Temperature Scaling of Conductance between Quantum Hall Plateaus

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Temperature Scaling of Conductance between Quantum Hall Plateaus.

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Abstract. - We show that tunnelling in the quantum Hall regime acquires the form of thermal activation above a crossover temperature T_1. The width of the conductance peak σ_{xx} between Hall plateaus scales then with temperature as T^{κ} with $\kappa = 3/7$ at $T < T_1$ and crosses over to $\kappa = 6/7$ at $T > T_1$. We find that T_1 decreases for longer-range potentials or for higher magnetic fields, hence at a given temperature range the apparent κ should decrease for samples with shorter-range scattering or in a given sample at higher Landau levels. These results are consistent with data showing $0.3 \lesssim \kappa \lesssim 0.8$ for different Landau levels and for the remarkable difference between $In_xGa_{1-x}As/InP$ and $GaAs/Al_xGa_{1-x}As$ heterostructures.

Temperature scaling of transport coefficients in the quantum Hall effect (QHE) is of considerable interest. The steps connecting plateaus of Hall conductance σ_{xy} as well as the peaks of the longitudinal conductance σ_{xx} become sharper with decreasing T. In a number of experiments [1-10] it was found that the width in Fermi energy ΔE (or in the filling fraction $\Delta\nu$) of the peaks in σ_{xx} shrinks as a power law T^{κ}. Experiments on $In_xGa_{1-x}As/InP$ and $GaAs/Al_xGa_{1-x}As$ heterostructures show that $\kappa = 0.42 \pm 0.04$ is universal [1-3] while other data [4-10] mainly on $GaAs/Al_xGa_{1-x}As$ heterostructures shows that κ depends on both density and type of doping and is also different for transitions between different Landau levels, yielding $0.3 \lesssim \kappa \lesssim 0.8$. The main distinction between the two types of heterostructures, as emphasized in [2], is the longer range of potential fluctuations in the $GaAs/Al_xGa_{1-x}As$ system; it is however not obvious why this generates a lower temperature range for scaling. Thus, the apparent power law [4-10] with a non-universal κ remains a controversial issue.

The first theoretical approach, based on scaling and universality in the QHE [11], considered [11, 12] an inelastic-scattering length $L_{in} \sim T^{-p/2}$ which acts as a finite size for the QHE. This length is compared with the localization length ξ which diverges when the energy E approaches the energy E_c of a delocalized state in each Landau band. It is well established that $\xi \sim E^{-7/3}$ (choosing $E_c = 0$), based on experimental data [5], theoretical considerations [13] and numerical simulations [14, 15]; thus $L_{in} \sim \xi$ yields $\kappa = p/2\nu$ and to account for the data $p = 2$ was assumed. This approach was extended [16] to include phonon scattering within a percolating cluster, leading to $\kappa = 3/7$.

A different theoretical approach was suggested by Polyakov and Shklovskii (PS) [17] based on formulating variable-range hopping (VRH) with a Coulomb gap. They find that $\sigma_{xx} \sim \exp[-(T_0/T)^{1/2}]$, with $T_0 \sim E^{\nu}$, *i.e.* $\kappa = 1/\nu = 3/7$. Recent data [6] supports the proposed T-dependence, however $\kappa = 0.62 \pm 0.04$ was found.

In the present work we extend the idea of Milnikov and Sokolov (MS) [13] who allow for quantum tunnelling between semi-classical percolating clusters [18]. When tunnelling is considered at a finite temperature, there is a crossover temperature T_1 above which the tunnelling rate acquires a form of thermal activation. We show that in the presence of a strong magnetic field T_1 is considerably reduced and can be in the temperature range where κ is measured. We then apply VRH either with a Coulomb gap or with semi-classical quantization and show that $\kappa = 3/7$ at $T < T_1$ crosses over to $\kappa = 6/7$ at $T > T_1$. Finally we resolve the controversial aspect of κ by showing its dependence on the range of scatterers and on the Landau level.

Consider an electron in two dimensions in a perpendicular magnetic field B. The electron with energy $E \neq 0$ follows semi-classical constant potential contours except for tunnelling events connecting different contours [13]. These tunnelling events occur across saddle-points in the potential which are on the infinite percolating cluster at $E = 0$. We note that this description neglects the long-range Coulomb interaction which for sufficiently slowly varying potentials may lead to metallic strips in the vicinity of the saddle-points [19, 20], as is the case due to screening of a gate voltage in quantum dots [21]. However, the experimental data [2, 4, 6] do show that $\Delta\nu$ shrinks as T^{κ} at least down to $\Delta\nu = 0.04$, hence limiting the existence of metallic strips to $\Delta\nu < 0.04$. We assume then that Coulomb effects merely lead to smoother effective local potentials [16, 19], at least in the samples under study. Furthermore, even if metallic strips are formed, tunnelling of quasi-particles can still be described by an effective potential; this was shown experimentally by tunnelling through a quantum dot [22] which decreases exponentially with the barrier height, as it should for non-interacting electrons. In contrast, the excitation spectrum which involves long-range properties needs Coulomb interactions explicitly, as shown below.

