Thermal and quantum creep of vortices trapped by twin boundaries and columnar defects

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We have analyzed the creep process in which a vortex loop is disconnecting from a twin boundary or a columnar defect and is then expanding, driven by a transport current. The shape of the vortex loop is in general an arc and not a semicircle. The contact angle α between the loop and the plane of the twin-boundary or the axis of the columnar defect is small if the trapping potential is small. The energy barrier resisting the vortex creep is strongly decreased as α^3 for small α. Quantum tunneling through this barrier is analyzed for a massless vortex motion governed by the Magnus force. The tunneling rate W is strongly enhanced for small contact angles with $\ln W \sim \alpha^5$.

I. INTRODUCTION

The process of dissipation of a supercurrent in the presence of vortices is possible if the vortex lines are crossing the stream lines of the supercurrent. Usually, however, this process is suppressed by an energy barrier which determines the rate of nucleation of new vortices or the rate of depinning of vortices. At higher temperatures the barrier is overcome by thermal fluctuations, but at lower temperatures a crossover to quantum tunneling through the barrier occurs and the dissipation ceases to depend on temperature. Recently an essential progress in experimental observation of the quantum tunneling of vortices has been achieved. The quantum nucleation of vortices in small orifices has been observed.[1] Also reports on quantum tunneling of vortices in high-T_c superconductors has been published.[2] It has stimulated new efforts of theorists to calculate the probability of quantum tunneling of vortices. The present paper is devoted to thermal and quantum creep of vortices trapped by twin boundaries (plane defects) and columnar (linear) defects in superconductors.

In order to describe the vortex nucleation and creep, either thermal or quantum, it is important to find the shape of the vortex line during the process which optimizes the process, i.e., provides its maximal intensity. This problem is analyzed in Sec. II. The outcome of the analysis is that creep occurs via disconnection of some circular segment of the vortex line which meets the plane or the line of the defect at disconnection points with a contact angle α. The angle α depends on the intensity of the potential which traps the vortex at the defect. For a weak trapping potential the angle α is small and the creep barrier is proportional to α^3. In this case approximation of the disconnected segment of the vortex line by the half-loop with $\alpha = \pi/2$, used in some papers, is unsatisfactory and it is very important to take into account the true value of the contact angle.

Section III is devoted to the creep via semiclassical quantum tunneling. The type of vortex dynamics is crucial for this analysis. Earlier estimates of quantum tunneling for superfluid vortices in the semiclassical approximation assumed that the vortex is massless and the driving force produced by the supercurrent is balanced by the Magnus force.[3–5] In superfluid hydrodynamics the Magnus force may be directly derived using the momentum conservation law (see, e.g., Ref. 6 and references therein). But for superconductors this derivation is not valid because of the interaction with the crystal lattice which breaks translational invariance. If the Magnus force is weak or is absent at all, then the mass of the vortex should be taken into account for the calculation of the quantum tunneling as was done in some recent theoretical publications.[7] Though Ao and Thouless[8] have given some general arguments in favor of the Magnus force, at the present moment there is no consensus in the theory on the magnitude of the Magnus force in superconductors where it is directly connected with the Hall effect (see discussion and references in Ref. 9). A possible way to resolve this problem is to compare the experiments on quantum tunneling with the predictions of the theory based on the vortex mass or the Magnus force. Therefore both approaches are worthy of a serious analysis. Quantum tunneling for the massless vortex has been calculated in Refs. 9–11 for some model potentials. In the present paper quantum tunneling is analyzed for a massless vortex trapped by twin boundaries or columnar defects.

In Sec. IV we discuss the dependence of the creep rate on the transport current in our theory compared to that derived for a vortex with an inertial mass, but without the Magnus force. Also the results of the present study are summarized.

