Interference in the presence of dissipation

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We study a particle on a ring in the presence of various dissipative environments. We develop and solve a variational scheme assuming low frequency dominance. We analyze our solution within a renormalization group (RG) scheme to all orders which reproduces a two loop RG for the Caldeira-Legget environment. In the latter case the Aharonov-Bohm (AB) oscillation amplitude is exponential in $-R^2$, where R is the ring's radius. For either a charge or an electric dipole coupled to a dirty metal we find that the metal induces dissipation, however the AB amplitude is $\sim R^{-2}$ for large R, as for free particles. Cold atoms with a large electric dipole may show a crossover between these two behaviors.

The problem of interference and dephasing in presence of dissipative environments is of significance for a variety of experimental systems and a fundamental theoretical issue. The experimental systems include mesoscopic rings embedded on various surfaces where Aharonov-Bohm (AB) oscillations can be measured,[1,2] and the related problem of decoherence at low temperatures.[3] A different type of experimental systems are cold atom traps created by atom chips,[4–6] or trapped excited atoms with huge electric dipoles.[7] The atom chip that produces a magnetic or electric trap for the cold atoms necessarily also produces noise. Our problem is then relevant for evaluating the interference amplitude of cold atoms or molecules in presence of such noise.

As an efficient tool for monitoring the effect of the environment we follow a suggestion by Guinea[8] to find the AB oscillation amplitude as function of the radius R of the ring, as measured by the curvature[9,10] $1/B_c$ of the ground state energy E_0 at external flux $\phi_x = 0$, i.e., $1/B_c = \partial^2 E_0 / \partial \phi_x^2|_0$. For free particles of mass M this amplitude is the mean level spacing $\sim 1/MR^2$. Two types of environments were suggested to lead to an anomalous suppression, i.e., a stronger decrease of the oscillation amplitude than $1/R^2$: system (i) is that of a Caledeira Legget bath and system (ii) of a charge in a dirty metal environment.

System (i) is relevant to the Coulomb blockade problem[9–12] as well as to quantum dots at a distance from metallic gates.[13] This problem has been extensively investigated by instanton methods,[14–17] by RG methods,[8,9,18] and by Monte Carlo (MC) methods.[9,10,19] All methods show that B_c increases exponentially with the dissipation strength α, i.e., $B_c \sim \alpha^{-\mu} e^{\pi^2 \alpha}$, with differences in the exponent μ. In second order renormalization group[9] (RG) $\mu = 2$, real time RG gives[18] $\mu = 6.5$, instanton methods give either[14] $\mu = 2$ or[15,17] $\mu = 3$, or[16] $\mu = 4$, while MC suggests[19] $\mu = 5$. This system was also studied by a variational approach[21] which was solved numerically showing a nonperturbative regime at strong α. Since $\alpha = \gamma R^2$ where γ is a friction coefficient, a length scale $\pi/\sqrt{\gamma}$ is identified,[8] beyond which the AB oscillations decay.

The system (ii) was investigated by RG methods[8] finding $B_c \sim R^{2+\mu'}$ with $\mu' \lesssim 1$ nonuniversal, while MC data[20] shows $\mu' \approx 1.8$. Furthermore, the MC data shows that at any finite temperature an exponential form appears, with a temperature independent length.

In the present work we study also the effective action for an electric dipole coupled to a dirty metal. This system (iii), which is relevant to experiments on cold atoms[7] or molecules, is found to induce dissipation on the dipole. We then solve these systems by a variational method. We test our method on system (i), which is extensively studied yet still controversial. Our aim is, however, to develop an efficient method for a large class of dissipative environments as in systems (ii) or (iii).

We show, within the variational method, that at zero temperature the effective mass B/R^2 of the zero winding number sector determines the curvature, i.e., $B = B_c$. We find that the variational method defines an RG scheme to all orders and it reproduces the known RG equation [9] to two loops in system (i). In systems (ii) and (iii), we find that the environment induces dissipation in the effective action, however the effective mass remains $B/R^2 \sim R^0$ for large R, as for free particles. As a measurable result, we show that giant Rydberg atoms[7] with huge dipole moments are sensitive probes of metallic environments. They allow a crossover from system (i) (at small R) with exponential decrease in its interference amplitude to a large R behavior with the much weaker $1/R^2$ behavior.

