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Quantum fluctuations in finite size Josephson junctions

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An effective Josephson coupling energy for a one-dimensional Josephson junction is renormalized due to quantum fluctuations of the phase difference. In a long junction at $T\to 0$ a Kosterlitz–Thouless phase transition takes place. The state with a logarithmically divergent phase–phase correlation function shows a nontrivial combination of phase disorder on a junction surface with phase order in the bulk. For finite size junctions the renormalized value of the Josephson coupling energy turns out to be strongly suppressed for small Josephson-to-charging energy ratio. The implications of this effect for Bloch oscillations are discussed.

The prediction [1] of the effect of Bloch oscillations in ultrasmall superconducting tunnel junctions induced substantial theoretical and experimental activity in the field (see e.g. refs. [2,3] for a review). Recently reliable experimental evidence of this effect was reported [4–6]. Most of these experimental results turn out to be in good quantitative agreement with the theory [7,8].

The usual starting point for theoretical investigations of quantum dynamics of a Josephson junction is the Hamiltonian [1–3]

$$H=\hat{Q}^2/2C-E_J\cos\hat{\varphi}\,, \tag{1}$$

where $\hat{Q}$ and $\hat{\varphi}$ are respectively the junction charge and the phase difference operators, C is the junction capacitance and E_J is the Josephson coupling energy. This Hamiltonian is written under the assumption that the junction cross section area S is very small, so that the phase difference φ is independent of space coordinates in the junction plane. This is indeed a natural assumption: the effect of Bloch oscillations can be observed only provided the condition $T\ll E_c=e^2/2C$ is fulfilled. Therefore the junction capacitance $C\propto S$ should be small enough to allow for an experimentally accessible temperature interval. Typical experimental parameters for "quantum" Josephson junctions are $S\sim 10^{-9}$–10^{-10} cm and $C\sim 10^{-14}$–10^{-16} F. Then if we assume the junction size in the x-direction L_x to be of the order of that in the y-direction L_y (x and y are coordinates in the junction plane) we can estimate $L_x\sim L_y\sim 10^{-5}$ cm. These values are of the order of (or even smaller than) the London magnetic penetration depth λ for bulk superconductors. On the other hand the typical variation scale for the phase difference in the junction plane is larger than λ [9]. Thus for two-dimensional (2D) small capacitance Josephson junctions with $L_x\sim L_y$ space fluctuations of the phase difference along the junction are not important and the Hamiltonian (1) is justified with sufficient accuracy.

In addition to 2D junctions modern nanolithographic technique allows one to fabricate 1D small capacitance tunnel junctions in which case one might have $L_x\gg L_y$. For these junctions the parameter L_y can be very small (e.g. $L_y\sim 10^{-6}$ cm) while L_x is relatively large, $L_x\sim 10^{-3}$ cm. Below we shall show that in this case the description of quantum effects within the framework of the point contact Hamiltonian (1) is not sufficient in general and one has to take space fluctuations of the phase into account. Space and time fluctuations of φ renormalize the Josephson coupling energy E_J and in the limit of large L_x and low T lead to the Kosterlitz–Thouless (KT) phase transition between space–time ordered and space–time disordered phase states. We shall discuss the physical consequences of this effect and compare our results with those of previous considerations [10,11].

Let us consider a 1D Josephson junction and express its grand partition function in terms of a path integral,

$$Z=\int D\varphi\,\exp(-S[\varphi])\,, \tag{2}$$

where

$$S[\varphi]=\int_0^{1/T} d\tau \int_0^{L_x} \frac{dx}{L_x}\,[(1/16E_c)(\partial\varphi/\partial\tau)^2 + \tfrac{1}{2}\lambda_J^2 E_J(\partial\varphi/\partial x)^2 + E_J(1-\cos\varphi)] \tag{3}$$

is the junction effective action [3], λ_J is the penetration depth of the magnetic field into the junction (Josephson penetration depth). The time derivative term in (3) describes a local charging energy with the corresponding local capacitance defined on the scale of the Debye length λ_D which is usually much smaller than any other scale of our problem. Then eq. (3) represents a summation of these capacitances in parallel so that E_c decreases with L_x, $E_c \propto 1/L_x$. In contrast the parameter $E_J \propto L_x$ increases with the junction size L_x.