II. ENERGETICS

The trapped segment of the vortex line coincides with the z axis, and the vortex loop, which is disconnected from the defect, is lying in the yz plane (Fig. 1). Within the local elasticity model, when direct interaction between distant parts of the vortex line is neglected (see discussion of this model in Ref. 12) the elastic energy of the vortex line is proportional to its length, i.e.,

$$E_0 = \int_{z_{\min}}^{z_{\max}} \left[\varepsilon\sqrt{1+(y'(z))^2} - \varepsilon_t\right] dz \ , \tag{1}$$

where $y' = dy/dz$ and the coordinates $z_{\max}$ and $z_{\max}$ are the contact points, at which the vortex line disconnects from the defect. Here

$$\varepsilon = \frac{\Phi_0^2}{(4\pi\lambda)^2} \ln\frac{r_u}{r_c} \tag{2}$$

is the line tension of the free segment of the vortex line, $\Phi_0 = hc/2e$ is the magnetic-flux quantum, λ is the London penetration depth, and r_c is the vortex core radius; the upper cutoff r_u in the logarithm argument is the smaller between two lengths, the curvature radius r and the London penetration length λ, and ε_t is the line tension of the trapped segment of the vortex line which is smaller than that for the free segment (ε). The difference $\varepsilon - \varepsilon_t$ is the pinning energy per unit vortex length.

If there is a supercurrent j_s along the x axis, the Lorentz force contributes the energy $(\Phi_0/c)[\hat{\mathbf{n}} \times \mathbf{j_s}] \cdot \mathbf{r}$ per unit length, where $\hat{\mathbf{n}}$ and $\mathbf{r}$ are the unit tangent vector and the position vector of the points on the vortex line, respectively. This energy is proportional to the area of the loop, i.e.,

$$E = E_0 - J_s\frac{\Phi_0}{c}\int_{z_{\min}}^{z_{\max}} y(z)\, dz \ . \tag{3}$$

Here and later on we assume that the supercurrent is uniform. Therefore the London penetration length λ should exceed the transverse size of the loop.

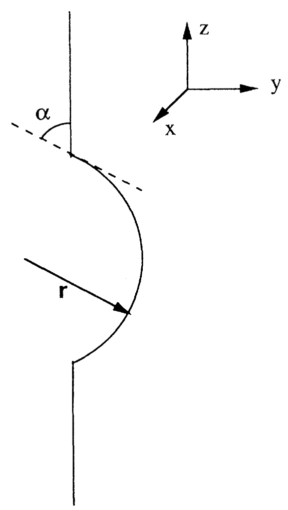

FIG. 1. Untrapping of the vortex line from the defect by formation of a loop. The vortex line is shown by the solid line with its vertical part pinned on the twin boundary (the xz plane) or on the columnar defect (the z axis).

Minimizing E with respect to $y(z)$ yields that the line curvature $(d/dz)[y'/\sqrt{1+(y')^2}]$ is a constant proportional to the constant Lorentz force. This means that the vortex loop is an arc. Thus for a given contact angle α between the trapped and the untrapped (free) segments (see Fig. 1) the elastic energy due to formation of a loop with curvature radius r is

$$E_0 = \varepsilon 2\alpha r - \varepsilon_t 2r\sin\alpha \ . \tag{4}$$

The total energy is given by

$$E = E_0 - j_s\frac{\Phi_0}{c}(2\alpha - \sin 2\alpha)\frac{r^2}{2} \ . \tag{5}$$

The contact energy $\varepsilon\sqrt{dz^2+dy^2} - \varepsilon_t dz$ is minimal with respect to the variation dz of the contact point if the contact angle α satisfies the relation

$$\cos\alpha = \frac{\varepsilon_t}{\varepsilon} \ . \tag{6}$$

This corresponds to the balance of the line-tension forces in the disconnection point,[12] similar to the balance equation for the surface-tension forces which determines the wetting angle. Equation (6) in the case of small α was used also in Refs. 13 and 14. Finally the energy of the vortex loop in the presence of the supercurrent is

$$E = (2\alpha - \sin 2\alpha)\left(\varepsilon r - j_s\frac{\Phi_0 r^2}{2c}\right) \ . \tag{7}$$

The energy is maximal at

$$r_m = \frac{c\varepsilon}{\Phi_0 j_s} = \frac{c\Phi_0}{16\pi^2\lambda^2 j_s}\ln\frac{r_u}{r_c} \ . \tag{8}$$

The maximal energy

$$E_m = \frac{1}{2}(2\alpha - \sin 2\alpha) r_m \varepsilon \tag{9}$$

determines the height of the energy barrier which suppresses the vortex creep. It is proportional to α^3 at small α. For the vortex lines pinned by twin boundaries the angle α is rather small (about 10° according to Ref. 13). The intensity of the thermal activated flux flow (TAFF), i.e., of the thermal creep, is proportional to $\exp(-E_m/kT)$ and thereby is strongly influenced by the contact angle α.

