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Thermodynamics of Two-Dimensional Josephson Junctions.

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Abstract. – We derive the effective free energy of a two-dimensional Josephson junction in the presence of an external current and predict that the junction has a phase transition at a temperature T_J below the bulk transition temperature T_c. In the range $T_J < T < T_c$ long-range phase correlation exists in the bulk, but is destroyed in the vicinity of the junction. The critical current I_c is reduced by thermal fluctuations; for a junction of size L, $I_c \sim L^{b(T)}$ where $b(T) < 0$ for $T_J < T < T_c$ (*i.e.* I_c vanishes at $L \to \infty$) while $0 < b(T) < 2$ for $T < T_J$. Our results may account for the absence of an observable supercurrent at temperatures below T_c in $YBa_2Cu_3O_x$- and $Bi_2Sr_2CaCu_2O_8$-based junctions.

Recent developments in fabrication of Josephson junctions has led to junctions with a large area, *i.e.* the junction size L (in either direction in the junction plane) is much larger than λ, the magnetic penetration depth in the bulk superconductors. Studies of trilayer junctions composed of [1] $YBa_2CuO_x/PrBa_2Cu_3O_x/YBa_2Cu_3O_x$ (YBCO junction) and of [2] $Bi_2Sr_2CaCu_2O_8/Bi_2Sr_2Ca_7Cu_8O_{20}/Bi_2Sr_2CaCu_2O_8$ (BSCCO junction) have shown anomalies of the critical current I_c. In particular in the YBCO junction [1] a zero-resistance state was achieved only below ~ 50 K, although the $YBa_2Cu_3O_x$ layers were believed to be superconducting already at the bulk critical temperature $T_c \approx 85$ K. Similarly, in the BSCCO junction [2], a supercurrent through the junction could not be observed above 30 K, although the $Bi_2Sr_2CaCu_2O_8$ layers remained superconducting up to $T_c \approx 80$ K. These remarkable observations are significant both as basic phenomena and for junction applications.

A free energy for a 2D junction was proposed by Josephson [3], and has since been studied and applied to various systems [4-7]. This free energy has the form [3]

$$F_0 = \int \mathrm{d}x\,\mathrm{d}y \left[\frac{\tau}{16\pi}(\nabla\varphi_J)^2 + \frac{E_J}{\lambda^2}(1 - \cos\varphi_J)\right], \tag{1}$$

where $\varphi_J(x, y)$ is the Josephson phase, E_J is the Josephson coupling energy in area λ^2 and (x, y) is the junction plane. If eq. (1) can be used to describe thermal fluctuations of $\varphi_J(x, y)$

it follows that the system undergoes a Berezinskii-Kosterlitz-Thouless–type phase transition [8] at a critical temperature $T_J \approx \tau$. As shown below, this implies the existence of a temperature interval $T_J < T < T_c$ at which the phase $\varphi_J(x, y)$ is disordered, although superconductivity exists in the bulk. Is the coexistence of a disordered phase on the surface and an ordered phase in the bulk a valid situation?

To study this fundamental question we need to extend Josephson's mean-field theory and derive from *ab initio* the free energy for a 2D junction, allowing also for the presence of external currents. We then proceed with the renormalization group (RG) analysis of our system and finally discuss the experimental data.

Consider first the problem of a superconductor in a Meissner state (*i.e.* no vortices in the bulk) which is bounded by a volume V. We wish to integrate out Gaussian thermal fluctuations in the bulk and obtain an effective free energy in terms of fields on the surface. Define $\vec{A}' = \vec{A} - (\phi_0/2\pi)\vec{\nabla}\varphi$, where $\vec{A}$ is the vector potential, φ is the superconductor's phase and ϕ_0 is the flux quantum (the arrow indicates a 3D vector, while an arrowless vector denotes a two-component vector parallel to the surface). The free energy of the superconductor

$$F = \frac{1}{8\pi}\int_V \mathrm{d}^3 r\left[\frac{1}{\lambda^2}\vec{A}'^2 + (\vec{\nabla}\times\vec{A}')^2\right] \tag{2}$$

