Quantum Hall transitions in field-induced spin density systems

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Abstract

Field-induced spin density wave (FISDW) systems exhibit coexistence phases between well-defined Quantum Hall plateaux phases with even integers $2N$ and $2N'$. We show that a disordered coexistence region accounts for the observed peaks in the longitudinal reisitivity as the field varies between plateaux. It also results in a random spin mixing which yields two energy-split extended states. The longitudinal resistance is expected to show two peaks with a temperature (T) dependent width $\sim T^\kappa$. The peak width should saturate below the non-nesting interlayer coupling of ≈ 40 mK. © 1999 Elsevier Science Ltd. All rights reserved.

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Organic conductors exhibit a cascade of (magnetic) Field-Induced Spin Density Wave Phases (FISDW) below about 1 K and in fields ranging from a few teslas to about 20 T. Each SDW phase shows a well-defined Hall plateau where the Hall resistance is $h/(2e^2N)$ with an integer N; see Refs. [1–3] for recent reviews. The sequence of integers is usually monotonic, the integer N decreasing by 1 as the field increases. The even integers $2N$ signify that both spin states are coupled by the spin density wave (SDW) and the quantum Hall (QH) phenomena are then degenerate in the spin states. Although the Quantized Hall Effect (QHE) in FISDW seems similar in many respects to the Integer QHE seen, e.g. in MOSFETS [4], it is different from the latter in many important ways: it is observed in an anisotropic 3D material; the effect would not exist in the absence of electron–electron interactions; furthermore, under specific conditions of pressure and field, the QHE changes sign: a negative plateau is inserted within the positive sequence of FISDW [5]; last, disorder plays no role in the phenomenon, contrary to the situation in the IQHE: the Fermi level is pinned in the middle of the SDW electronic gap between extended states by the broken symmetry phenomenon, not by disorder. In fact the very existence of FISDW phases is made possible because very clean samples are available. This is a

prerequisite for the observation of FISDW: the electronic mean free path has to be larger than the magnetic length $2\pi/G = \phi_0/(Bb)$ (where ϕ_0 is the flux quantum, B the magnetic field and b the interchain distance in the most conducting plane).

The FISDW phase diagram is well understood within the so-called Quantum Nesting Model [1–3,6–8]: in materials with open Fermi surfaces and good nesting properties, electronic motion under magnetic field becomes 1D and periodic; this opens up gaps between Landau bands when electron–electron interactions stabilize a SDW phase. Minute changes in the electronic dispersion relation, such as can be caused by applying pressure, may result in stabilizing phases with negative Hall numbers [9].

The change in order parameter from sub-phase in the sequence of FISDW is a discontinuous jump of the SDW wave vector parallel component by one inverse magnetic length G (in the usual monotonic sequence) or by an integer number of G's (in the case of a transition from a positive Hall number to a negative one). As a result, all phase transitions are weakly first order. Hysteresis is observed quite generally both in transport properties [5] and thermodynamic ones [1–3].

Although the Quantum Nesting Model accounts in a satisfactory way for nearly all aspects of the phase diagram (except perhaps at very large fields), for the existence of Hall plateaux, their sequence, and the rare occurrence of negative Quantum Hall numbers, it fails to account correctly

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for the longitudinal Hall resistance at the transition between FISDW phases. A naive interpretation of the model would predict a small discontinuous change of ρ_{xx} at the transition between plateaux at low temperatures, reflecting the small discontinuous change in electronic gap at the Fermi level, and the activated nature of dissipation processes. The experimental situation is quite different: ρ_{xx} exhibits spikes at the transition between plateaux [5], in a manner similar, at first sight, to what is observed in the usual MOSFET IQHE [4].

The purpose of this paper is to suggest that spikes in ρ_{xx} can be understood on the basis of the thermodynamics of the first-order transition between FISDW: we consider the coexistence region between Hall plateaux N and N' and show that it exhibits critical phenomena specific to these QH states. We propose then the following scenario. The first-order transition between plateaux N and N' is driven by a balance of the interaction between electrons and Landau quantization in a magnetic field. Within the coexistence region we assume that nucleation of the new phase is a slow dynamic process and, therefore, the system is composed of isolated clusters of the N' phase embedded in a continuum of the N phase. On the time scale of electronic transport, these clusters are randomly quenched. Uncoupled chiral gapless states are formed at the cluster boundaries. As the field increases the coexisting phases have an increased fraction of the N' phase until a percolation threshold is achieved at some field, beyond which the N phase forms isolated clusters within the N' phase.[2]

