{\omega_q}\left(\frac{m}{\pi}\right) \quad (3)$$

where (m/π) is the density of states for a free 2-D electron gas. Note that the phase space reduction, restricting phonons to have small q_z, yields a smaller λ for the same $g(\mathbf{q})$ than the corresponding λ in 3-D systems. Utilizing the deformation potential of bulk silicon and incorporating the finite extension of the electron wave function in the z direction as discussed in [5], we estimate λ to be of order 0.03. Using the Laguerre polynomials, $\mathscr{L}^m_n(x)$, the coupling (for $n' > n$) can be written as [6]

$$|g_{\mathbf{q},n,n'}|^2 = \frac{\lambda\pi\omega_q}{m}\frac{n!}{n'!}(\tfrac{1}{2}q^2l^2)^{n'-n}[\mathscr{L}^{n-n'}_n(\tfrac{1}{2}q^2l^2)]^2 \times \exp(-\tfrac{1}{2}q^2l^2). \quad (4)$$

From this relation we can obtain useful information about the relevant phonons. For a magnetic field of $H \sim 7$ T, (as in the typical experiment [7–9]) $l \sim 100$ Å. Thus the exponential factor in equation (4) is extremely small unless $q \lesssim (\sqrt{2}/l)$. The prefactor in equation (4) shifts the important q to higher values. This is significant if n near the Fermi level is high, e.g. in 3-D systems $n \sim 10^3$ and all the phonons must be considered. However, for typical inversion layers with electron density of $n_s \sim 10^{12}$ cm^{-2}, only a few Landau levels are filled (~ 2 levels for $H \sim 7$ T, $m^*_b = 0.2 \times$ electron mass). In this situation the exponent in equation (4) dominates and the important phonons have frequency $\sim \omega_0$, where $\omega_0 = v_s\sqrt{2}/l$. Typically, therefore the temperature dependence associated with the excitation of these phonons should be observable down to temperatures of a few degrees.

As a specific example of these considerations we evaluate the cyclotron mass and linewidth to lowest order in λ. A straightforward perturbation theory gives divergent terms at $\omega = \omega_c$ and one has to sum infinite series. Instead, we use the memory function technique [10] which is equivalent to summation of these infinite series, although it involves just the lowest order perturbation calculation. The current operator is,

$$J_+ = (J_x + iJ_y) = i\sqrt{(2m\omega_c)} \sum_{n,k} \sqrt{(n+1)}\, c^+_{n+1,k} c_{n,k}, \quad (5)$$

and $J_- = J^\dagger_+$. The conductivity, which is related to the J_-, J_+ correlation function, is written in the form

$$\sigma_{-+}(\omega, H, T) = \frac{2iNe^2}{m} \frac{1}{\omega - \omega_c + M_{-+}(\omega, H, T)} \,. \quad (6)$$

For $\omega \neq \omega_c$ one can expand with respect to M and identify it to a given order in perturbation theory. However the form (6) has the correct resonance structure even at ω_c *provided* M is a well behaved function at ω_c. Therefore if $M_{-+}(\omega)$ from low order perturbation theory is well behaved at $\omega = \omega_c$, one can assume analytic continuation and use $M_{-+}(\omega_c)$ in equation (6). We proceed to evaluate $M_{-+}(\omega)$ to first order in λ by using the equation of motion procedure [10]. We

obtain

$$M_{-+}(\omega, H, T) = \frac{1}{2Nm\omega}[\phi_{-+}(\omega) - \phi_{-+}(\omega = 0)] \quad (7)$$

where

$$\phi_{-+}(\omega, H, T) = \frac{-1}{\pi} m\omega_c \sum_{nn'} \sum_{\mathbf{q}} q^2 \{f(n)[1 - f(n')]$$
$$\times [1 + N(\omega_q)] - f(n')[1 - f(n)]N(\omega_q)\}$$
$$\times \{[\omega_c(n - n') - \omega_q + \omega]^{-1}$$
$$+ [\omega_c(n - n') - \omega_q - \omega]^{-1}\} |g_{\mathbf{q},n'n}|^2 \quad (8)$$

$f(n)$ is the Fermi function at energy $\hbar\omega_c(n + 1/2)$, and $N(\omega_q)$ is the Bose function.

In general the renormalized cyclotron frequency ω_c^* is given by the pole of equation (6). However if the frequency dependent $M_{-+}(\omega)$ is used to solve for the pole, ω_c^* will include higher order terms in λ which are beyond the validity of our approximations. Consequently we evaluate $\omega_c^* = \omega_c - \mathrm{Re}\, M(\omega_c)$. Note that for impurity scattering [10–12] the result is similar to equation (8) except that in the denominator ω_q is replaced by zero. In that case $M_{-+}(\omega)$ is not well behaved at $\omega = \omega_c$ and the memory function technique is not useful; direct summation of perturbation series is necessary [11]. The acoustic phonon dispersion provides a continuum energy variable in equation (8) so that $M_{-+}(\omega_c)$ is well behaved at ω_c.