yields the partition sum as a functional integral on the 3-component vector $\vec{A}'$ and on its boundary values $\vec{A}'(\boldsymbol{r}_s) = \boldsymbol{A}_1(\boldsymbol{r}_s)$ with the weight $\exp[-F/T]$, where $\boldsymbol{r} = \boldsymbol{r}_s$ defines the surface of the superconductor. We shift $\vec{A}' \to \vec{A}' + \delta\vec{A}'$ where now $\vec{A}'$ is *the* solution of $\delta F/\delta\vec{A}' = 0$, *i.e.* the London equation $\lambda^2\vec{\nabla}\times\vec{\nabla}\times\vec{A}' = -\vec{A}'$ subject to the boundary conditions, while $\delta\vec{A}'(\boldsymbol{r}_s) = 0$. Note that the London equation implies $\vec{\nabla}\cdot\vec{A}' = 0$ and $\lambda^2\vec{\nabla}^2\vec{A}' = \vec{A}'$. Since $F(\vec{A}' + \delta\vec{A}') = F(\vec{A}') + F(\delta\vec{A}')$ the integration on the bulk fluctuations $\delta\vec{A}'$ yields a constant independent of the boundary values $\boldsymbol{A}_1$.

Consider now two superconductors with surface fields $\boldsymbol{A}_i(x, y)$, magnetic fields $\boldsymbol{H}_i(x, y) = \vec{\nabla}\times\vec{A}_i'$ and phases $\varphi_i(x, y)$, $i = 1, 2$, respectively, which form a 2D junction (fig. 1). Boundary conditions at the barrier are obtained [4] by integrating both $\vec{A}$ and $\vec{\nabla}\times\vec{A}$ along the loop in fig. 1, assuming that within the barrier $\boldsymbol{H} = \boldsymbol{H}(x, y)$. This yields

$$\begin{cases} \boldsymbol{H}_1(x, y) = \boldsymbol{H}_2(x, y), \\ \boldsymbol{A}_1(x, y) - \boldsymbol{A}_2(x, y) = \mathrm{d}\boldsymbol{H}(x, y)\times\hat{z} - (\phi_0/2\pi)\nabla\varphi_J(x, y), \end{cases} \tag{3}$$

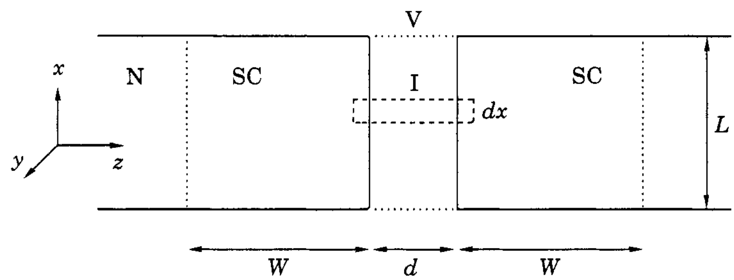

Fig. 1. – Junction geometry. Dotted lines separate superconductors (SC) from normal metals (N) or insulator (I) from vacuum (V). The dashed box is used for deriving the boundary conditions equation (4).

where d is the thickness of the barrier, $\hat{z}$ a unit vector in the z-direction and $\varphi_J(x, y)$ is the Josephson phase

$$\varphi_J(x, y) = \varphi_1 - \varphi_2 - (2\pi/\phi_0) \int_{-d/2}^{d/2} A_z \, dz \,. \tag{4}$$

We proceed now to study two limiting cases depending on the width W of each superconductor (fig. 1). In case I, $W \gg \lambda$, the external current becomes confined to a layer of thickness λ near the superconducting-vacuum surfaces for $|z| \gg \lambda$. Thus we can consider the external current $j^{ex}(x, y)$ to be in the z-direction and to obey London's equation $\lambda^2 \nabla^2 j^{ex} = j^{ex}$. Since the total current $\sim \vec{A}'(r)$ becomes $\hat{z} j^{ex}$ for $|z| \gg \lambda$, the solution of $\lambda^2 \nabla^2 \boldsymbol{A}' = \boldsymbol{A}'$ has the form ($z = 0$ at the barrier)

$$\begin{cases} \boldsymbol{A}'(r) = \boldsymbol{A}_1(x, y) \exp[-z/\lambda] \,, \\ A_z'(r) = \lambda \exp[-z/\lambda] \nabla \cdot \boldsymbol{A}_1(x, y) - (4\pi\lambda^2/c) j^{ex}(x, y) \,, \end{cases} \tag{5}$$

where $\boldsymbol{A}_1(x, y)$ is assumed to be slowly varying on the scale of λ.