The finite size of an N cluster implies that not all the electronic states participate in forming the SDW, i.e. there should be $2N$ gapless edge states, as in the usual QH system. In fact Yakovenko and Goan [1–3] have explicitly constructed such edge states for the FISDW. They show that the SDW couples opposite $\pm k_F$ states which are N chains apart where k_F is the Fermi wave-vector along the quasi-1D chains (x-direction). Hence the last N chains of either $+k_F$ or $-k_F$ near the edge are uncoupled, i.e. gapless chiral electronic 1D liquids. When an N phase is embedded in an N' phase the latter produces $2N'$ edge states with opposite chirality around the N phase. The SDW can then couple some of these states, so that only $|N - N'|$ chains remain gapless. The density of states of the gapless states is then $N(0) = |N - N'|/(\pi v_F l)$ per unit area, where l is the transverse size of a cluster.

Transport in the coexistence region is determined by the gapless states which are scattered by the random SDW clusters. The problem is then similar to the two dimensional (2D) QH systems in which two spin states are randomly

coupled, e.g. by spin orbit coupling. Here, the two spin states are coupled by the SDW fluctuating field. This coupling leads to two nondegenerate extended states [10,11], which yield peaks in ρ_{xx}; the temperature dependence of these peaks has been extensively studied in the 2D QH systems [12,13] and yields information on the criticality of QH states [14,15].

Consider the Hamiltonian in presence of a SDW order parameter with amplitude Δ and phase θ. This Hamiltonian has been applied successfully for the FISDW phases [1–3]. We represent the Hamiltonian in a spinor state

$$[u(x, y) \exp(ik_F x + ik_z z), v(x, y) \exp(-ik_F x + i(k_z + \pi/c)] \tag{1}$$

for the right and left moving electrons. In all the FISDW phases the SDW has wavevectors π/c in the least conducting z direction parallel to the magnetic field [7,8]; hence the SDW couples states with momentum k_z only to those with momenta $k_z + \pi/c$. This is a result of perfect nesting in the z direction, i.e. the one-electron dispersion relation along k_z is dominated by a $t_c \cos(k_z c)$ term which allows perfect matching of the opposite Fermi surfaces when k_z is shifted by π/c [7,8,16]. In contrast, the SDW wavevector components Q_x, Q_y depend on the magnetic field and jump discontinuously between the N phases. In particular $Q_x = 2k_F - NG$. The Hamiltonian, with Pauli matrices τ_i, $i = 1, 2, 3$ in this spinor space, has the form [1–3]

$$\mathcal{H} = \{ -iv_F \partial_x - t_c \cos(k_z c) \} \tau_3 + \Delta \tau_1 \exp[i\tau_3(NGx - \theta)]$$

$$- f(k_y b - Gx) \tag{2}$$

Here v_F is the Fermi velocity, and $f(k_y b)$ represents the electron dispersion in the k_y direction with the wavevector k_y shifted by a vector potential. Note that the $t_c \cos k_z c$ form is essential in obtaining a coupling between the two states in the spinor Eq. (1) and $t_c \cos[(k_z + \pi)c] = -t_c \cos k_z c$ is used to obtain its τ_3 form in Eq. (2). As a result, by a unitary transformation $U = \exp[ixt_c \cos(k_z c)/v_F]$ the t_c term can be eliminated and has no effect on the mean field level [6–8,12,13]. Non-nesting terms, e.g. $t_c' \cos(2k_z c)$, cannot be simultaneously transformed away.

In the coexistence phase the SDW order corresponds to a random mixture of N and N' phases and Δ, θ become space dependent. We assume first that the disorder depends only on x, y, i.e. the clusters are correlated in the z-direction. This is reasonable since as discussed above, variations in Q_x induce variations in Q_y but not in $Q_z = \pi/c$. Hence disorder in the x- and y-directions is an inherent feature of the coexistence, while the clusters can remain correlated in the z-direction. The last term in the Hamiltonian is then $f(-i\partial_y b - Gx)$.