III. DYNAMICS

Until now we considered a stationary problem and the analysis referred both to the twin boundaries and the columnar defects. Dynamic properties are different for these two types of defects. In the case of an ideally flat twin boundary the problem is invariant with respect to translations of the vortex line along the boundary (the direction of the x axis). The vortex loop can have a uniform z-independent velocity $\dot{x}$ in the x direction. We consider this type of motion with x as a collective coordinate. The equation of motion for the massless vortex contains the Magnus force. The Lagrangian for the vortex line of arbitrary shape, but confined in the plane parallel to the yz plane is

$$L = hn_s \int_{z_{\min}}^{z_{\max}} \dot{x} y(z)\, dz - E\{y(z)\} - V\{y(z), x\} \ , \quad (10)$$

where n_s is the condensate density of the Cooper pairs, i.e., a half of the superconducting electron density, the function $y(z)$ determines the shape of the disconnected segment ($z_{\min} < z < z_{\max}$) of the vortex line, $E\{y(z)\}$ is the energy for the case of the ideal boundary which is a functional of $y(z)$ and does not depend on x [Eq. (3)], and $V\{y(z), x\}$ is the energy of the interaction with defects of the twin boundary. The kinetic term in the Lagrangian proportional to the velocity $\dot{x}$ is responsible for the Magnus force, and the part of the energy E which is proportional to the supercurrent [see Eq. (3)] yields the Lorentz force in the equation of vortex motion.

Equation (10) shows that the momentum $\partial L/\partial \dot{x}$ conjugate to x is proportional to the loop area $\int y(z)\, dz$. Our final goal is to derive the probability of the quasiclassical tunneling for the case of arbitrary contact angle α. But earlier we consider simpler cases for which one has well-defined models of the interaction $V\{y(z), x\}$.

A. Nucleation of the straight-vortex near-plane superconductor border

In this case the Lagrangian and the energies per unit length are

$$L = hn_s \dot{x} y - E(y) - V(y, x) \ ,$$
$$E(y) = \frac{\Phi_0^2}{(4\pi\lambda)^2} \ln \frac{y}{r_c} - j_s \frac{\Phi_0}{c} y \ . \quad (11)$$

The potential $V(y, x)$ can be exactly determined for a semicylindrical bulge at the plane. Then the problem of vortex motion is solved using the images of vortices in the plane and in the cylindrical surface. Suppose that x and y are the coordinates of the straight vortex. Then x and $-y$ are the coordinates of the vortex image in the plane $y = 0$. The images of the cylindrical surface of the bulge with radius R are given by the coordinates $x_1 = xR^2/(y^2 + x^2)$, $y_1 = yR^2/(y^2 + x^2)$ and $x_2 = x_1$, $y_2 = -y_1$. Only the interaction between the vortex itself and its images contributes to the total energy. The interaction with the image in the plane yields the term proportional to $\ln(y/r_c)$. If the vortex is located at distances from the bulge large compared to the radius R of the bulge, the effect of the two images in the cylinder may be taken into account in the dipole approximation. Then the energy of the interaction between the vortex and the vortex dipole inside the bulge is

$$V(y, x) = -\frac{\Phi_0^2}{(4\pi\lambda)^2} \frac{2y^2 R^2}{(x^2 + y^2)^2} \ . \quad (12)$$