We evaluate ω_c^* for temperatures $\omega_0 \ll T \ll \omega_c$, so that the Fermi factors are temperature independent, while $N(\omega_q) = (T/\omega_q)$ since the important phonons have $\omega_q \lesssim \omega_0$. Also, since typically $\omega_c \sim 10$ to 40 K and $\omega_0 \sim 2$–5 K, the assumed range of temperatures covers most of the relevant experimental temperatures. Neglecting terms with $(\omega_0/\omega_c) \ll 1$, we obtain,

$$\left(\frac{m_{cr}^*}{m_b^*}\right) = \left\{1 + \lambda\left[1 - (2\nu + 1)\frac{f(\nu)[1 - f(\nu)]}{\nu + f(\nu)} + \frac{2T}{\omega_c}\right.\right.$$
$$\left.\left.\times \left(1 - \tfrac{1}{2}\frac{f(0)}{\nu + f(\nu)}\right)\right]\right\} \quad (9)$$

where ν is the number of filled Landau levels, and the Fermi level lies near the $(\nu + 1)$ level: (the electron density is $N = g_v(\nu + f(\nu))(m\omega_c/\pi)$ where g_v is the valley degeneracy). For $T \lesssim \omega_0$ the temperature dependence departs from linearity due to the low temperature phonon occupation function. At $T = 0$, the last term in equation (9) goes to zero.

Consider next the width, $\Gamma_{\mathrm{RES}}^{e\text{–}ph}$, of the cyclotron resonance. The summation in equation (8) corresponds to sharp Landau levels. Therefore, when $\omega = \omega_c$, the imaginary part restricts ω_q to be a non-zero multiple of ω_c yielding a very small contribution $[\sim \exp(-\omega_c/\omega_0)^2]$. However, if we assume that the contributions of the electron–phonon and electron-impurity interactions lead to additive corrections for $M(\omega)$ (this is correct [10] to leading order in λn_i, where n_i is the impurity concentration) then the impurity contribution to the resonance width is given by the previous calculations [11], while the electron–phonon effect includes corrections due to impurities (leading to a Landau level width of $\sim \Gamma_L$) without the impurity vertex corrections. Therefore we can define a resonance width due to impurity correction, $\Gamma_0 \sim \sqrt{2}\Gamma_L$ [11] and use this width in the evaluation of $M(\omega_c)$. The temperature dependent part of equation (9) does not change when Γ_0 is introduced (if $\Gamma_0 \ll \omega_c$). The e–ph contribution to the resonance width for $\omega_c \gg T \gg \omega_0$, Γ_0 is then,

$$\Gamma_{\mathrm{RES}}^{e\text{–}ph} = 2\lambda T(\omega_c/\Gamma_0) \quad (10)$$

so that, the total resonance width is given by

$$\Gamma_{\mathrm{RES}} = \Gamma_{\mathrm{RES}}^{e\text{–}imp} + \Gamma_{\mathrm{RES}}^{e\text{–}ph} \quad . \quad (11)$$

Thus the width is a linear function of temperature, down to very low temperatures of a few degrees ($\sim \omega_0$). For $T \lesssim \omega_0$, it departs from linear dependence and saturates at a value governed by the e-impurity scattering (and e–e, if appropriate).

Equations (9) and (10) provide the density, temperature and magnetic field dependence of the cyclotron effective mass and resonance linewidth, respectively. The cyclotron effective mass shows a leading order correction given by λ. In analogy with the well known result of the Fermi liquid theory in 3-dimensions, such a leading order correction was predicted by one of us to occur in two dimensions as well, and employed [3] to place an upper bound on λ via comparison with the observed mass enhancement. The second and third terms inside the square brackets reflect the corrections due to magnetic field and temperature dependence. These terms involve the Fermi functions, thus giving rise to an oscillatory behavior (see Fig. 1) caused by the Landau levels sweeping past the Fermi energy as the magnetic field is varied. This is the analogue of the quantum oscillations found for short ranged impurity scattering [11]. The explicit temperature dependence is seen in the third term and arises from the phonon occupation function. The linear temperature dependence is a consequence of the regime $\omega_0 < T \ll \omega_c$. We note that the mass enhancement due to phonons increases with temperature, in conformity with the experiments [10]. It is worth noting that in the same regime of temperatures, the electron–impurity and electron–electron interactions give no temperature dependence [12] while for higher temperatures, the effective mass is found to decrease with temperature, both features in contradiction to relevant experiment [9].

