Phase Transitions for a Collective Coordinate Coupled to Luttinger Liquids

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We study various realizations of collective coordinates, e.g., the position of a particle, the charge of a Coulomb box, or the phase of a Bose or a superconducting condensate, coupled to Luttinger liquids with N flavors. We find that for a Luttinger parameter $(1/2) < K < 1$ there is a phase transition from a delocalized phase into a phase with a periodic potential at strong coupling. In the delocalized phase the dynamics is dominated by an effective mass, i.e., diffusive in imaginary time, while on the transition line it becomes dissipative. At $K = (1/2)$ there is an additional transition into a localized phase with no diffusion at zero temperature.

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Diffusion and propagation of massive particles surrounded by a bath is one very challenging problem of condensed matter. Historically, it started with the celebrated Brownian motion [1] in which the interactions between a classical particle and the microscopic motion of the classical bath lead to a diffusion, connected by the Einstein relation to a finite friction.

This problem gets incredibly more complicated when the bath becomes quantum. Indeed the excitations of the bath can lead, by Anderson orthogonality effects, to a modification of the motion of the quantum particle or the collective coordinate coupled to the bath [2]. One of the realizations of such a problem is the polaron problem [3] where the interaction with the vibrations of the lattice leads to an increase of the mass of the particle and even potentially to self-trapping. This type of problem has recently benefitted from the recent progress in cold atomic systems [4]. Indeed, in such systems impurities in quantum baths can be realized in a variety of manners ranging from Fermi or Bose mixtures to ions in condensates, and at various dimensionalities [5–14].

A situation of special interest is provided by a one-dimensional bath for which the bath-bath correlations can become highly nonuniversal; i.e., they acquire an interaction dependent power-law correlations characteristics of a Luttinger liquid (LL) [15]. In that case, special effects can potentially occur, as is clear from the static impurity case [16] and mobile ones coupled to single baths [17,18]. In particular, it was shown recently [19] that this led to a new universality class for the motion of the impurity, for which, in particular, subdiffusion can occur. This very rich situation was explored further. On the theory side, diffusive [20–22], kicked [23,24], and driven impurities [25–27] were considered. On the experimental side, driven impurities [12], mixtures of ^{87}Rb and ^{41}K [13], and ^{87}Rb experiments with local addressability [14] were successful implementations of the one dimensional problem.

In this Letter, we study the physics of a collective coordinate coupled to N Luttinger baths ($N \gg 1$), e.g., a particle position, charge of a Coulomb box, a phase of a Bose-Einstein condensate (BEC), or a phase of a superconducting grain, see Fig. 1. These potential experimental realizations are further examined before the conclusions. We solve this system allowing for both LL density fluctuations and LL interaction and derive a novel localization-delocalization transition, as summarized in Fig. 2. The localized and delocalized phases are separated by a line on which the motion is simply diffusive. We note that the collective coordinate represents a small system with correlations decaying in

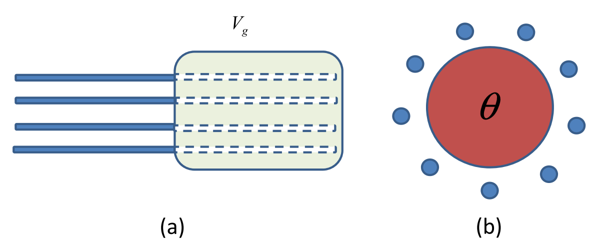

FIG. 1 (color online). Illustrations of collective coordinates coupled to LLs: (a) The environment of a few LLs enter a finite Coulomb box that is under a gate voltage V_g and where the total charge interacts with an effective capacitance. (b) A BEC condensate with phase θ is Josephson coupled to bosonic LLs, shown by their cross section as they cross the figure plane. In a condensed matter context, the figure could also represent a superconducting grain coupled to one-dimensional superconducting wires, which will act as LLs.

time. The periodic and localized phases are, therefore, of particular significance since the collective coordinate acquires long range order due to its interaction with the LLs.