The time dependent angular position $\theta_m(\tau)$ of a particle on the ring has in general a winding number m so that $\theta_m(\tau) = \theta(\tau) + 2\pi m \tau / \beta$, where $\theta(0) = \theta(\beta)$ has periodic boundary condition and β is the inverse temperature ($\beta \to \infty$ below). In presence of ϕ_x the partition sum has the form

$$Z = \sum_m e^{2\pi i m \phi_x - 2\pi^2 m^2 MR^2 / \beta} Z_m,$$

$$Z_m = \int \mathcal{D}\theta e^{-S_1\{\theta(\tau)\} - S_{\text{int}}\{\theta(\tau) + 2\pi m \tau / \beta\}}, \qquad (1)$$

where in presence of a general dissipative bath the effective action can be written in terms of Fourier coefficients[8] α_n

$$S_1\{\theta\} = \int_0^\beta d\tau \frac{1}{2} MR^2 \left(\frac{\partial \theta}{\partial \tau} \right)^2,$$

$$S_{int}\{\theta_m\} = \sum_n \alpha_n \int_0^\beta \int_0^\beta d\tau d\tau' \frac{\pi^2\beta^{-2}\sin^2\left\{\frac{n}{2}[\theta_m(\tau)-\theta_m(\tau')]\right\}}{\sin^2[\pi\beta^{-1}(\tau-\tau')]} \tag{2}$$

and α_n depend on the type of bath. At $\tau \to \tau'$ (or at high frequencies ω) one can expand the $\sin^2(\cdots)$ in Eq. (2) and then $S_{int} \to \Sigma_n \alpha_n n^2 \int d\omega |\omega||\theta_m(\omega)|^2$, identifying a dissipative system. The Caldeira Legget bath has a single α_n with $\alpha_1 = \gamma R^2$ while a charged particle in a dirty metal bath has[8,20] $\alpha_n \approx \frac{2\gamma}{\pi r}\ln(r/n)$ for $1 < n \lesssim r$ and $\alpha_n \approx 0$ otherwise; here l is the mean free path, k_F is the Fermi wave vector, $r = R/l$, and $\gamma = 3/(8k_F^2 l^2)$.

As a new realization of Eq. (2) we consider an electric dipole p perpendicular to the plane of the ring and coupled to a dirty metal. The interaction with the fluctuating electric field $\mathbf{E}(\mathbf{r},\tau)$ is $p\int_0^\beta E_z[\mathbf{R}(\tau),\tau]d\tau$, where $\mathbf{R}(\tau) = R[\cos\theta(\tau), \sin\theta(\tau)]$ is the particle's position on the ring. After a Gaussian average we have $S_{int} = \frac{1}{2}p^2 \int\int d\tau d\tau' f(\mathbf{X}, \tau - \tau')$, where $\mathbf{X} = \mathbf{R}(\tau) - \mathbf{R}(\tau')$ and

$$f(\mathbf{X},\tau) = \langle E_z[\mathbf{R}(\tau),\tau]E_z[\mathbf{R}(0),0]\rangle = \sum_{q,\omega_n} e^{i\mathbf{q}\cdot\mathbf{X}-i\omega_n\tau}\frac{4\pi q_z^2}{q^2\epsilon(i|\omega_n|,q)} \tag{3}$$

with $\epsilon(i|\omega_n|,q)$ being the dielectric function[20] $\omega_n = 2\pi n/\beta$. The frequency sum yields the form (2) with

$$\frac{3}{8k_F^2 l^2}\frac{p^2}{e^2 l^2}\left\{1 - \left(4r^2\sin^2\frac{z}{2}+1\right)^{-3/2}\right\} = \sum_n \alpha_n \sin^2 n\frac{z}{2}. \tag{4}$$

Hence, for large r, $\alpha_n \sim \frac{1}{r}\left(1 - \frac{n^2}{r^2}\right)$ for $n < r$ and $\alpha_n \approx 0$ otherwise.

The variational method[22] for Z_m finds the best Gaussian approximation, i.e., $S_0 = \frac{1}{2\beta}\Sigma_{\omega_n}G^{-1}(\omega_n)|\theta(\omega_n)|^2$ so that the variational free energy $\beta F_{var} = \beta F_0 + \langle S_1 + S_{int} - S_0\rangle_0$ is minimized; here $\langle\cdots\rangle_0$ is an average with respect to $\exp(-S_0)$ and F_0 is the free energy corresponding to S_0. The method is tested below by RG results, where available, and is found to reproduce these RG results. The interaction term is then

$$\langle S_{int}\rangle_0 = \beta\sum_n \alpha_n \int_0^\beta \frac{d\tau}{2\tau^2}\{1 - \cos(2\pi nm\tau/\beta)$$
$$\times e^{-n^2\int_\omega G(\omega)[1-\cos(\omega\tau)]}\}, \tag{5}$$