The probability of quasiclassical tunneling is proportional to $\exp(-2\, \mathrm{Im} S/\hbar)$, where $S = \int L\, dt$ is the action along the trajectory at the energy fixed ($E = 0$ in our case) in the complex plane. The trajectory should connect two states on both sides of the barrier. The following procedure is similar to that in Ref. 3. At $V = 0$ there are two trajectories for $E = 0$ which do not cross one another since both do not depend on x: $y = r_c \approx 0$ and $y = y_f$ where

$$y_f = \frac{c\Phi_0}{16\pi^2 \lambda^2 j_s} \ln \frac{c\Phi_0}{\lambda^2 j_s r_c} \ . \quad (13)$$

These two values of y are solutions of the equation $E(y) = 0$. When the interaction $V(y, x)$ is switched on, the two trajectories cross at imaginary values of $x = ik$. In the limit of a small size $R \ll y_f$ of the bulge the interaction does not affect the second trajectory $y = y_f$, but the first trajectory becomes $y = k$ and crosses the second one at the point $x = ik = iy_f$, $y = y_f$. Then the logarithm of quasiclassical tunneling is given by[15]

$$-\ln W = \frac{2\, \mathrm{Im} S}{\hbar} = \frac{2}{\hbar} \mathrm{Im} \int_0^{x_f} [p_2(x) - p_1(x)]\, dx \ , \quad (14)$$

where $x_f = iy_f$, $p_1(x)$ and $p_2(x)$ are the values of the vortex momentum $p(x) = dn_s h y(x)$ along the first and the second trajectories, respectively, and d is the thickness of the superfluid layer along the z axis. Equation (14) can also be derived by realizing that the imaginary part of the action S comes from the Magnus term $\propto \dot{x}$ in Eq. (11), the other terms being constant along the trajectory. Finally one obtains

$$-\ln W = 2\pi n_s d y_f^2 \ . \quad (15)$$

Thus the probability logarithm is on the order of the number of the superfluid electrons in the volume dy_f^2. This volume has the following meaning: If a vortex is created at the distance y_f from the border, most important changes of the velocity occur in the area y_f^2 where the velocity induced by the nucleated vortex is proportional to the inverse distance from the vortex. Thus dy_f^2 is the volume of the region of such a slow decrease of the velocity induced by the vortex.

The case of a straight vortex near the plane border has also been analyzed by Ao and Thouless,[10] but for another potential $V = \frac{\Phi_0^2}{32\pi^2\lambda^2} k_x x^2$. Taking into account both potentials, ours and that of Ao and Thouless, one can see that the two quasiclassical trajectories remain and weakly affect one another. If $k_x \ll 1/y_f^2$, our trajectory yields a larger probability and thereby is governing the quasiclassical tunneling.

B. Tunneling from twin boundary

For a circular loop the Lagrangian is

$$L = \dot{x} p(r) - E(r) - V(r, x) \ , \quad (16)$$

where $E(r)$ is given by Eq. (7) and

$$p(r) = \frac{1}{2}(2\alpha - \sin 2\alpha) hn_s r^2 \quad (17)$$

is the vortex loop momentum conjugate to the coordinate x, proportional to the loop area as shown above. Since $V(r,x)$ is weak at $r > R$, our previous considerations on x being z independent and on the circular shape of the vortex loop are valid. First let us consider the case of strong pinning. This corresponds to the limit of $\varepsilon_t \to 0$, when supercurrents cannot cross the twin boundary, so that the latter is equivalent to a free surface and the contact angle is $\alpha = \pi/2$; i.e., the vortex line meets the twin boundary normal to its plane. Then the problem is identical to that analyzed by Volovik:[3] the vortex nucleation near the plane wall. Volovik considered a semicircular vortex of radius r which nucleates around a semispherical bulge of radius R. The potential V for the bulge may be found exactly in terms of spherical functions, but for the limit of a small bulge the exact expression is not necessary: It is enough to know that the potential V is singular in the limit $r^2 + x^2 \to 0$. Then the path in the (ix, r) plane is the same as in the (ix, y) plane for the problem of the straight vortex. Instead of the final value $y = y_f$ after tunneling of the straight vortex, one has the final value $r = r_f$ of the loop radius after tunneling [a root of the equation $E = 0$ where E is from Eq. (7)]:

$$r_f = \frac{c\Phi_0}{8\pi^2\lambda^2 j_s} \ln \frac{c\Phi_0}{8\pi^2\lambda^2 j_s r_c} \ . \tag{18}$$

Taking into account the relation Eq. (17) between the momentum p and the loop radius r, Eq. (14) yields for the probability of the tunneling

$$-\ln W = \frac{4\pi^2}{3} n_s r_f^3 \ . \tag{19}$$

The probability logarithm does not depend on the amplitude of the potential, but the latter can influence the preexponential factor which is not considered here.