We proceed to describe the localization properties of the gapless states in presence of a random distribution of the two coexisting SDW order parameters. We recall first the description of (spinless) electrons in random 2D QH systems. The transition between QH plateaux is a quantum

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[2] A hint about the interpretation of ρ_{xx} spikes at the transition between Hall plateaux in terms of a mixed phase is present in Ref. [3] above, which refers in that context to the following paper: I. Ruzin, S. Feng, Phys. Rev. Lett. 74 (1995) 154. The latter work analyses transport properties in the "usual" quantum Hall effect systems.

percolation transition which involves tunneling and interference between clusters. It leads to a well-known localization length ξ which diverges as the electronic Fermi energy E approaches a percolating value E_c as $\xi \sim |E - E_c|^{-\nu}$ with $\nu \approx 2.4$ (close to 7/3) [4,10]. Quantum percolation signifies here the presence of an extended state, while at energies away from percolation the states are localized. Note, however, that in the QH system the clusters result from a given random potential while in the SDW system the disordered clusters themselves are generated by the magnetic field which drives the first-order transition. In both cases the magnetic field drives the Fermi energy across a percolation point and the dynamics of gapless modes in the SDW system is similar to that of electrons in a random potential. In a layered QH system with weak hopping t between layers, an uncorrelated disorder between layers [17] leads to a finite width of extended states (scaling as $(t)^{1/\nu}$) and the actual localization exponent becomes $\nu \approx 1.45$.

The case with two spin states requires some care in identifying their symmetry. For example, if $f(k_y b) = t_b \cos(k_y b)$, i.e. perfect nesting also in the y-direction, the Hamiltonian would anti-commute with $K = \tau_2 \exp(-i\tau_3 \theta)$ leading to particle–hole symmetry. Such symmetries are essential for identifying universality classes of QH systems. For example, in a random super conductor [18] an electron–hole symmetry is exact and leads to degenerate extended states and distinct critical exponents; the symmetry operation in the latter case is anti-linear, while the SDW type (approximate) symmetry defines yet another symmetry class [19]. Although this SDW symmetry is approximate in the uniform SDW state, it breaks down in the random coexisting phase. The phase θ which signifies an SDW translation is now randomly x, y dependent and the operator $K = \tau_2 \exp[-i\tau_3 \theta(x, y)]$ no longer anti-commutes with the Hamiltonian. The coexistence phase is then identified as a $U(2)$ symmetry class [10,11] in which the extended states are nondegenerate and have the usual exponent $\nu \approx 2.4$.

Consider now the localization problem of gapless states in the presence of a random SDW which mixes the two spin states. We propose that this is equivalent to the QH $U(2)$ system [10,11] with a random scalar potential and a random spin-flip coupling which mixes the two spin states. The latter system exhibits "repulsion" between extended states, i.e. even if the spin states were degenerate (i.e. no Zeeman term) the $U(2)$ mixing produces two non degenerate energies of extended states. A Zeeman term will further increase the splitting. In the SDW problem the magnetic field drives also the "landscape" of the random potential so that the splitting between critical field values corresponds to situation that the SDW fluctuation $\langle (\delta\Delta)^2 \rangle^{1/2}$ changes by ΔE, the energy splitting of the two extended states. Since fluctuations $\delta\Delta$ relate to the unknown kinetics of the first-order transition, we cannot estimate the splitting of the fields H_c.

We can, however, evaluate the critical behaviour near one H_c since the situation there is equivalent to a "spinless"

particle localization. As the field approaches a percolation point H_c the localization length diverges as $\xi \sim |H - H_c|^{-\nu}$ and the resulting extended state will produce a peak in ρ_{xx}. To estimate the width of this peak at finite temperatures we consider the states at half maximum of ρ_{xx} as localized states and evaluate their conductance via variable range hopping, similar to the QH treatment [14,15]. The excitation energy for a hop is either a Coulomb energy or determined by the level spacing of the edge states. In view of the huge dielectric constant of the SDW state ($\sim 10^9$ or $\sim 10^3$ in the x- and y-directions, respectively [21]) we consider an excitation energy which is dominated by the level spacing $\approx 1/N(0)r^2$ at distance r. Hence

$$\rho_{xx} \sim \exp\left[\frac{1}{N(0)r^2 T} - \frac{r}{\xi} \right]. \tag{3}$$

Minimizing with respect to r yields

$$\ln \rho_{xx} \sim [(H - H_c)^{2\nu}/N(0)T]^{1/3}$$

i.e. for a constant $N(0)$ the width of ρ_{xx} is $|H - H_c| \sim T^{1/2\nu} = T^{\kappa}$, so that $\kappa = 1/(2\nu)$. Hence $\kappa = 0.21$ for correlated disorder in the c-direction while $\kappa = 0.34$ for uncorrelated disorder. Furthermore at a given $H \neq H_c$ this predicts the Mott law $\ln \rho_{xx} \sim T^{-1/3}$. (Note that, in contrast, when Coulomb interactions dominate electron–hole levels, $\kappa = 1/\nu$ [14,15].)