where $\int_\omega = \int d\omega/2\pi$ and the variational equation $\delta F_{var}/\delta G(\omega_n) = 0$ becomes

$$G^{-1}(\omega) = MR^2\omega^2$$
$$+ 2\sum_n \alpha_n n^2 \int_{1/\omega_c}^\infty d\tau \frac{1-\cos(\omega\tau)}{\tau^2}\cos(2\pi nm\tau/\beta)$$
$$\times e^{-n^2\int_{\omega_1}G(\omega_1)[1-\cos(\omega_1\tau)]}. \tag{6}$$

Here the limit $\beta \to \infty$ is taken (except for the m dependent term) and a cutoff ω_c is introduced to control the short time behavior. This cutoff represents a high frequency limit of the bath degrees of freedom.

In the following we will study the variational equation with $m=0$. To justify this, we show now that the effective mass B of the $m=0$ system is indeed what is needed to find the AB oscillation amplitude at $\beta \to \infty$. The effective mass is defined by $G^{-1}(\omega) = B\omega^2$ in the limit $\omega \to 0$ and is identified from Eq. (6) at $\beta \to \infty$ as

$$B = MR^2 + \sum_n \alpha_n n^2 \int_0^\infty d\tau e^{-n^2\int(d\omega/2\pi)G(\omega)[1-\cos(\omega\tau)]}. \tag{7}$$

We use here $\langle m^2\rangle \sim \beta$ [Eq. (1)] and convergence in τ due to the exponent in Eq. (6) $\sim \exp(-n^2\tau/B)$. Hence $\cos(2\pi nm\tau/\beta) \to 1 + O(1/\beta)$ in Eq. (6) and the effective mass B is m independent.

The AB oscillation amplitude is usually measured by the curvature $1/B_c = -\partial^2 E_0/\partial^2\phi_x|_0 = 4\pi^2\langle m^2\rangle/\beta|_0$. To identify B_c we expand $\ln Z_m = -\beta F_{var}(m)$ in m/β and since at $m=0$ we have $\partial G(\omega)/\partial m = 0$ and $\partial F_{var}(m)/\partial G = 0$ (the variational condition) the leading term is from expanding Eq. (5)

$$\beta F_{var}(m) = \beta F_{var}(0) + \frac{4\pi^2}{\beta}(B-MR^2)m^2 + O(m^4/\beta^3). \tag{8}$$

The effect of S_{int} in the partition sum Eq. (1) is therefore to replace the factor $2\pi^2 MR^2 m^2/\beta$ by $2\pi^2 Bm^2/\beta$, i.e., the response to an external flux is that of a free particle with a mass renormalized to B/R^2. Our task is therefore to study the $m=0$ system and find this renormalized mass.

We note that at finite $\beta < B/n^2$ the τ integrals are not suppressed by the exponential factor and $\langle S_{int}\rangle_0 \approx \pi^2\Sigma_n \alpha_n n|m|$, corresponding to instanton trajectories.[14–16] Hence for large α_n only the lowest m sectors contribute, in contrast with low temperatures where $\langle m^2\rangle \sim \beta/B \to \infty$; This defines a crossover temperature $\beta^* \approx B/n^2$.

We assume now that the frequency integral in Eq. (6) is dominated by low frequencies and derive a simplified variational equation. Consider first the regime $\omega < \omega_c$, but ω is not too small, i.e., $\ln(\omega_c/\omega) \approx 1$. This is a perturbative regime where the significant low frequencies are not yet manifested. For large dissipation coefficients α_n the term in the exponent of Eq. (6) is small, and $G^{-1}(\omega) = MR^2\omega^2 + \pi\omega\Sigma_n\alpha_n n^2$. This form shows that the ω^2 term acts as cutoff ω_c' in $\int_\omega G(\omega)$ with $\omega_c' = \pi\Sigma_n\alpha_n n^2/(MR^2)$; we choose ω_c as the lowest of the original ω_c and ω_c' and ignore the bare ω^2 term. The next order in perturbation is then

$$G^{-1}(\omega) = \pi\omega\sum_n \alpha_n n^2\left[1 - \frac{n^2}{\pi^2\sum_m \alpha_m m^2}\ln\frac{\omega_c}{\omega}\right] \tag{9}$$

which identifies the perturbation parameter, i.e., the perturbative ω range is large for system (i), while for systems (ii) and (iii) the $n \approx r$ terms require $\ln\omega_c/\omega \ll 1$.