Now we consider the general case of arbitrary α. We assume that there is a local defect in the symmetry plane of the vortex loop like in the case of the semicircle. We expect that the interaction with this defect is also singular when the vortex loop approaches the defect, and the distance which governs this singularity is $\sqrt{\rho^2 + x^2}$ where $\rho = r(1 - \cos\alpha)$ is the shortest distance between the vortex line and the defect. Then the second trajectory is determined by the condition $\rho^2 + x^2 = 0$, i.e., $r = ix/(1 - \cos\alpha)$, and the two trajectories are crossing at the point where $r = r_f$ and $x = x_f = ir(1 - \cos\alpha)$. Finally one obtains from Eq. (14)

$$\begin{aligned} -\ln W &= 2\pi n_s (2\alpha - \sin 2\alpha) \text{Im} \int_0^{x_f} \left[r_f^2 - r(x)^2 \right] dx \\ &= (1 - \cos\alpha)(2\alpha - \sin 2\alpha) \frac{4\pi}{3} n_s r_f^3 \ . \end{aligned} \tag{20}$$

In the limit of small α, $\ln W$ is proportional to α^5; i.e., tunneling is strongly enhanced for small α.

C. Tunneling from columnar defect

In the case of the columnar defect the translational invariance along the axis x is broken: The vortex cannot move freely in the x direction, but rotates around the columnar defect (the z axis). In the presence of a transport supercurrent the circular shape of the vortex loop is not maintained since there is a component of the Lorentz force out of the loop plane. Nevertheless, we adopt a model which assumes that the vortex loop during rotation remains an arc with the same contact angle α, but with another curvature radius r. This means that the numerical factors in our final expressions are not exact. In our model the energy in the presence of the supercurrent is given by [cf. Eq. (7)]

$$\begin{aligned} E &= (2\alpha - \sin 2\alpha) \left(\varepsilon r - \frac{\Phi_0}{2c} j_s r^2 \cos\theta \right) \\ &= (2\alpha - \sin 2\alpha) \left[\frac{\Phi_0^2}{(4\pi\lambda)^2} r \ln \frac{r}{r_c} - \frac{\Phi_0}{2c} j_s r^2 \cos\theta \right] \ , \end{aligned} \tag{21}$$

where θ is the rotation angle; i.e., the loop plane (including the position vector $\mathbf{r}$; see Fig. 1) forms an angle θ with the yz plane.

The kinetic term in the Lagrangian can be written in the form

$$\frac{1}{2} h n_s \int (\dot{x} y - \dot{y} x) dz = J(r)\dot{\theta} \ . \tag{22}$$

Substitution of x,y by r, θ, via

$$\begin{aligned} x(z,\theta) &= [\sqrt{r^2 - z^2} - r\cos\alpha] \sin\theta \ , \\ y(z,\theta) &= [\sqrt{r^2 - z^2} - r\cos\alpha] \cos\theta \ , \end{aligned} \tag{23}$$

identifies the angular momentum J of the vortex loop,

$$J = f(\alpha) h n_s r^3 \ , \tag{24}$$

where

$$f(\alpha) = \sin\alpha - \frac{1}{3}\sin^3\alpha - \alpha\cos\alpha \ . \tag{25}$$

The Lagrangian is then

$$L = \dot{\theta} J(r) - E(r,\theta) \ , \tag{26}$$

where the energy $E(r,\theta)$ is given by Eq. (21).

We need the trajectory at $E(r,\theta) = 0$, which connects the points ($r = r_c \approx 0$, $\theta = 0$) and ($r = r_f$, $\theta = 0$) where r_f is given by Eq. (18). This trajectory consists of two parts:

(i) Along the first one the angle θ increases from 0 to the imaginary infinite values keeping $r = r_c \approx 0$. The contribution of this part of the trajectory to the action is negligible.