It is convenient for us to rescale the time coordinate as $z=\lambda_J\omega_J\tau$, $\omega_J=\sqrt{8E_J E_c}$, and rewrite the action (3) as follows,

$$S[\varphi]=\frac{E_J}{T}\int_0^{L_x}\frac{dx}{L_x}\int_0^{L_z}\frac{dz}{L_z} \times\{\tfrac{1}{2}\lambda_J^2[(\partial\varphi/\partial x)^2+(\partial\varphi/\partial z)^2]+1-\cos\varphi\}\,, \tag{4}$$

$L_z=\lambda_J\omega_J/T$ is the "size" of our system in the z-direction. Equation (4) defines an effective 2D sine-Gordon model (see e.g. ref. [12] for a review). The effect of quantum fluctuations of φ can be treated within the framework of a standard renormalization group (RG) technique. Starting from the shortest length scale λ_0 of our problem (which will be defined below) we successively integrate out fluctuations of $\varphi(x, z)$ with wavelengths between a and $a+da$ making the scale a larger and larger. This procedure results in a renormalization of the initial parameters of our problem. For not very large E_J (such that $\bar{y}=\lambda_0^2 E_J/\omega_J L_x\lambda_J \ll 1$) we arrive at the RG equations [12]

$$\frac{d\bar{y}}{\bar{y}}=2(1-\bar{x})\frac{da}{a}\,, \tag{5a}$$

$$d\bar{x}=-2\gamma^2\bar{y}^2\bar{x}^3\frac{da}{a}\,, \tag{5b}$$

where we defined $\bar{x}=L_x\omega_J/8\pi\lambda_J E_J$ and γ is a numerical coefficient of order one which depends on the choice of the cutoff procedure. An infinite 2D system ($L_x\to\infty$, $T\to 0$) shows a KT phase transition which takes place at $\bar{x}=x_c=1/(1-\gamma\bar{y})$. Below we shall assume $\bar{y}$ to be much smaller than one and drop the term $\gamma\bar{y}$ in the expression for x_c. Then for $\bar{x}>1$ the quantity $\bar{y}$ (and thus E_J) scales out to zero with increasing a. It means that for $E_J/E_c<L_x^2/8\pi^2\lambda_J^2$ and $T\to 0$ quantum fluctuations of the phase φ in a long 1D Josephson junction destroy the effect of Cooper pair tunneling and therefore two superconducting electrodes become effectively decoupled from each other. On the other hand, for $\bar{x}<1$ (or, equivalently, for $E_J/E_c>L_x^2/8\pi^2\lambda_J^2$) the quantity E_J scales to a finite value and the superconductors remain correlated.

The existence of a disordered phase state of a 1D Josephson junction has an interesting physical consequence. Indeed for $\bar{x}>1$ the phase–phase correlation function diverges logarithmically in space–time,

$$\langle\varphi(0,0)\varphi(x,z)\rangle \propto \ln[(x^2+z^2)/\lambda_0^2]\,, \tag{6}$$

i.e. quantum fluctuations destroy long range order in either time or space directions or in both. As to the space disorder it means that the phase of *each* superconductor is disordered near the junction plane while it obviously remains ordered in the bulk. In other words, long range phase order in the bulk *does not* prevent phase disorder on a surface. This nontrivial situation is due to quantum fluctuations of the supercurrent component normal to the junction plane. This component survives only in the vicinity of this plane and vanishes at a distance of the order of the screening length λ outside this plane. Then fluctuations at the junction decouple from those in the bulk and lead to a coexistence of surface disorder with the ordered state inside the superconductor. We can add that for an isolated superconductor the absence of the current component normal to the junction plane results in a constraint which prevents any surface fluctuations of the phase.

Note that similar results were obtained for 2D Josephson junctions [9] and layered superconductors [13]. In those cases, however, a disordered phase state on a junction surface is due to classical thermal fluctuations of the Josephson current. For 2D Josephson junctions thermal fluctuations of the phase difference φ also lead to a suppression of E_J and to a KT phase transition between ordered $(T<T_J)$ and disordered $(T>T_J)$ phase states [9].

In contrast to superconductors the correlation between two bulk magnets (described e.g. by the XY model) weakly coupled through some surface will be enforced by the long range order in the bulk. In this case there is no screening length analogous to the London length λ for superconductors and fluctuations on a surface cannot be decoupled from those in the bulk.