We proceed now to the significant range of $\omega \ll \omega_c$ and assume the general form

$$G^{-1}(\omega) = f(\omega), \quad \omega_0 < \omega \ll \omega_c,$$
$$G^{-1}(\omega) = B\omega^2, \quad \omega < \omega_0. \tag{10}$$

It is convenient, for $\omega > \omega_0$, to study a derivative of Eq. (6) for which the oscillating $\sin(\omega\tau)/\tau$ is replaced by 1 while the τ integration acquires a cutoff $\tau < 1/\eta_1\omega$, such that the result should not be sensitive to $\eta_1 \approx 1$.

We assume now that the integral in the exponent of Eq. (6) is dominated by $\omega_1 > 1/\tau$ where the $\cos \omega_1\tau$ averages to zero. For $\omega \gtrsim \omega_0$ this requires $1/B\omega_0 \ll \int_{\omega_0}^{\omega_c} d\omega_1/f(\omega_1)$ [condition (i)]. Introducing a second cutoff uncertainty η_2 we obtain in terms of $\omega_2 = 1/\tau$

$$f'(\omega) = 2\omega \sum_n \alpha_n n^2 \int_{\eta_1\omega}^{\omega_c} \frac{d\omega_2}{\omega_2^2} e^{-n^2 \int_{\eta_2\omega_2}^{\omega_c} d\omega_1/\pi f(\omega_1)}. \quad (11)$$

Taking $d/d\omega$ we obtain our main equation for $f(\omega)$,

$$f'(\omega) = \pi\eta \sum_n \alpha_n n^2 e^{-n^2 \int_\omega^{\omega_c} d\omega_1/\pi f(\omega_1)}, \quad (12)$$

where $\omega f''(\omega) \ll f'(\omega)$ is assumed [condition (ii)]. Here $\eta_1 = 2/(\pi\eta)$ and $\eta_1\eta_2 = 1$ are chosen, to connect smoothly with the perturbative regime where $f'(\omega_c) = \pi\eta\sum_n \alpha_n n^2$; from Eq. (9)

$$\eta = 1 + \frac{\sum_n \alpha_n n^4}{(\pi\sum_n \alpha_n n^2)^2}. \quad (13)$$

A similar analysis for the range $\omega < \omega_0$ leads to $f'(\omega_0) = \eta' B\omega_0$ [note the similarity of Eqs. (7) and (12)] with η' reflecting cutoff uncertainties. Hence Eq. (12) is to be solved with the boundary conditions [η given by Eq. (13)]

$$f(\omega_c) = \pi\omega_c \sum_n \alpha_n n^2, \quad f(\omega_0) = B\omega_0^2,$$

$$f'(\omega_c) = \eta\pi \sum_n \alpha_n n^2, \quad f'(\omega_0) = \eta' B\omega_0. \quad (14)$$

We show now that the variational Eq. (12) can be solved by an RG process. The latter identifies a change in the cutoff $d\omega_c = \omega_c' - \omega_c$ combined with a change in the couplings $d\alpha_n = \alpha_n' - \alpha_n$ such that Eq. (12) for $f'(\omega)$ is unchanged. In terms of $dl = -d(\ln \omega_c)$ this yields an RG equation to all orders [provided $\eta(\alpha_m)$ is known]

$$\frac{d\alpha_n}{dl} + \frac{\alpha_n}{\eta}\sum_m \frac{\partial\eta}{\partial\alpha_m}\frac{d\alpha_m}{dl} = -\frac{n^2\alpha_n}{\pi^2\sum_m \alpha_m m^2}, \quad (15)$$

which to lowest order ($\eta = 1$) agrees with the RG proposed by Guinea.[8] In system (i) with a single α we obtain to order $1/\alpha$ (and $\alpha \equiv \alpha_1$ for brevity)

$$\frac{d\alpha}{dl} = -\frac{1}{\pi^2} - \frac{1}{\pi^4\alpha} \quad (16)$$

which amazingly is precisely the two loop RG result[9] (requiring 14 diagrams). For system (ii) $\sum_n \alpha_n n^4/(\sum_n \alpha_n n^2)^2 = O(1)$ is independent of the large parameter r. Hence there is no expansion parameter for the RG, yet the variational method is useful as shown below.