We wish to evaluate the probability of tunnelling through a potential barrier of the form $V(y) + K_x x^2/2$, where $V(y)$ is a non-linear potential with a metastable state at, say, $y < 0$ and a local maximum at $y = 0$, *i.e.* $V(y) = -K_y y^2/2$ near $y = 0$. In this case $E < 0$ and the electron tunnels through the saddle-point at $(0, 0)$. The tunnelling is described by the Euclidean action [23] (here $\beta = 1/T$)

$$S = \int_0^{\beta\hbar} \mathrm{d}\tau \left[\frac{1}{2} m\dot{r}^2 + i \frac{eB}{c} \dot{x}y + V(y) + \frac{1}{2} K_x x^2 \right]. \tag{1}$$

The aim is to evaluate the free energy $F = -T \ln \mathrm{Tr}[\exp[-S/\hbar]]$ and find its Imaginary part. This $\mathrm{Im}\, F$ represents the decay rate of the metastable state [24, 25] at $y < 0$, *i.e.* the tunnelling probability through the barrier. We need to integrate all closed trajectories so that the $x(\tau)$ coordinate can be integrated out, leading to a one-dimensional tunnelling problem,

$$S_{\mathrm{eff}} = \frac{1}{2} \beta\hbar \sum_n \left[\left(\frac{eB}{c} \right)^2 \frac{\nu_n^2}{m\nu_n^2 + K_x} + m\nu_n^2 \right] y_n y_{-n} + \int_0^{\beta\hbar} \mathrm{d}\tau\, V(y(\tau)), \tag{2}$$

where $y(\tau) = \sum_n y_n \exp[i\nu\tau]$ and $\nu_n = 2\pi n/\beta\hbar$. This type of tunnelling problem in one dimension was extensively studied [24, 25]. At high temperatures one can linearize $V(y)$ around its

local maximum leading to $S_{\text{eff}} = (1/2)\beta\hbar \sum_n \lambda_n y_n y_{-n}$, where

$$\lambda_n = \left(\frac{eB}{c}\right)^2 \frac{v_n^2}{mv_n^2 + K_x} + mv_n^2 - K_y \,. \tag{3}$$

At high temperatures $\lambda_n > 0$ for all $n \neq 0$ but $\lambda_0 < 0$ leads to $\operatorname{Im} F \neq 0$; the tunnelling rate has then an activated form [25] $\sim \exp[-|E|/T]$. Below a temperature T_1 also λ_1 becomes negative and the full non-linear potential with a bounce-type trajectory must be considered; the tunnelling rate then approaches its $T = 0$ form. Thus T_1 for which $\lambda_1(T_1) = 0$ signals the crossover between thermal activated form and low-T tunnelling. It is remarkable that T_1 is independent of details of the non-linear potential except for its curvature at the local maximum.

Equation (3) shows that the magnetic field tends to lower T_1. In particular in the limit of strong fields where $T_1 \ll \hbar\omega_c = \hbar eB/mc$

$$T_1 = \frac{1}{2\pi} l^2 \sqrt{K_x K_y}\,, \tag{4}$$

where $l = \sqrt{\hbar c/eB}$ is the magnetic length. We note that Ohmic dissipation [23] can also be included leading to a $\sim l^4$ correction, *i.e.* a small effect at high fields.

We proceed now to formulate the problem of localization in the QHE. Consider a classical percolation problem [18] describing the motion of an electron's guiding centre. The electron propagates without decay on a constant energy surface which is the perimeter of a connected cluster. The cluster size ξ_p diverges as E approaches the energy of the delocalized state with $\xi_p \sim E^{-4/3}$. The electron can propagate to a distance $r \gg \xi_p$ by tunnelling between clusters [13], with each tunnelling event contributing to the decay of the wave function. The number of these events is r/ξ_p since the distribution of clusters on scales $r \gg \xi_p$ is homogeneous. Each tunnelling event reduces the wave function by a factor of $\exp[-a|E|]$ (where $a = 1/2T_1$), at least for $a|E| \gtrsim 1$ (in fact for $E \to 0$ $\exp[-a|E|] \to 1$ describes the correct resonance effect of approaching an extended state). The total decay is then

$$(\exp[-aE])^{r/\xi_p} = \exp[-r/\xi] \tag{5}$$

so that $\xi \sim \xi_p/E \sim E^{-7/3}$, *i.e.* $\nu = 7/3$ (here and in the following replace E by $|E|$ if $E < 0$). This derivation, which is essentially given by MS [13], can be extended to finite temperatures, and in particular to $T > T_1$. The tunnelling rate is then $\sim \exp[-E/T]$ so that the probability of finding an electron at distance r is $\sim [\exp[-E/T]]^{r/\xi_p}$. This is the analogue of the $T = 0$ correlation $\exp[-r/\xi]$.