Apart from the obvious similarity between the problem of ref. [9] and that discussed here, there are several significant physical differences between them. Perhaps the most important one is that contrary to the case of ref. [9] the quantum problem (2)–(4) is essentially anisotropic with respect to space and time coordinates. E.g. the "space volume" L_x is in general by no means linked to the "time volume" L_z. Therefore depending on the relation between L_x and L_z one might expect the existence of a crossover between the effective 2D and 1D behaviours of our model. This effect will be considered below.

As to the macroscopic quantum effects in small capacitance tunnel junctions [1–8] the experimentally relevant limiting case is $L_z \gg L_x$ or, equivalently, $T \ll \lambda_J\omega_J/L_x$. Indeed the temperature is usually expected to be sufficiently low (typically $T\sim 10^{-2}$–10^{-1} K) and therefore for typical junctions one can estimate the corresponding "time length" as $L_z \gtrsim 10^{-1}$ cm which turns out to be much larger than $L_x \sim 10^{-3}$ cm. As a result fluctuations of the phase φ are effectively two-dimensional only within the scale interval $\lambda_0 < a < L_x$, while for $a > L_x$ or, equivalently, for the frequency range $\omega\sim 1/\tau \lesssim \lambda_J\omega_J/L_x$ we arrive at the effective point junction problem with the renormalized Josephson coupling energy E_J^r,

$$S[\varphi] = \int d\tau\, [(1/16E_c)(\partial\varphi/\partial\tau)^2 + E_J^r(1-\cos\varphi)]\,. \qquad (7)$$

The effect of the charging energy renormalization is small for $\bar{y} \ll 1$ and we shall neglect it here and below. To evaluate the parameter E_J^r we proceed with the RG equations (5). For $\bar{x} > 1$ we start renormalization at $a\sim\lambda_0$ and stop it at $a\sim L_x$. Making use of (5) we get for $E_J/E_c < L_x^2/8\pi^2\lambda_J^2$

$$E_J^r = E_J(\lambda_0/L_x)^{2\bar{x}}\,. \qquad (8)$$

For $E_J/E_c > L_x^2/8\pi^2\lambda_J^2$ $(\bar{x}<1)$ the parameter $\bar{y}$ increases with increasing the length scale a. In this case renormalization should be stopped either at L_x or at the correlation length of our problem depending on which scale is smaller. Here we define the correlation length or, equivalently, the renormalized Josephson penetration length λ_J^r as a scale at which the condition $\bar{y}\sim 1$ is satisfied. Then combining this condition with (5) and choosing the numerical factor to match λ_J^r with the conventional Josephson length λ_J in the limit $\bar{x}\to 0$ we get

$$\lambda_J^r = \lambda_J(\lambda_J/\lambda_0)^{\bar{x}/(1-\bar{x})}\,, \quad \bar{x}<1\,. \qquad (9)$$

For $L_x < \lambda_J^r$ the correlation length λ_J^r (9) is irrelevant and as in the case $\bar{x}>1$ one should stop renormalization at $a\sim L_x$ thus reproducing the result (8) also for $\bar{x}<1$. On the other hand for $L_x > \lambda_J^r$ we stop renormalization at $a \sim \lambda_J^r$ and find for $\bar{x}<1$

$$E_J^r = E_J(\lambda_0/\lambda_J)^{2\bar{x}/(1-\bar{x})}\,. \qquad (10)$$

In this case the renormalized effective action has the form

$$S[\varphi] = \int d\tau\, [(1/16E_c)(\partial\varphi/\partial\tau)^2 + \tfrac{1}{2}\lambda_J^2 E_J(\partial\varphi/\partial x)^2 + E_J^r(1-\cos\varphi)]\,. \qquad (11)$$

Both results (8) and (10) show that high frequency quantum fluctuations of φ in a 1D Josephson junction with $L_x > \lambda_0$ decrease the effective Josephson coupling energy E_J^r in comparison to that of a point junction. This effect is particularly pronounced for $\bar{x} \gg 1$. In the case described by eqs. (7), (8) at low frequencies $\omega \lesssim \lambda_J\omega_J/L_x$ the junction behaves as a point one with the corresponding effective Hamiltonian

$$H=\hat{Q}^2/2C-E_J^r\cos\hat{\varphi}\,, \qquad (12)$$

This in turn means that the effective band structure of the problem is sensitive to the junction size being renormalized in accordance with E_J^r (8). Therefore a corresponding modification of the theory [1-3,7,8] is needed for the case of finite size Josephson junctions.