The boundary condition equation (3) determines $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ in terms of φ_J and j^{ex} leaving just a scalar phase φ_J as an independent degree of freedom. The junction free energy F_J consists of eq. (2) for both superconductors in addition to the tunnelling term coupling states at the junction surface and leading to the Josephson term $\sim \cos \varphi_J$. Substitution of eq. (5) in eq. (2) yields the free-energy equation (1) with $\tau = \tau = \phi_0^2/(4\pi^2 \lambda)$ for $d \ll \lambda$.

In the presence of a given external current one needs the Gibbs free energy $G(\vec{\boldsymbol{H}})$ in terms of $\vec{\boldsymbol{H}}$, where $\vec{\nabla} \times \vec{\boldsymbol{H}} = (4/\pi c) \, \vec{j}^{ex}$ is defined outside the sample. The standard derivation of $G(\vec{\boldsymbol{H}})$ [9] assumes that j^{ex} does not flow through the sample, otherwise j^{ex} or $\vec{\boldsymbol{H}}$ are not uniquely defined inside the sample. Our generalized $G(\vec{\boldsymbol{H}})$ involves a *surface* term

$$G(\vec{\boldsymbol{H}}) = F - (1/4\pi) \int \vec{\boldsymbol{A}} \times \vec{\boldsymbol{H}} \cdot d\sigma \,, \tag{6}$$

where $d\sigma$ is the surface of the superconductors; minimization with respect to $\vec{\boldsymbol{A}}$ yields the correct flow of energy, *i.e.* the Poynting vector. Writing $\vec{\boldsymbol{A}}$ in terms of φ_J as above finally yields

$$F_J = F_0 - \frac{\phi_0}{2\pi c} \int dx \, dy \, j^{ex}(x, y) \, \varphi_J(x, y) \,. \tag{7}$$

Note that the Gibbs free energy yields a term which is not invariant under $\varphi_J \rightarrow \varphi_J + 2\pi$; this symmetry breaking corresponds to the flow of external current. For a uniform j^{ex} the Gibbs term reduces to the previously known form [5, 10, 11].

We have bothered to present a detailed derivation of eq. (7) in order to show that i) this fluctuation energy is decoupled from the phase fluctuations in the bulk of the superconductors, no matter how large they are, and ii) that it is a free energy valid for all configurations of $\varphi_J(x, y)$ and not just those which solve the mean-field equation $\delta F_J / d\varphi_J = 0$, *i.e.*

$$(E_J/\lambda^2) \sin \varphi_J = (\tau/8\pi) \nabla^2 \varphi_J + (\phi_0/2\pi c) j^{ex} \,. \tag{8}$$

This equation may be interpreted as equating the Maxwell current with the tunnelling current $\sim \sin \varphi_J$. Equation (8) however is *not* a boundary condition in the previous sense; configurations $\varphi_J(x, y)$ which do not satisfy eq. (8) are allowed in the partition sum, while eq. (8) should be satisfied only after thermal averaging, *i.e.* $\langle \delta F_J / \delta \varphi_J(x, y) \rangle = 0$. An equivalent way of studying thermal averages is to add to eq. (8) time-dependent terms like dissipative and random force terms. The time average, which includes configurations not satisfying eq. (8), is by the ergodic hypothesis equivalent to the partition sum, *i.e.* a functional integral over φ_J with the weight $\exp[-F_J/T]$.

Case II involves thin superconductors, *i.e.* $\xi \ll W \ll \lambda$, where ξ is the coherence length; $\xi \ll W$ is needed to avoid vortex fluctuations in the superconductors so that the bulk T_c is valid [12]. The equilibrium state for the case II was studied in ref. [13].