For concreteness, the following presentation uses the particle coordinate language. We consider a particle of mass $\tilde{M}$ coupled to a LL with a contact interaction $\tilde{H}_{\rm int} = g\rho(\tilde{X})$ where $\tilde{X}$ is the operator measuring the impurity position and $\rho(x)$ the density is the LL. We study this model in the large N limit, so the impurity becomes coupled to N independent LLs and the interaction becomes $H_{\rm int} = g\sum_{i=1}^{N}\rho_i(\tilde{X})$. The action of the system can be computed by a cumulant expansion in powers of g and only the second order cumulant remains when $g^2N = O(1)$. Indeed, the fourth order cumulant is of order $g^4N \sim 1/N$ and can be neglected. Using the expression of the density in a LL [15] $\rho(x,\tau) = \rho_0 - (1/\pi)\partial_x\phi(x,\tau) + \rho_0[e^{2i[\pi\rho_0 x - \phi(x,\tau)]} + {\rm H.c.}]$ where $\phi(x,\tau)$ is the bosonic phase, and performing the Gaussian integration over the LL Hamiltonian, the action becomes

$$S_{\rm eff} = \frac{M}{2}\int_\tau (\dot{X})^2_\tau - \frac{\eta\Lambda^2}{2\pi}\int_\tau\int_{\tau'}\frac{\cos(X_\tau - X_{\tau'})}{[\Lambda(\tau-\tau')]^{2K}}, \quad (1)$$

where we have used the dimensionless variables $X = 2\pi\rho_0\tilde{X}$ and $M = \tilde{M}/(2\pi\rho_0)^2$, $\eta = 2\pi g^2\rho_0^2 N/\Lambda^2$, τ is the imaginary time, u the velocity of excitation in the LL, and K the Luttinger parameter that controls the power-law decay of the correlation functions. A frequency cutoff $\Lambda = u/\alpha$ is used to have a dimensionless coupling η where $\alpha \approx 1/\rho_0$ is the natural momentum cutoff of the LL. In the above expression, only the oscillating (backscattering) term in the density has been retained. Indeed, the $\partial_x\phi(x,\tau)$ interaction can be integrated, leading at long times to $(X_\tau - X_{\tau'})^2/(\tau-\tau')^4$, i.e., an ω^3 term in frequency which can be neglected relative to the bare kinetic energy term of the impurity $M\omega^2$. We have used that for a LL, one obtains [15] $\langle e^{i\phi(X_\tau,\tau) - i\phi(X_{\tau'},\tau')}\rangle_{\rm LL} \sim \{[X(\tau) - X(\tau')]^2 + u^2(\tau-\tau')^2\}^{-K}$. We have also made the additional assumption, which will be verified in what follows, that the impurity is less than ballistic and, thus, that $\langle (X_\tau - X_{\tau'})^2\rangle \ll u^2(\tau-\tau')^2$.

To solve for the thermodynamics of (1), first, we consider a renormalization group (RG) process [28] valid for large η, which was also applied to the $K = 1$ case [29]. The action (1) is approximated by its short time form where it becomes Gaussian

$$S_0 = \frac{1}{2}\int_\omega [M\omega^2 + \eta C_K \Lambda^{2-2K}|\omega|^{2K-1}]|X(\omega)|^2, \quad (2)$$

where $\int_\tau (1 - \cos(\omega\tau)/\tau^{2K}) = -2\Gamma(1-2K)\times \sin(K\pi)|\omega|^{2K-1} \equiv \pi C_K|\omega|^{2K-1}$, so that $C_K = 1 - 0.85(K-1) + O(K-1)^2$. The cutoff Λ is replaced by Λ' and the interaction is averaged with S_0 in the small frequency interval $\Lambda' < \omega < \Lambda$ leading to $d\Lambda/\pi C_K\eta\Lambda$. The action then has a renormalized coefficient $\eta^R(\Lambda')^{2-2K}$ where

$$\eta^R = \eta\left\{1 + \left[(2-2K) - \frac{1}{\pi\eta}\right]\ln\frac{\Lambda}{\Lambda'}\right\}, \quad (3)$$