To estimate the typical value of the ratio E_J^r/E_J let us define the minimal length scale λ_0 at which the junction can be described by the action (3). As it was already discussed in ref. [9] the space gradient term in the effective action (3) has the form $\frac{1}{2}\lambda_J^2 E_J(\partial\varphi/\partial x)^2$ provided λ_0 exceeds the London penetration depth λ. Another restriction for λ_0 comes from the adiabaticity condition $\omega\ll 2\Delta$ (Δ is the superconducting gap) for the Josephson coupling energy term $-E_J\cos\varphi$ (see e.g. ref. [3]). It yields $\lambda_0\gg\lambda_J\omega_J/2\Delta$. Combining these restrictions with the obvious inequality $\lambda_0>L_y$ (which allows us to describe the junction by means of a 1D model) we get

$$\lambda_0>\max(\lambda, L_y, \lambda_J\omega_J/2\Delta)\,. \qquad (13)$$

Here we estimate $\lambda\sim 10^{-5}$ cm, $L_y\sim 10^{-6}$–10^{-7} cm, $\lambda_J\sim 10^{-2}$–10^{-3} cm, $\omega_J/2\Delta\sim 0.1$–1 and thus find $\lambda_0\gtrsim 0.1\lambda_J$. Therefore for $L_x\sim\lambda_J$ and $\bar{x}\gtrsim 1$ the renormalized value E_J^r (8) turns out to be much smaller than E_J. The consequence of this effect might be e.g. strong renormalization of the effective bandwidth δ: even for $E_J\gg E_c$ one might reach the opposite limit $E_J^r\ll E_c$ and thus the bandwidth for a finite size junction $\delta\sim E_c$ becomes much larger than that for a point junction, $\delta\propto\exp(-8E_J/\omega_J)$. This in turn increases the threshold current I_{th} for Bloch oscillations [1-3]. Also Zener tunneling [3,8] becomes much more intensive of one increases the junction size keeping the parameters E_J and E_c fixed.

Here the following comment is in order. Strictly speaking one has to modify the action (3) to include the effect of an external current I_x and/or an external circuit into consideration. It is easy to see, however, that this modification affects only the low frequency part of our problem while the high frequency renormalization of E_J discussed here remains unchanged. Indeed combining the results of refs. [3,9,14] one can write the corresponding modified effective action $\tilde{S}$ as

$$\tilde{S}[\varphi]=S[\varphi]-\int d\tau\, I_x\bar{\varphi}(\tau)/2e+S_D[\varphi]\,, \qquad (14)$$

where $S[\varphi]$ was defined in (3), $\bar{\varphi}(\tau)$ is the space average of the junction phase

$$\bar{\varphi}(\tau)=\frac{1}{L_x}\int_0^{L_x} dx\,\varphi(x,\tau) \qquad (15)$$

and $S_D[\varphi]$ is a dissipative contribution from a (part of an) external circuit. The precise form of $S_D[\varphi]$ depends on the details of the setup. Usually it is Ohmic at reasonably low frequencies in which case we have

$$S_D[\varphi]=\frac{\alpha T}{4\pi}\sum_n|\omega_n|\,|\bar{\varphi}(\omega_n)|^2\,, \qquad (16)$$

$\alpha=\pi/2e^2R$, R is the effective resistance of external leads and $\omega_n=2\pi nT$. Then it is necessary to check that the frequency scale $\omega>\omega_J\lambda_J/L_x$ involved in the space–time renormalization of E_J is separated from a substantially lower frequency scale of Bloch oscillations $I_x/2e$. For the practically important parameter region $E_J^r\lesssim E_c$ oscillations occur $\alpha\ll(E_J^r/E_c)^2$ and $I_{th}<I_x<I_{cr}$, i.e. the maximum frequency of Bloch oscillations [8]

$$\omega_{max}=I_{cr}/2e=1/2RC+(\sqrt{\pi TE_J^r}/4e^2R)^{2/3}$$

is still much lower than $\omega_J\lambda_J/L_x$ for any reasonable value $L_x\lesssim\lambda_J$. Scale separation becomes even more pronounced for $E_J^r>E_c$. This allows us to conclude that our RG analysis remains valid also in the presence of an external bias I_x.