The solution for $0 < z \ll \lambda$ is

$$A'(\boldsymbol{r}) = \boldsymbol{a}_1(x, y) \exp[-z/\lambda] + \boldsymbol{b}_1(x, y) \exp[z/\lambda], \tag{9}$$

where $\boldsymbol{a}_1$ and $\boldsymbol{b}_1$ are slowly varying. The external current and geometry determine an external field $\boldsymbol{H}^{ex}$ at $z = \pm W$ such that $\nabla \times \boldsymbol{H}^{ex} = (4\pi/c) j^{ex}$. Matching $\nabla \times \boldsymbol{A}'$ at $z = \pm W$ and using the boundary condition at the junction equation (3) one obtains for the free energy the form of eqs. (1), (7), except that now τ is reduced by the factor W/λ from case I, *i.e.* $\tau = \phi_0^2 W/(4\pi^2 \lambda^2)$. The factor W/λ represents the reduced volume of magnetic fluctuations which extends to $|z| = W$ rather than to $|z| \approx \lambda$ as in case I. Note that j^{ex} is now determined by the current source, *e.g.* j^{ex} may be uniform.

We proceed to evaluate the effects of thermal fluctuations in eq. (7) which is the celebrated 2D sine-Gordon system [8]. The usual RG procedure with $j^{ex} = 0$ can be applied provided that j^{ex} is localized in space, *i.e.* $j^{ex} = 0$ on intermediate scales between λ and L. This is the natural situation for case I, where j^{ex} is localized near the superconductor-vacuum surfaces; in case II j^{ex} should be applied only to part of the junction cross-section, which seems to be the case in the relevant experiments [1, 2]. If j^{ex} is not localized the following RG still holds if L is not too large or if $|T - T_J|$ is not too small. We proceed with length scale renormalization of the $j^{ex} = 0$ case and stop at either the relevant correlation length or at the sample size L. Finite j^{ex} is then treated by mean-field theory for the renormalized free energy.

RG integrates fluctuations of $\varphi_J(x, y)$ with wavelengths between a and $a + \mathrm{d}a$, the initial scale being λ. The parameters $\bar{x} = T/\tau$ and $\bar{y} = E_J/T$ are renormalized, to second order in $\bar{y}$, by the differential equations [8],

$$\mathrm{d}\bar{y}/\bar{y} = 2(1 - \bar{x})\, \mathrm{d}a/a, \tag{10a}$$

$$\mathrm{d}\bar{x} = -2\gamma^2 \bar{y}^2 \bar{x}^3\, \mathrm{d}a/a, \tag{10b}$$

where γ is of order 1, depending on the cut-off smoothing procedure. Equation (10) defines a phase transition at $1/\bar{x} = 1 - \gamma\bar{y}$. Note, however, that τ itself is T-dependent, since $\lambda(T) = \lambda'(1 - T/T_c)^{-1/2}$ near T_c. Thus the solution of $\tau(T)/T = 1 - \gamma\bar{y}$ defines a transition temperature T_J which is below T_c. For $T_J < T < T_c$, $\bar{y}$ is irrelevant and E_J scales to zero, *i.e.* the superconductors are decoupled by the thermal fluctuations in the junction. For $T < T_J$, $\bar{y}$ is relevant and E_J scales to a finite value, *i.e.* the superconductors are correlated.

The range $T_J < T < T_c$ is remarkable: the superconducting phases are disordered near the junctions (with correlation functions which decay as a power law) while these phases remain ordered in the bulk. This situation is due to fluctuations in the Josephson current; in fact for a single isolated superconductor the restriction for zero outcoming currents amounts to an additional constraint which eliminates all surface fluctuations.

Note also that a similar junction for an X-Y model of a magnet does not yield a phase transition, *i.e.* long-range order in the bulk enforces long-range order on the surface. Superconductors, in contrast, have additional gauge fields which result in a finite screening length λ; fluctuations at the junction can then decouple from those in the bulk and enable the loss of coherence at the surface in the temperature interval $T_J < T < T_c$. (We note that application of these methods to long (1D) quantum junction leads to renormalization of Bloch oscillations [14].)

We proceed to estimate T_J and find the effect on the critical current. The solution for $T = \tau(T)$ yields T_J which is close to T_c, *i.e.* for typical high-T_c systems [1, 2] $T_c - T_J = 10^{-6}$ K for case I while $T_c - T_J \approx 10^{-1}$ K for case II. We expect, however, that effects of disorder will reduce T_J significantly, as in the dual system of disordered 2D *X-Y* magnets [15, 16]. This reduction in T_J is in fact needed to avoid the critical region near T_c and justify our assumption on the absence of thermally excited vortices in the bulk.