with $C_K \to 1$ to 1st order in either $1/\eta$ or $1-K$. Hence, if $K \geq 1$, η^R flows to small values, while if $K < 1$, there is an unstable fixed point at $\eta_c = 1/(2\pi(1-K))$. $\eta > \eta_c$ flows to large values, while $\eta < \eta_c$ flows to smaller values of η. One can integrate (3) when $\eta < \eta_c$ down to $\eta^R \approx 1$ below which the RG is not controlled. The new cutoff is interpreted as an effective mass [29,30] M^*

$$\frac{1}{M^*} \approx \Lambda[1 - \pi\eta(2-2K)]^{1/(2-2K)}, \quad (4)$$

which for $\pi\eta(2-2K) \ll 1$, but $\pi\eta \gg 1$, i.e., far from the transition point, represents an exponentially large mass $M^* \sim e^{\pi\eta}$, as for the $K = 1$ case [29,30].

To supplement this scenario, and study the properties of the three resulting phases, we follow a variational scheme [31] where we find the best quadratic action approximating the original action (1). The corresponding Green's function $1/f(\omega)$ is a solution of the self-consistent equation

$$f(\omega) = M\omega^2 + \frac{2}{\pi}\eta\Lambda^{2-2K}\int_0^\infty d\tau\frac{1-\cos\omega\tau}{\tau^{2K}} \times e^{-\int_0^\Lambda (1-\cos\omega'\tau)/(\pi f(\omega'))}. \quad (5)$$

First, we note that at $\omega \approx \Lambda$ the solution is $f(\omega) - M\omega^2 \sim |\omega|^{2K-1}$. In the following, we focus on $\omega \ll \Lambda$ and on $K < 1$. As a first option, we consider $f(\omega) = \eta^* C_K\omega^{2K-1}/\Lambda^{2K-2}$. The integral in the exponent converges as $\tau \to \infty$, so it is $\int_0^\Lambda d\omega'/[\pi f(\omega')] = [\pi\eta^* C_K(2-2K)]^{-1}$; hence, (5) reduces to

$$\eta^* = \eta e^{-[\pi\eta^* C_K(2-2K)]^{-1}}. \quad (6)$$

This equation has solutions only if η is sufficiently large, i.e., $\pi C_K\eta(2-2K) > e$. A second possible solution is $f(\omega) = \eta^*|\omega|$. The exponent behaves as $\int_0^\Lambda (1-\cos\omega\tau)/(\pi\eta^*\omega) = 1/(\pi\eta^*)\ln\Lambda\tau$, since the τ integral is dominated by long τ, hence,

$$\eta^*\omega = \frac{2}{\pi}\eta\Lambda^{2-2K-1/\pi\eta^*}\int_0^\infty d\tau\frac{1-\cos\omega\tau}{\tau^{2K+1/\pi\eta^*}} = \eta\omega, \quad (7)$$

which is a consistent solution on a line $(1/\pi\eta) = 2(1-K)$. The third possible solution is similar to the bare one $f(\omega) = M^*\omega^2$, then the exponent behaves as $\int_0^\infty (1-\cos\omega\tau)/(\pi M^*\omega^2) = |\tau|/2M^*$, leading to $M^* \approx M$ for intermediate or weak coupling. The variational scheme can be shown to be related to an RG process [31] from which the fixed point line Eq. (3) is reproduced. We note that both the RG and the variational method are valid as weak coupling expansions where the coefficients in Eq. (3) are small, i.e., large η and small $|1-K|$.