We proceed to solve the nontrivial Eq. (12) for the Caldeira-Legget system. Differentiating Eq. (12) we obtain $f''(\omega) = \frac{f'(\omega)}{\pi f(\omega)}$ which upon integration yields

$$f'(\omega) = \pi^{-1}\ln[Kf(\omega)], \quad (17)$$

where from the boundary conditions (14)

$$K = \frac{e^{\pi^2\alpha\eta}}{\pi\alpha\omega_c}. \quad (18)$$

A solution of Eq. (17) requires an asymptotic expansion of the log integral $\int df/\ln f$, an expansion that is not available in standard textbooks. We develop here a large α expansion using the following idea: In terms of $f(\omega) = \omega g(K\omega)$ we have

$$g(x) + xg'(x) = \pi^{-1}\ln[xg(x)]. \quad (19)$$

The boundary condition at $\omega = \omega_c$ can be written as

$$g\left(\frac{e^{\eta\pi^2\alpha}}{\pi\alpha}\right) = \pi\alpha. \quad (20)$$

We claim that if the function $g(K\omega)$ is chosen such that it does not depend explicitly on α, except through its argument $K(\alpha)$, then a useful large α expansion is generated. The boundary condition (20) becomes a functional relation involving $\eta(\alpha)$. To show our claim we use the boundary condition (20) $g(x_c) = \pi\alpha$ at $x_c = K\omega_c$, its derivative $x_c g'(x_c) = \pi\frac{K(\alpha)}{K'(\alpha)}$, and $f'(\omega_c) = \pi\eta\alpha = g(x_c) + x_c g'(x_c)$ to yield

$$\eta = 1 + \frac{K(\alpha)}{\alpha K'(\alpha)} = 1 + \frac{1}{\pi^2\alpha\eta - 1 + \pi^2\alpha^2\dfrac{d\eta}{d\alpha}}. \quad (21)$$

This relation generates a large α expansion with the leading form $\eta = 1 + (\pi^2\alpha)^{-1} + O(\alpha)^{-2}$, consistent with the perturbation expansion (13). It is remarkable that the initial values for Eq. (17) as given by the perturbation expansion are precisely such as to allow for an asymptotic expansion of Eq. (21).

We note that combining the RG equation (15) with (21) leads to yet another remarkable relation

$$\frac{d\alpha}{dl} = \alpha[1 - \eta(\alpha)], \quad (22)$$

where $\eta(\alpha)$ solves Eq. (21). Hence $1 - \eta(\alpha)$ is the exact β function within the variational method. In view of its success in reproducing the two loop result[9] it may hold even to higher orders in RG for the original action (2).

For $\omega < \omega_c$ we choose $\bar\alpha(\omega)$ such that $K[\bar\alpha(\omega)]\omega_c = K(\alpha)\omega$. The boundary condition (20) with $\bar\alpha$ produces then the solution $f(\omega) = \omega g(K\omega) = \pi\omega\bar\alpha(\omega)$ with $\bar\alpha(\omega)$ the solution of

$$K\omega = \frac{e^{\pi^2\bar\alpha(\omega)\eta[\bar\alpha(\omega)]}}{\pi\bar\alpha(\omega)}. \quad (23)$$

Inverting this relation we find

$$f(\omega) = \pi\omega\bar\alpha(\omega) = \frac{\omega}{\pi\eta}\ln(\pi\bar\alpha K\omega)$$

$$= \frac{\omega}{\pi\eta}\ln\left(\frac{K\omega}{\eta\pi}\ln(\pi\bar\alpha K\omega)\right) = \frac{\omega}{\pi\eta}\ln\left[\frac{K\omega}{\eta\pi}\ln\left(\frac{K\omega}{\eta\pi}\cdots\right)\right] \quad (24)$$

and at least two ln embeddings are needed for a large α solution.

To identify the effective mass B we note that the boundary condition for Eq. (17) at ω_0 is $K\omega_0 = e^{\eta' \pi B \omega_0}/B\omega_0$ so that $g(K\omega_0) = B\omega_0$ becomes $g(e^{\eta' \pi B \omega_0}/B\omega_0) = B\omega_0$. This equation does not involve the large parameter α, hence $B\omega_0 \approx 1$, $K\omega_0 \approx 1$, and the effective mass is

$$B \approx \frac{1}{\omega_0} \approx \frac{e^{\pi^2 \alpha}}{\alpha \omega_c}. \tag{25}$$

Condition (i) for Eq. (12) is satisfied if $\ln \alpha_1 \gg 1$, while condition (ii) requires $\ln \omega/\omega_0 \gg 1$; this is valid in the exponentially large range $\alpha\omega_0 < \omega < \omega_c$ while in the relatively small range $[\omega_0, \alpha\omega_0]$ $O(1)$ changes occur in Eq. (12).