The final step in our analysis is to use VRH at both $T < T_1$ and $T > T_1$. As claimed by PS, the only possible mechanism of transport in a strongly localized electron system is VRH. In looking for the width of σ_{xx} we are indeed concerned with states away from the delocalized ones and VRH should apply. PS proceed by using the level spacing δE for transport to a distance r as given by Coulomb interaction, *i.e.* $\delta E = e^2/\varepsilon r$, where ε is the dielectric constant. Thus for $T < T_1$

$$\sigma_{xx} \sim \max\left\{\exp\left[-\left(\frac{e^2}{\varepsilon r T} + \frac{r}{\xi}\right)\right]\right\} = \exp[-(T_0/T)^{1/2}], \tag{6}$$

where $T_0 \approx 4e^2/\varepsilon\xi \sim E^{7/3}$. Thus σ_{xx} is suppressed at energies for which $T < T_0(E)$ which yields $\Delta E \sim T^{\kappa}$, with $\kappa = 3/7 = 0.42$. This is the PS result which indeed accounts for some of the data [1-3].

Consider next $T > T_1$. The quantum decay rate $\exp[-r/\xi]$ is replaced now by $\exp[-Er/\xi_p T]$, *i.e.*

$$\sigma_{xx} \sim \max\left\{\exp\left[-\left(\frac{e^2}{\varepsilon r T} + \frac{Er}{\xi_p T}\right)\right]\right\} = \exp[-T_0'/T], \tag{7}$$

where now $T_0' = 2(e^2 E/\varepsilon\xi_p)^{1/2} \sim E^{7/6}$. Hence $\Delta E \sim T^{\kappa}$, with $\kappa = 6/7 = 0.84$ so that the apparent κ is sensitive to the temperature interval when it is near the crossover temperature T_1.

Note that consistency of VRH and of our localization scheme requires tunnelling events between many clusters so that for the energies at the half-width the optimal hopping length r should satisfy $r \gg \xi_p$. We find that for relevant system parameters this is indeed the case, *e.g.* for $T \approx 0.2$ K, $\varepsilon \approx 13$ and $\xi_p \approx 10^3$–10^4 Å we obtain $r/\xi_p \approx 10$–100. Equation (7) is therefore not a standard activation to the $E = 0$ extended state [26] for which $T_0' = E$. In our approach, while single tunnelling events have an $\exp[-E/T]$ form, critical cluster behaviour is still relevant for $T > T_1$ and VRH involves jumps between many clusters to a distance $r \gg \xi_p$.

We consider now the possibility that the level spacing is dominated by semi-classical quantization and not by the Coulomb energy. A classical trajectory at energy E travels along the cluster perimeter of length [27] $t \sim \xi_p^{7/4} \sim E^{-7/3}$. A cluster area s is also proportional to t so that the area between trajectories at $t(E + \Delta E)$ and $t(E)$ is $\sim (\mathrm{d}t/\mathrm{d}E)\Delta E$. Since x, y become conjugate variables in the semi-classical limit, this area is quantized to be $2\pi l^2$, *i.e.* the local level spacing is $(\delta E)_{\mathrm{loc}} \sim \mathrm{d}E/\mathrm{d}t \sim E^{10/3}$. On scales $r \gg \xi_p$ the average local spacing is reduced by the number of clusters $(r/\xi_p)^2$, *i.e.*

$$\delta E \approx (\delta E)_{\mathrm{loc}}\, \xi_p^2/r^2 = bE^{2/3}/r^2\,, \tag{8}$$

where b is a constant. This level spacing is in the spirit of the original Mott derivation of VRH, considering a local $(\delta E)_{\mathrm{loc}}$ which is reduced by a volume factor r^{-d} (in d dimensions) for hops to a distance r. The peculiarity of the QHE enters into the form of $(\delta E)_{\mathrm{loc}}$.