Thus, the above methods lead to three different possible behaviors for the system: (i) At $1 - K = 1/(2\pi\eta)$ the particle propagator has the friction form $(\eta|\omega|)^{-1}$; i.e., the nonlinearity of the cosine and long range effect balance each other to produce an equivalent action with $\eta|\omega||X(\omega)|^2$. (ii) The case $1-K<1/(2\pi\eta)$ flows to small η and eventually to an $M^*\omega^2$ form, with $\langle (X_\tau - X_0)^2\rangle \sim |\tau|$, which corresponds to a delocalized phase. The effective mass M^* is identified by the RG flow, as in Eq. (4). Note that even in this delocalized phase, some effects of the underlying quasilong range periodicity of the LL with the wave vector $2\pi\rho_0$ are still felt by the particle. Indeed, its correlation at that periodicity is only very slowly decaying $\langle \cos X_\tau \cos X_0\rangle \sim \tau^{-2K}$. This indicates that the particle has a much greater chance to be found at some particular places on the chain. This can be understood qualitatively by the argument that the particle moves in the "charge density wave" of wave vector $2\pi\rho_0$ provided by the LL; hence, the particle diffuses predominantly by tunneling between lattice sites spaced by $1/\rho_0$. On the mathematical side, this property which is apparent in a first order calculation in η [31] is in fact known in general in the context of XY models with long-range interactions [32]. (iii) The case $1-K>1/(2\pi\eta)$ flows to large η with eventually $f(\omega)\sim\omega^{2K-1}$; i.e., S_0 of Eq. (2) is a fixed point action. From this form, one could naively expect the correlations of $\langle [X_\tau - X_0]^2\rangle$ to be convergent and, thus, this phase to be a localized one. The situation is, in fact, more subtle, and we discuss this phase in more detail below.

A summary of the various regimes can be found in Fig. 2, and the corresponding correlation functions are indicated in Table I. At finite temperatures T and after analytic continuation to the retarded response at real time t [15], we find the replacements $|\tau|^{-2K}\to \sin\pi K e^{-2K\pi T t}$, $|\tau|^{2K-2}\to\sin\pi(1-K)e^{-(2-2K)\pi T t}$, and $\ln|\tau|\to Tt/\eta$, i.e., diffusion in real time on the dissipative line.

For $K<1$, we complement the above analysis by a mean-field approach similar to the one used in the context of XY models with long-range interactions [33]. We take $h=\langle\cos X_\tau\rangle$ as an order parameter. The interaction term

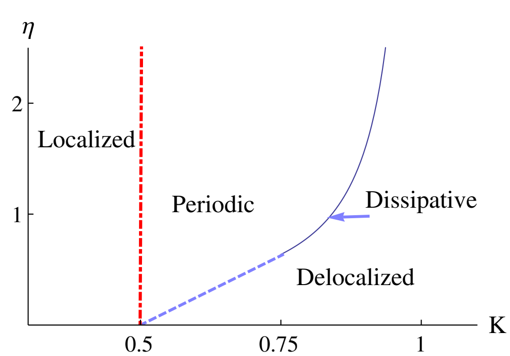

FIG. 2 (color online). Phase diagram for an impurity in a bath of LLs as a function of the LL parameter K and the interaction parameter between the impurity and the bath η. Four regimes can occur (see text) in which the impurity is delocalized, just dissipative, periodically localized, or localized. Dashed lines indicate boundaries out of the control of the perturbative RG. The corresponding correlation functions are given in Table I.

TABLE I. Correlations of the phases in Fig. 1 at $T=0$.

Correlation	Delocalized	Dissipative	Periodic	Localized
$\langle\cos X_\tau\rangle$	0	0	constant	1
$\langle\cos X_\tau \cos X_0\rangle$	$\sim|\tau|^{-2K}$	$\sim|\tau|^{-(2-2K)}$	constant	1
$\langle (X_\tau - X_0)^2\rangle$	$\sim|\tau|$	$\sim\ln|\tau|$		0