(ii) Along the second part of the trajectory the angle θ returns from $i\infty$ to 0, whereas $r = r_f/\cos\theta$. The quasiclassical probability of tunneling is determined mostly by the integral over this part of the trajectory:

$$\begin{aligned} -\ln W &= \frac{2\,\text{Im}S}{\hbar} = \frac{1}{\pi} f(\alpha) n_s r_f^3 \text{Im} \int_{i\infty}^0 \frac{d\theta}{\cos^3\theta} \\ &= \frac{1}{\pi} f(\alpha) n_s r_f^3 \int_0^\infty \frac{d\phi}{\cosh^3\phi} = \frac{\sqrt{\pi}}{3} f(\alpha) n_s r_f^3 \ . \end{aligned} \tag{27}$$

It is easy to estimate that this yields the probability of the same order as in Eq. (20), but now we cannot control the numerical factor as explained in the beginning of this subsection. In any case Eq. (27) is a lower bound for W.

Let us check that the probability logarithm given by Eqs. (20) and (27) is of order of the number of the Cooper pairs in the region where the velocity field is induced after vortex tunneling as in the case of the straight vortex, Eq. (15). The maximal distance of the vortex line from the defect after tunneling is $r_f(1-\cos\alpha) \approx \alpha^2 r_f$ and the length of the unpinned segment of the vortex line is on the order of αr_f. This vortex line induces the velocity in the region of the cross-section area $\sim (\alpha^2 r_f)^2$ and of the length αr_f. It yields the volume $\sim \alpha^5 r_f^3$ which is present in Eqs. (20) and (27). Thus the probability logarithm for quantum tunneling of the massless vortex is on the order of the number of the Cooper pairs in the region where the velocity field is induced after vortex tunneling. This holds independently of the model of vortex interaction or the type of defect and is also confirmed by the results of Refs. 9 and 11. Earlier this statement was derived from the theory which did not refer to the quasiclassical theory for one collective degree of freedom of the vortex, but instead directly calculated the probability in terms of the many-body wave function for the weakly nonideal Bose gas.[4]

IV. DISCUSSION AND CONCLUSION

Both for the twin boundary and the columnar defect the probability logarithm for quasiclassical tunneling is proportional to r_f^3. Since $r_f \propto 1/j_s$, the probability logarithm is inversely proportional to j_s^3. At the same time for the quantum creep of the vortex with the inertial mass, but without the Magnus force, the probability logarithm should be inversely proportional to j_s^2. Indeed, the vortex loop may be considered as a particle with the mass $M \propto r_f$ tunneling through the barrier of the height $V \propto r_f$ and of width $\sim r_f$. Then the logarithm of the probability for quasiclassical tunneling $\propto \sqrt{MV} r_f \propto r_f^2 \propto 1/j_s^2$. According to Ref. 7, in the case of single-vortex collective pinning of the vortex with the inertial mass and without the Magnus force the probability logarithm (the effective Euclidean action) is inversely proportional to $j_s^{1/2}$. Thus observation of quantum creep as a function of the supercurrent may provide information on what reactive force, the force of inertia or the Magnus force, is more important for the vortex dynamics. Since in our theory we ignored dissipation and interactions of a tunneling vortex with other vortices and the point pinning centers, the results of the theory are relevant for low magnetic fields and very pure materials.

In summary, we have determined the shape of the vortex loop which is formed in the process of thermal and quantum creep of vortices trapped by twin boundaries and columnar defects. We have then calculated the formation energy and the probability of quantum creep for a single massless vortex. We have found that the contact angle at which the loop meets the defect influences the intensity of the creep; namely, it is strongly enhanced for small contact angles. We have also found that the probability for quantum creep is inversely proportional to j_s^3. At the small supercurrent j_s this yields a creep rate smaller than that for the case when the vortex motion is governed by an inertial mass. The j_s dependence can therefore determine to what extent the vortex has a mass.

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