Consider now VRH with eq. (8), *i.e.*

$$\sigma_{xx} \sim \max\left\{\exp\left[-\left(\frac{bE^{2/3}}{r^2 T} + \frac{r}{\xi}\right)\right]\right\} = \exp[-1.5(T_s/T)^{1/3}], \tag{9}$$

with $T_s = 2bE^{2/3}/\xi^2 \sim E^{16/3}$, *i.e.* $\kappa = 3/16 = 0.19$. Allowing for Coulomb interactions, the dominant mechanism is the one with larger level spacing at the optimal r. The ratio of these level spacings is then $\sim T_0^{1/2}/T_s^{1/3} \sim E^{-11/18}$ so that for $E \to 0$ the Coulomb level spacing dominates.

At $T > T_1$, the semi-classical level spacing yields

$$\sigma_{xx} \sim \max\left\{\exp\left[-\frac{1}{T}\left(\frac{bE^{2/3}}{r^2} + \frac{Er}{\xi_p}\right)\right]\right\} = \exp[-T_s'/T], \tag{10}$$

where $T_s' = 1.5(2bE^{8/3}\xi_p^{-2})^{-1/3} \sim E^{16/9}$, *i.e.* $\kappa = 9/16 = 0.56$. Again the Coulomb level spacing dominates since $T_0'/T_s' \sim E^{-11/18}$.

We conclude that the PS result is valid at temperatures $T < T_1$, while as temperature increases the apparent exponent κ crosses over from 3/7 to 6/7 around T_1. Since T_1 depends on the potential curvature $K_x K_y$ and on the magnetic field, we expect that in a given temperature interval the apparent exponent depends on the potential via the type and

concentration of dopants, and for a given sample κ can depend on the Landau level being probed since different levels correspond to different magnetic fields.

To estimate T_1 for $GaAs/Al_xGa_{1-x}As$ we consider an impurity concentration of $n_d \sim 10^{10}$–10^{11} cm^{-2} and a Coulomb potential $e^2/\varepsilon r$, with [3] $\varepsilon \approx 13$ and $r \approx n_d^{-1/2}$. For $l \approx$ $\approx$ 60–100 Å we estimate from eq. (4) $T_1 \approx 0.1$–1 K. In general we expect that increasing n_d or decreasing the magnetic field will increase T_1 and therefore decrease the apparent κ towards its low-temperature value of 3/7. Most of the data on Si- and Be-doped samples [4] is consistent with our result; in particular all the three sheet doped samples of Koch *et al.* [4] show that κ decreases with increasing Landau level; a similar decrease is found in Si-MOSFET samples [8].

To test the dependence of T_1 on the curvature $K_x K_y$ we compare $GaAs/Al_xGa_{1-x}As$ with $In_xGa_{1-x}As/InP$ heterostructures where alloy scattering with a short-range λ of potential fluctuations is dominant [1,2]. For short-range potentials T_1 is higher and the expected $\kappa = 0.42$ is indeed found in the whole measurable interval [1,3,9].

The most direct evidence for a crossover temperature was found by Wei *et al.* [2] on $GaAs/Al_xGa_{1-x}As$ heterostructures. Their data on the $N = 0 \downarrow$ Landau level shows a clear crossover at $T \approx 0.2$ K from $\kappa \approx 0.4$ at lower T to $\kappa \approx 1$ at higher T.

Furthermore, for Landau levels $N = 1 \uparrow$ and $N = 1 \downarrow$ $\kappa \approx 0.4$ fits a larger temperature range implying a higher T_1. All these features are fully consistent with our result eq. (4) for T_1. Thus we account for the most controversial aspect of the data on κ, *i.e.* the apparent universality of κ for alloy scattering [1-3] *vs.* non-universal κ in samples with slowly varying potentials [4-10].

Wei *et al.* [2] argued that an inelastic length l_{in} comparable to λ can account for their data. However, the Hall plateaus which persist even at ~ 1 K [4] indicate a longer l_{in}; in fact, finite-size studies show [5] that $l_{in} > 10^5$ Å at 0.1 K so that $\lambda \approx 10^3$ Å [4] is much shorter even near 0.2 K.

A recent study [5] has fitted data on σ_{xx} to the form of eq. (6); however, $\kappa = 0.62$ was found and the authors mention that a fit with an activated behaviour (eq. (7)) cannot be ruled out. We suggest that the temperature range which was studied is near T_1, so that an intermediate κ between 6/7 and 3/7 is found. Clearly extending the temperature interval can clarify the situation. Furthermore, fitting data from other samples with $\kappa = 0.42$ to the form of eq. (6) should give a much better fit.

In conclusion, we have shown that tunnelling phenomena in QHE are sensitive to temperature and can modify the apparent exponent of the data. We account for the dependence of κ on Landau level index and resolve the controversy of universal κ [1-3] *vs.* a non-universal one [4-10]. Further experiments can probe the full temperature dependence of the width of σ_{xx} and clarify the peculiar role of tunnelling in the QHE.

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