in (1) decouples as $\eta\Lambda^{2-2K}h\int_\tau \cos X_\tau \int_{\tau'}|\tau-\tau'|^{-2K} = \eta\Lambda h(1/(2K-1))\int_\tau \cos X_\tau$. The self consistency relation, linear in h, is $1=\eta\Lambda(1/2K-1)\int_{\tau'}\langle\cos_\tau \cos_{\tau'}\rangle_0 = 4\eta M\Lambda(1/2K-1)$; it yields the critical line $\eta_c=(2K-1)/(4M\Lambda)$ above which $\langle\cos X_\tau\rangle\neq 0$. We expect the mean field result to be more reliable near $K=1/2$, where the range of the interaction increases. As K increases from $K=1/2$, fluctuations will increase the critical value, eventually joining the transition line with the variational form $\eta_c=1/(2\pi(1-K))$ near $K=1$. Note that mean field exponents become valid [33] when $K<3/4$, e.g., $\langle\cos X_\tau\rangle\sim\sqrt{\eta-\eta_c}$. In Fig. 2, we plot the transition line as an interpolation between the mean field at $K<3/4$ and the variational form at $K>3/4$. We see that the point $K=1/2$ plays an important role, not captured by the variational or RG approaches. Below this point, the interaction is so long range that an ordered phase would exist, within the mean-field solution, for arbitrary strength of the coupling η.

In the periodic phase, instanton excitations must *a priori* be considered since one would have many degenerate minima of the order parameter. Such instantons are known for the $K=1$ case [34,35]. Assuming an instanton with width τ_0, the interaction term in (1) has the form $\eta(\Lambda\tau_0)^{2-2K}B_K$ while the mass term is $\sim M/\tau_0$; hence, the action is minimized at $\Lambda\tau_0\sim(M\Lambda)/[(1-K)\eta B_K]^{1/(3-2K)}$ for $K<1$; the numerical prefactor B_K is known at $K=1$, $B_1=\pi$. Note that the mass term and $K\neq 1$ set a finite scale for τ_0, unlike the $K=1$ case. The instanton action is then

$$S_{\rm inst}\approx M\Lambda\left(\frac{(2-2K)\eta B_K}{M\Lambda}\right)^{1/(3-2K)}\frac{3-2K}{2-2K}. \qquad (8)$$

Such instantons mean that the coordinate X_τ can tunnel between neighboring minima of the ordered $\langle\cos X_\tau\rangle$. Assuming independent instantons, this would imply that $\langle (X_\tau - X_0)^2\rangle = D|\tau|$ has a finite diffusion constant, $D\sim e^{-S_{\rm inst}}$.

In particular, we consider $K\to 1/2$ and an instanton localized at $\tau=0$. The dominant contribution for the instanton center at $|\tau|<\tau_0$ comes from $|\tau'|>\tau_0$ that involves $|X_\tau|\gg|X_{\tau'}|$ and $\int_{|\tau'|>\tau_0}|\tau'|^{-2K}\sim 1/(2K-1)$, which diverges at $K\to 1/2$, hence,

$$S\left(K\to\frac{1}{2}\right)=\frac{1}{2}M\int_\tau(\dot X)^2_\tau+\frac{\eta\Lambda(\Lambda\tau_0)^{1-2K}}{\pi(2K-1)}\int_\tau(1-\cos X_\tau)+S'. \qquad (9)$$

S' comes from the instanton tails where X_τ, $X_{\tau'}$ are small (up to 2π) and comparable. This action is similar to the well known sine-Gordon system, identifying $B_K \sim (2K-1)^{-1}$ whose instanton (or soliton) solution has a width $\tau_0 \sim (2K-1)^{1/2}$ and action $S_{\rm inst} \sim (2K-1)^{-(1/2)}$. Assuming independent instantons, the diffusion constant would diverge at $K = 1/2$, i.e., $\ln D \sim (2K-1)^{-(1/2)}$. We propose that the whole range of the periodic phase in Fig. 2 has instanton solutions with a finite action, with an explicit solution provided by the sine-Gordon system at $K \to 1/2$. However, given the long range form of the interaction within the tail term S', to ascertain the correct behavior of $\langle (X_\tau - X_0)^2 \rangle$ at large time requires further study of how these instantons interact, which is left for the future.

Next, we consider the system at $K < 1/2$. This case has been studied in the context of discrete XY models [36–38] and was shown to have a phase transition in the limit that the coupling vanishes as a power of the system size, which in our case is $\beta = 1/T$; i.e., there is a critical value for $\eta(\beta)^{1-2K}$. Hence, at $T = 0$ the system is fully ordered and $\langle \cos X_\tau \rangle = 1$. Furthermore, instanton excitations would involve the effective coupling $\eta(\beta)^{1-2K}$; hence, they will have diverging action. Extending the mean-field analysis to $K < 1/2$ yields a critical temperature T_x where $(1-2K) \times (T_x/\Lambda)^{1-2K} = 2M\Lambda\eta$; fluctuations would render T_x into a sharp crossover temperature.