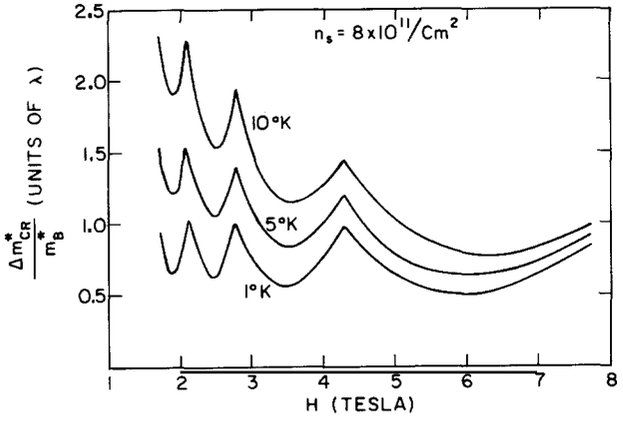

Fig. 1. The shift in the cyclotron mass in units of λ [equation (10) as a function of magnetic field for various temperatures]. Note the increase with temperature, decrease with magnetic field (or external frequency) and the quantum oscillations.

The magnetic field and temperature dependence of the mass enhancement is shown in Fig. 1 for $n_s = 8 \times 10^{11}\,\mathrm{cm}^{-2}$, in units of the dimensionless electron–phonon coupling strength, λ. The quantum oscillations show the largest variation at lower fields and high temperature, the overall behavior being a gradual decrease in the effective mass with increasing magnetic field at a fixed temperature. The absolute magnitude of the variations in the effective mass are difficult to estimate in the absence of either a reliable first principles model and estimate of λ, or of relevant and uncontroversial data from which λ may be extracted. However, the only two [9, 13] reported data on the temperature dependent linewidth are in agreement and may be employed to obtain a first estimate of λ.

The linewidth given by equation (11) is consistent with the linear temperature dependence reported for Si(100) inversion layers in references 9 and 13. The total linewidth is found to be of the form $\omega_c(A + BT)$, with both A and B being essentially the same in both the reported data for $8 < T < 45$ K. The observed departure from linear temperature dependence for $T < 8$ K is consistent with our estimated value of $\omega_0 \sim 6$ K. To make a first estimate of λ, we identify the contribution (BT) with equation (10). We note however that equation (10) involves a resonance width Γ_0 arising from e-impurity interaction, but without vertex corrections. Though the resonance width $\Gamma_{RES}^{e\text{-imp}}$ due to impurities is in general larger than the resonance width Γ_0 [11], taking the value ($\omega_c A$) as a lower bound on Γ_0, an estimate of λ is possible from the resonance linewidth. From the data, $A \approx 0.15$ and $B \approx 7.5 \times 10^{-3}$, so that at $n_s = 5 \times 10^{11}\,\mathrm{cm}^{-2}$ and $\omega_c = 3.71$ meV we find $\lambda \gtrsim 0.026$. Such a value of λ is in close agreement with the estimate based on bulk deformation potential noted earlier. When employed in equation (10), it gives a small temperature dependent enhancement of the mass. This would be consistent with the data of Kennedy *et al.*, but, at face value, not with the data of Külbeck and Kotthaus. However, a value of Γ_0 about three times larger would be sufficient to explain the data of Külbeck and Kotthaus, if such a significant mass enhancement were confirmed experimentally. This could be the case if long range impurity scattering is important, as is expected at low densities. In this case $\Gamma_{RES}^{e\text{-imp}} \gg \Gamma_0$ [11] and we cannot evaluate Γ_0 from experimental data. However at higher densities ($\gtrsim 10^{12}\,\mathrm{cm}^{-2}$) we expect $\Gamma_{RES}^{e\text{-imp}} = \Gamma_0$ and then one can estimate λ from both equations (11) and (9), yielding a test on the present theory.

A significant feature of the linewidth expression, equation (11), is its dependence on the magnetic field. If Γ_0 were independent of the magnetic field, the linewidth contribution due to phonons would be proportional to the magnetic field, H. However, the resonance width caused by short ranged impurity scattering according to Ando, is determined by the Landau level width, which in turn is proportional to $\sqrt{H}$. Hence in this regime the linewidth contribution due to phonons shall also be proportional to $\sqrt{H}$. Thus the observation of $\sqrt{H}$ dependence of the resonance linewidth is completely consistent with the phonon scattering in the presence of short ranged impurity scattering. It is important to note though that at $n_s \lesssim 7 \times 10^{11}\,\mathrm{cm}^{-2}$, the resonance linewidth shows departures from $\sqrt{H}$ behavior, thus indicating that indeed $\lambda > 0.026$ at $n_s = 5 - 7 \times 10^{11}\,\mathrm{cm}^{-2}$.

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