This derivation assumes that $N(0)$ is non-critical and smooth near percolation, i.e. $N(0)$ of the edge states is determined by the coexisting clusters rather than by the percolation path that the edge states choose to take. Indeed, $N(0)$ is non-critical in the usual QH systems [4,10]. An alternative derivation of the temperature scaling is based on limiting ξ by an inelastic length $L_\phi \sim T^{p/2}$ [12,20] i.e. the width of ρ_{xx} is $\sim (L_\phi)^{-1/\nu} T^{p/2\nu}$ with $p/2\nu \simeq 0.36$ for the single layer system [20].

We consider finally the issue of interlayer coupling. Within our 2D random system the effective 2D coupling is the deviation from nesting in the z-direction which is rather small 40 mK [22]. Thus the ρ_{xx} width should saturate below 40 mK. If, however, the clusters are not correlated in the c-direction, then $t_c \approx 10$ K would lead to a large width for ρ_{xx} and critical behaviour below ≈ 10 K could not be seen. Recent data on ρ_{zz} [23] has shown no special features at the transition between plateaus in the ClO_4 salt. This is consistent with correlated clusters in the z-direction. In this case the hopping term t_c in Eq. (2) can be eliminated and σ_{zz} depends only on the much smaller t_c'. In the presence of random point impurities [24] t_c cannot be strictly gauged away, though for extremely clean samples either a renormalized t_c or t_c' are the relevant scales.

The explanation we have given for the behaviour of the longitudinal resistivity at the transition between FISDW phases is based on the first-order nature of this transition and on the consideration of chiral edge electronic liquid states which should exist at the boundary of a finite cluster of phase N embedded in a sea of phase N'. Our analysis has

various consequences: the intensity of the longitudinal resistivity spike at its maximum at the transition between plateaux of a monotonic sequence, i.e. when $N - N' = 1$ should not depend on field, only on temperature, since it only depends on the number of percolating, channels at the transition. It should be much larger at the transition between phases with different signs of the Hall effect, since the number of dissipative edge channels is $|N - N'| \gg 1$. In fact the width of the ρ_{xx} peak, following Eq. (3) has a factor $\delta H \propto [N(0)]^{1/2\nu} \propto |N - N'|^{\kappa}$, i.e. increasing with $|N - N'|$.

Indeed, experiments exhibit much larger spikes at the transition between phase $N = -2$ and its neighbours (i.e. with $|N - N'| = 4$ or 5) than between phases for which $N - N' = 1$ [5].

In $(TMTSF)_2PF_6$, a single spike is observed at all transitions, indicating that the spin splitting we expect is too small to be resolved. However, magnetocaloric and partial transport data on $(TMTSF)_2ClO_4$ [25] shows a splitting of the transitions above 4.5 T. Those splittings have been explained phenomenologically on the basis of a Landau–Ginzburg expansion and the hypothesis of a repulsive coupling between neighbouring phases. A different possible interpretation of this splitting is the spin effect we have discussed above. Higher sensitivity data on ρ_{xx} are required to test our interpretation. However, one should keep in mind that the physics of the ClO_4 salt is made complicated by the anion ordering problem: depending on the sample cooling rate, transverse Bragg reflections may significantly alter the single-particle dispersion relation and thus the nesting properties of the electronic spectrum, in comparison with the PF_6 compound, besides introducing significantly different electronic relaxation times, different SDW pinning potentials, etc.

The explanation we have given for the behaviour of the longitudinal resistivity at the transition between Ultra Quantum Crystal phases (i.e. transition between FISDW subphases [26,27]) resolves a long-standing problem in the "standard model" (i.e. Quantum Nesting Model) approach to the Ultra Quantum Crystal phenomenon. It is based on the first-order nature of this transition and on the consideration of chiral edge electronic liquid states which should exist at the boundaries of a finite cluster of phase N embedded in a sea of phase N'. The coexistence regions of the phase diagram, characterized by a random mixture of clusters of one phase embedded in the other are the regions where ρ_{xy} varies rapidly with field from one Quantum Hall plateau to the other; we have shown how this leads naturally to dissipation and to peaks in ρ_{xx}, where disorder is induced only by the phase coexistence at the first-order transition. Using a variable range hopping approach, we have derived critical exponents for the width of the ρ_{xx} peaks and predict spin splitting of these peaks.

An intriguing possibility is that the split transition observed in ClO_4 salt corresponds to the resolved spin splitting limit. More detailed experimental data are needed to check the validity of our critical exponents' predictions.

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