Note that renormalization of the bandwidth [10] and the critical current [11] of a finite size Josephson junction has been already investigated before in the limit $E_J\gg E_c$ within the framework of an instanton technique. In this limit the results obtained here essentially coincide with those obtained in refs. [10,11]. E.g. combining eq. (8) with the expression for the renormalized bandwidth $\delta^r\propto\exp(-\sqrt{8E_J^r/E_c})$ and expanding in powers of $\bar{x}$ we immediately reproduce the result of ref. [10],

$$\delta^r=\delta[1+(L_x/\pi\lambda_J)\ln(2\Delta L_x/\omega_J\lambda_J)]\,.$$

It is worth pointing out that the technique of refs.

[10,11] allows us to study finite size renormalization effects only provided they are small enough. In contrast, the RG technique developed here makes it possible to proceed in a wide parameter region including the limit $E_J \ll E_c$ in which renormalization effects become strong.

As we already discussed, the problem of a 1D quantum Josephson junction can be reduced to that of a point junction (12) provided $L_x \lesssim \lambda_J^r$. For $\bar{x}<1$ and $L_x > \lambda_J^r$ the renormalized effective action still depends on both time and space coordinates. At the space–time scale $x<L_x$, $z<L_x$ the phase φ is ordered. However, for $L_x<z<L_z$ the behavior of the system is entirely different. At this scale the time coordinate is the only one which matters and the system becomes effectively one-dimensional again. Hence, at $T \propto 1/L_z \to 0$ and small $\omega < \omega_J \lambda_J / L_x$ the phase–phase correlation function diverges as $\langle \varphi\varphi \rangle \propto 1/\omega^2$ (or $\langle \varphi\varphi \propto 1/|\omega|$ for $\alpha \neq 0$) and the problem again can be mapped onto that of a point junction. This in turn means that in the low temperature limit macroscopic quantum phenomena (Bloch oscillations, Zener tunneling etc.) in principle can occur even in long ($L_x > \lambda_J^r$) but finite Josephson junctions. In this case, however, the corresponding temperature interval as well as other relevant parameters (e.g. the threshold current I_{th} and the amplitude of Bloch oscillations) schrink exponentially with $E_J \propto L_x$. In a infinite junction, $L_x \to \infty$, for $\bar{x}<1$ the phase remains ordered (while the conjugate charge variable is disordered) at any scale and quantum effects are essentially suppressed.

For the sake of completeness let us briefly discuss the results for a "semiquantum" case $\lambda_0 < L_z < L_x$. In this case we again proceed with the RG equations (5), stop renormalization at $\alpha \sim L_z$ and reproduce eq. (8) in which one should substitute L_z instead of L_x. For $\lambda_0 \sim \lambda_J \omega_J / 2\Delta$ and $\lambda_J \omega_J / L_x < T < 2\Delta$ this equation yields

$$E_J^r = E_J (T/2\Delta)^{2\bar{x}}, \quad \lambda_J^r = \lambda_J (2\Delta/T)^{\bar{x}}. \tag{17}$$

For the scale $L_z < x < L_x$ quantum fluctuations are irrelevant and we arrive at a 1D classical problem with a free energy functional

$$F[\varphi] = E_J^r \int_{L_z}^{L_x} \frac{dx}{L_x} \left[\tfrac{1}{2} (\lambda_J^r)^2 (\partial\varphi/\partial x)^2 + 1 - \cos\varphi \right]. \tag{18}$$

In conclusion, we showed that high frequency quantum fluctuations in a finite size 1D Josephson junction may substantially decrease the Josephson coupling energy in comparison to that of a 2D junction *with the same* cross section area. This effect is particularly pronounced for $L_x > \lambda_0$ and $E_J/E_c \ll L_x^2 / 8\pi^2 \lambda_J^2$. It opens up a possibility to vary the parameter E_J by changing the junction geometry. For a long junction at $T \to 0$ we predict a KT phase transition between disordered ($\bar{x}>1$) and ordered ($\bar{x}<1$) phases. In a disordered phase E_J scales out to zero, i.e Cooper pair tunneling between superconducting electrodes is suppressed by quantum fluctuations. A nontrivial feature of this phase is the coexistence of a disordered state on a junction surface with the ordered one in the bulk.

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