The phase transition is strictly defined only for infinitely large junctions. If the junction size L is finite, integration of eq. (11) must terminate at $a < L$. Excluding the narrow interval $|\tau/T - 1| < \gamma E_J/T \ll 1$, renormalization of $\bar{x}$ can be neglected and integration of eq. (10a) yields a renormalized E_J, $E_J^R = E_J(a/\lambda)^{2(1-T/\tau)}$.

When $T_J < T < T_c$ (*i.e.* $\tau < T$) E_J is renormalized towards zero. In a finite sample the renormalized system scales into a point contact junction (*i.e.* a uniform φ_J) with the effective free energy

$$F_J^{\text{eff}} = E_J\left(\frac{L}{\lambda}\right)^{2(1-T/\tau)} \cos\varphi_J - \frac{\phi_0}{2\pi c} I_{\text{ex}} \varphi_J, \tag{11}$$

where $I_{\text{ex}} = \int \mathrm{d}x\,\mathrm{d}y\, j_{\text{ex}}$. The critical current is therefore decreasing with L,

$$I_c \approx I_{c1}^0 (L/\lambda)^{-2T/\tau}, \tag{12}$$

where $I_{c1}^0 = (2\pi c/\phi_0) E_J (L/\lambda)^2$ is the mean-field critical current for a point junction.

The regime $T < T_J$ has also interesting renormalization effects. If L is sufficiently large, the scaled a of eq. (10) reaches a correlation length λ_J^R at which E_J corresponds to strong coupling, *i.e.* $E_J^R = \tau/4\pi$. Thus, $\lambda_J^R = \lambda_J(\lambda_J/\lambda)^{T/(\tau-T)}$ which is longer than the conventional Josephson length $\lambda_J = \lambda(\tau/4\pi E_J)^{1/2}$. The effective area for the current is $L\lambda_J^R$ so that

$$I_c \approx I_{c2}^0 (4\pi E_J/\tau)^{T/2(\tau-T)}, \tag{13}$$

where $I_{c2}^0 \approx c\tau L/2\phi_0\lambda_J$ is the mean-field critical current for a large junction. Note that even if the exponent in (13) becomes very small at $T < T_J$, a sufficiently small E_J can lead to an observable reduction of I_c. Finally if $L < \lambda_J^R$ renormalization must stop at $a = L$, leading to eq. (12); now, however, $T/\tau < 1$ and I_c approaches I_{c1}^0 when $T \ll \tau$.

The experiments on YBCO [1] and BSCCO [2] junctions with sizes L = 5–50 μm show that in a range of 20–50 K below T_c there is no measurable supercurrent through the junction. A point junction would be thermally disordered if $\phi_0 I_c/2c < kT$, *i.e.* $I_c < 1$ μA at ~ 80 K; however, the observed I_c at low temperatures are $\approx$ 150–400 μA and at ≈ 0.4–$0.8T_c$ should still be much higher than 1 μA.

We propose therefore that since $L \gg \lambda$ space-dependent fluctuations are relevant to the experiments [1, 2]. These junctions correspond to case II with $W \approx 0.1$ μm and $\lambda' \approx 0.2$ μm so that $T_c - T_J \approx 0.05$ K; we propose that effects of disorder [15, 16] are responsible for a larger $T_c - T_J$, consistent with the large sample dependence in the YBCO system [1].

A junction of type I may be realized as a YBCO edge junction [17] with $W = 100$ μm. In fact, Polturak *et al.* [17] have found that the critical current I_c for their weakest junction is decreasing faster as T approaches T_c than $I_c(T)$ of other junctions. This can be due to thermal fluctuations which are efficient even below T_J and are enhanced when E_J is small (eq. (13)).

In conclusion, we have shown that the study of large-area junctions is significant for two reasons. First, it provides a test for a new type of phase transition in which phases are disordered on a junction while retaining long-range order in the bulk. Second, thermal fluctuations in the junction should be considered in device design and applications. Furthermore, we suggest that our theory is related to a number of observed anomalies in the critical current of large-area junctions.

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