We conclude that the effective mass is exponentially large, Eq. (25), and that the power of the prefactor is $\mu = 2$, as in the second order RG result[9] [we use the cutoff $\omega_c = \omega_c'$ $= \pi\alpha/MR^2$ here, which adds 1 to the power of α in Eq. (25)]. The crossover temperature is $1/\beta^* \approx \omega_0$ where $\langle m^2 \rangle$ drops from the divergent $\sim \beta/B$ to a small $\sim 1/\alpha$ value at high temperatures.

We proceed now to multi-α_n problems. A generalized asymptotic expansion in the parameter $(\Sigma_n \alpha_n n^2)^2/\Sigma_n \alpha_n n^4$ is possible. Replacing $\alpha_n \rightarrow \gamma\alpha_n$ we obtain that Eq. (12) is equivalent to

$$\eta = 1 + \frac{1}{\gamma\pi^2 \left(\gamma\dfrac{d\eta}{d\gamma} + \eta \right) \dfrac{\left(\sum_n \alpha_n n^2 \right)^2}{\sum_n \alpha_n n^4} - 1}. \tag{26}$$

It is again remarkable that the perturbation expansion (22) allows an asymptotic expansion of Eq. (26) for large γ, i.e., from Eq. (13) $\gamma^2 d\eta/d\gamma \sim O(\gamma^0)$ produces higher order terms.

We show now that for the charge or dipole in a dirty metal environment, systems (ii) and (iii), the dependence on the radius is $B \sim R^2$ as for the free particle. For both cases and for $r \gg 1$ $\alpha_n = r^{-1}\alpha^*(n/r)$ with $\alpha^*(n/r)$ decaying to 0 after a large number of terms $n \approx r$. Hence the action in Eq. (2) has the form, for the $m=0$ winding number

$$S_{\text{int}}\{\theta_0\} = \sum_{n=1}^{n=r} (1/r)\alpha^*(n/r)\bar{S}\{n\theta_0(\tau)\} \rightarrow \int_0^1 dx \, \alpha^*(x)\bar{S}\{x\bar{\theta}_0(\tau)\}, \tag{27}$$

where $\theta(\tau)$ is rescaled, $\bar{\theta}_0(\tau) = r\theta_0(\tau)$. The action (including the free term S_1) is then r independent and therefore the effective mass for $[\bar{\theta}_0(\tau)]^2$ is r independent, which after unscaling yields $B \sim r^2$. We rely here on the variational scheme only to the extent that it shows that B_c can be deduced from the $m=0$ sector, i.e., $B=B_c$. For an actual solution for $f(\omega)$ we can imagine starting from a large γ and integrating Eq. (26) to an actual value of $\gamma \lesssim 1$. Since Eq. (26) is r independent for large r, the resulting η will also be r independent.

We note that for $r \lesssim 1$ the dipole problem reduces to that of the Caldeira Legget system (i) with $\alpha = \frac{9r^2}{4k_F^2 l^2}\left(\frac{p}{el}\right)^2$. Hence for large p, as for the giant Rydberg atoms,[7] one can be in the regime of large α showing the exponential dependence of Eq. (25). Upon further increase of r a crossover to the free particle form is predicted with $B \sim r^2$. We also note that the instanton crossover temperature $\beta^* = B/r^2$ is r independent. At temperatures above $1/\beta^*$ we have $\langle m^2 \rangle \sim e^{-\gamma' r}$ with $\gamma' = \frac{3\pi}{2k_F^2 l^2}$ for a charged particle, as in Ref. 20, while $\gamma' = \frac{3\pi}{k_F^2 l^2}\left(\frac{p}{el}\right)^2$ for a charge dipole.

In conclusion, we developed and solved a variational scheme for a large class of dissipative systems. The solution provides an RG of a high order, provided that a coefficient η is derived to high order in the perturbative regime. We also found an efficient asymptotic expansion for the relevant type of differential equations. We applied our method to the Caldeira-Legget system and found that its AB amplitude behaves as $\sim R^2 e^{-\pi^2 \gamma R^2}$. For a charged particle or a charged dipole in a dirty metal environment we find for large R an AB amplitude of $\sim R^{-2}$ as for free particles, in contrast with previous results.[8,20] Huge charge dipoles[7] in a dirty metal environment can show a variety of behaviors and provide therefore a valuable probe of dissipative environments.

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