Let us conclude this part by noting that the hypothesis made at the beginning to neglect $X_\tau - X_0$ compared to τ is, indeed, justified in all the phases. Furthermore, note that although the results of the present Letter are derived in the large-N limit, we, of course, expect them to extend to a finite number of components as well. For example, for the Coulomb box case, deviations due to finite N appear at exponentially small temperatures [39].

Finally, we discuss possible realizations of our model with various collective coordinates that are potential candidates for experimental studies. (i) A first example that yields our action (1) is a fermion Coulomb box [40]. Following the Ambegaokar-Eckern-Schön mapping [41], one introduces a phase X_τ such that $\dot{X}_\tau$ measures the charge in the box while the charging energy corresponds to $1/M$. The kernel in (1) is then $\sum_\alpha G_{\alpha,i}(\tau - \tau')\sum_k G_{k,i}(\tau' - \tau)$ where i is the channel index, α, k are internal quantum numbers of the dot and LL, respectively, and the Green's functions are for either free fermions on the dot, $\sim 1/(\tau - \tau')$ or for fermions in the LL (with Luttinger parameter K_f) $\sim |\tau - \tau'|^{-(1/2)(K_f + 1/K_f)}$. Hence, an effective action of the form (1) with $2K = 1 + (1/2)(K_f + 1/K_f)$, realizing only $K > 1$ cases.

(ii) A variation of realization (i) is a system of LLs that terminates in a Coulomb box, i.e., a region where all LLs have long range Coulomb interactions with an effective capacitance, as illustrated in Fig. 1(a). In this case, a boundary Green's function [15] is needed $G_{x=0,i}(\tau - \tau') \sim |\tau - \tau'|^{-1/K_f}$, hence, $K = 1/K_f$ and the interesting regime of Fig. 1 with $K < 1$ is realizable with attractive interactions $K_f > 1$. In case that Coulomb box region is a normal metal, we obtain $2K = 1 + (1/K_f)$.

(iii) A 3rd realization [Fig. 1(b)] corresponds to a BEC with a phase θ_τ that weakly couples to bosonic LLs with boson operators $\Psi_n(\tau)$ as $g e^{i\theta_\tau}\Psi_n(\tau)$ + H.c. The average now involves the boson's Greens function $\sim |\tau - \tau'|^{1/2K_b}$, $K_b \to \infty$ for noninteracting bosons and K_b decreases to 1 for on-site repulsion $U \to \infty$. Hence, (1) is realized with $K = 1/4K_b$ and the localized regime (Fig. 1) with $K < 1/2$ is realizable.

(iv) In analogy with the BEC, a superconducting grain can Josephson couple to superconducting one-dimensional wires [Fig. 1(b)]. For attractive short range interactions, $2K = 1/K_\rho$ and $K < 1$ can be realized by fermions (spin full in this example) with long range repulsive or attractive interactions allowing for the interesting regime $K < 1$. This case could potentially be realized with the new superconducting $LaAlO_3/SrTiO_3$ nanostructures [42].

(v) Finally, the mobile impurity case may be realized by an impurity confined in between LL chains forming, e.g., a hexagon. In this case, the interesting $K < 1$ regime is realized by repulsive fermion interactions.

In conclusion, we have studied the physics of LL environments that couple to a collective coordinate such as an impurity position, charge of a Coulomb box, a phase of a BEC, or that of a superconducting grain. We have shown that the coupling to the bath leads to various phases for the collective coordinate ranging from delocalized, dissipative, periodic, and localized. Our results are summarized in Fig. 2 and Table I, showing the distinctions among the various phases. We believe that the large set of realizations for the collective coordinate and the various phase transitions will stimulate further research.

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